

Novel stability approach using Routh-Hurwitz criterion for brain computer interface applications

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Abstract.

BACKGROUND: The stability criterion approach is very important for estimating precise behavior before or after fabricating brain computer interface system applications.

OBJECTIVE: A novel approach using the Routh-Hurwitz standard criterion method is proposed to easily determine and analyze the stability of brain computer interface system applications. Using this developed approach, we were able to easily test the stability of technical issue using simple programmed codes before or after brain computer interfaces fabrication applications.

METHODS: Using a MATLAB simulation program package, we are able to provide two different special case examples such as a first zero element and a row of zeros to verify the capability of our proposed Routh-Hurwitz method.

RESULTS: The MATLAB simulation program provided efficient Routh-Hurwitz standard criterion results by differentiating the highest coefficients of the s and a .

CONCLUSION: This technical paper explains how to use our proposed new Routh-Hurwitz standard condition to simply ascertain and determine the brain computer interface system stability without customized commercial simulation tools.

Keywords: Stability approach, Routh-Hurwitz, brain computer interface, system stability, standard criterion method

1. Introduction

The importance of stability in a brain computer interface systems or applications is unquestionable because the neuro signals generated by human brain cannot be detected by unstable brain computer interface systems [1]. As a result, the impulse response of the output systems is infinite, although the brain computer interface system input is quite limited [2]. For instance, unstable brain computer interface systems often result in a certain amount of physical impairment or damage, that becomes pricy [3]. To better understand these brain computer interface systems, it is essential and indispensable to understand why the stability issue of the brain computer interface system matters so much and what is physically implemented in a MATLAB program software (MathWork Inc., Natick, MA, USA).

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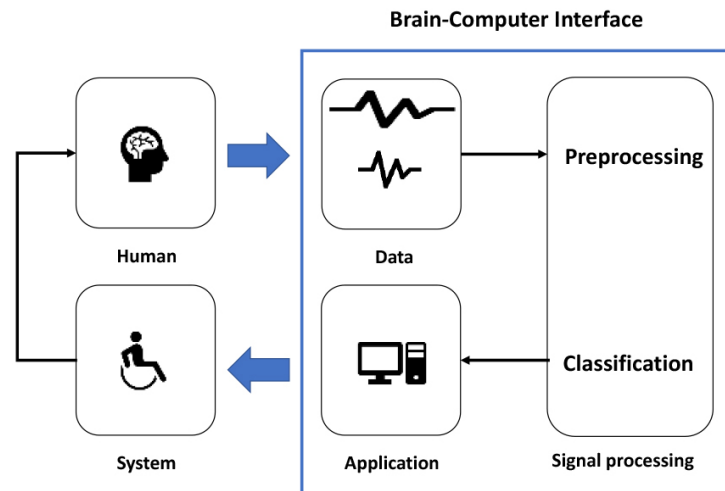


Fig. 1. A fundamental stable signal processing block diagram in brain computer interface systems.

We are also able to consider the stability issues for other medical imaging systems or instruments such as X-ray, computed tomography, magnetic resonance imaging, and positron emission tomography modalities because the medical imaging systems must provide stable output signals to be processed all the time [1]. To design such a circuit in the medical systems, several parameters, such as power dissipation, input and output impedance matching, speed, voltage swing, supply voltage, bandwidth, linearity, noise level, and voltage gains, were measured to show the stability of the developed analog circuit [4]. The high voltage class-F power amplifier was properly designed to have a wide bandwidth and high voltage gain with stable S-parameters of input and output reflection coefficients for portable acoustic system applications [5].

If any coefficients are zero values or below zero values, the designed engineering systems are in unbalanced conditions [6]. It is crucial to note, however, that even though all of the coefficients of the characteristic equation are positive, the proposed engineering system could be definitely in unstable conditions [7]. Figure 1 shows the fundamental stable signal processing diagram in the brain computer interfaces because of how stable systems in the brain computer interface applications are very important to process the obtained proper biomedical signals generated from the human brain [1]. As shown in the blue marked box, the brain computer interface system is a kind of black box so the system engineers or academic researchers typically do not know how to obtain and process the biomedical signals precisely without any simulation tool.

Section 1 describes the fundamental concept of the brain computer interface system or application. Section 2 describes how to implement Routh-Hurwitz codes in the simulation tool. Section 3 describes the implemented results of the Routh-Hurwitz program codes using the MATLAB simulation tool. Section 4 is the conclusion of the paper.

2. Methods

The Routh-Hurwitz standard criterion method is composed of two major phases or steps [8]: First, we need to build up a data table called a Routh table. Second, we are able to examine the designed table to determine the pole numbers of such closed-loop systems placed in the imaginary axis, left hand plane, or

the right half plane [8]. Let $G(s)$ be the open-loop gain and $H(s)$ be the feedback gain, then the transfer function to check the stability $T(s)$ becomes [8]:

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} \tag{1}$$

Equation (1) is simplified with a numerator equation $N(s)$ and a denominator equation $D(s)$:

$$T(s) = N(s)/D(s) \tag{2}$$

The Routh-Hurwitz standard condition focuses on the characteristic equation, which is the denominator polynomial $D(s)$ [8]. There are required conditions to be questionably stable for the given $D(s)$ [9]. From the $D(s)$ polynomial equation in Eq. (2), let n be the order of the polynomial i.e., the highest exponent power of s variable in the $D(s)$ polynomial equation $D(s)$. The polynomial equation $D(s)$ can then be shown in Eq. (3) [8].

$$D(s) = a_n s^n + a_{n-1} s^{n-1} + \dots + a_2 s^2 + a_1 s^1 + a_0 \tag{3}$$

There are two rules for the $D(s)$ polynomial equation to continue the analysis. If it satisfies both rules, then a Routh array equation is made with a_n coefficient of $D(s)$ polynomial equation. For each row, the last columns with zeros are shown.

$$\begin{matrix} s^n \\ s^{n-1} \end{matrix} \left| \begin{array}{ccc} a_n & a_{n-2} & 0 \\ a_{n-1} & a_{n-3} & \dots & 0 \end{array} \right|$$

Therefore, on the one hand, if the n variable is odd, the top row would consist of all odd coefficients. The rest of the array is filled:

$$\begin{matrix} s^n \\ s^{n-1} \\ s^{n-2} \\ \vdots \\ s^0 \end{matrix} \left| \begin{array}{ccc} a_n & a_{n-2} & 0 \\ a_{n-1} & a_{n-3} & 0 \\ b_{n-1} & b_{n-3} & \dots \\ c_{n-1} & c_{n-3} & \end{array} \right|$$

The empty field of the table indicates the ellipsis mark for repetitions. The number of elements in the table depends on the highest order of the brain computer interface differential equation. The order of the brain computer interface system is even so the end of the first row is non-zero. If the order of the brain computer interface system is odd, it is zero. Then, b , c , and other coefficients of the matrix are defined until row s^0 is reached. To fill them out, we used the following formula:

$$b_{n-1} = \frac{-1}{a_{n-1}} \left| \begin{array}{cc} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{array} \right| \quad b_{n-3} = \frac{-1}{a_{n-1}} \left| \begin{array}{cc} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{array} \right| \tag{4}$$

The leftmost element is called the pivot element in the row. Therefore, in the row b element, the element is a_{n-1} , and in the row c element, the element is b_{n-1} , etc., until it extends to the matrix or array bottom. There is another simpler example with a k , l , m , and n Routh array for determining the element of the current row of a and b :

$$\left| \begin{array}{cc} k & m \\ l & n \\ a & b \end{array} \right|$$

where k is the leftmost two-row elements above the current row (a), m is the two-row elements up above the current row and one column to the right element k , l is the obtained pivot element one row in front of

the current row, and n is the element one row up above the current row and one column to the right of the current element l . Then, coefficient a , which is assumed below l (same as the b_{n-1} position above), will become as follows:

$$a = \frac{(lm) - (kn)}{l} \tag{5}$$

3. Results and discussion

There are special or unusual cases related to the Routh-Hurwitz stability criteria [7], such as if the first term in any column row of the array is zero. In that case, we could avoid it by assuming an infinitely small value (ε) that tends to zero above zero during the calculation process [8]. By replacing zero with a symbol (ε), we could calculate all the Routh array elements, after which we must put on the limitation for each element with ε symbol.

In brain computer interface systems, there are two types of transfer functions. One is an open-loop transfer function $G(s)$ with feedback gain $H(s)$, and the other is a closed-loop transfer function $T(s)$ [10]. To verify the stability of the given brain computer interface system, we need the closed-loop transfer function but it is practically or physically impossible to directly obtain in real cases. Under this circumstance, we will assume the feedback gain is unity and then obtain the closed-loop transfer function as given by Eq. (1). The assumption is less or more tough but many brain computer interface systems suffice these limitations in real cases. For example, Eq. (6) shows a third-order linear differential equation derived from a standard closed-loop receiver system model integrated with several filter stages in brain computer interface systems for simple mathematical analysis. Therefore, we applied the proposed stability approach to check the stability in the brain computer interface system.

If all the elements of any row in the array are supposed to be zero cases of the row of zeros, the system can be said to have symptoms of marginal stability. If we do not have a sign change in the new table using the supplementary equation, then it will indicate the given Routh system is stable. Problems related to the Routh-Hurwitz standard criterion method for stability of the brain computer interface system are explained with special cases for example numbers 2 and 3.

1. We assume a specific application system of the brain computer interface system using characteristic equations:

$$D(s) = s^3 + 2s^2 + 4s + 3 \tag{6}$$

From Eq. (6), all the coefficients of the application systems are not zero, no sign changes and all of the coefficients in the systems are supposed to be positive. Thus, we then further proceed to build up the Routh array; in addition, we need to fill these wanted values.

$$\begin{array}{l} s^3 \left| \begin{array}{ccc|ccc} 1 & 4 & 0 & s^3 & 1 & 4 & 0 \\ s^2 & 2 & 3 & 0 & s^2 & 2 & 3 & 0 \\ s^1 & b_{n-1} & b_{n-3} & 0 & s^1 & 5/2 & 0 & 0 \\ s^0 & c_{n-1} & c_{n-3} & 0 & s^0 & 3 & 0 & 0 \end{array} \right. \end{array}$$

From this calculated array above, certainly, all of the signs of the first column in the brain computer interface systems are not supposed to be negative and there are no sign variances, and as a result, there are no poles of the brain computer interface systems.

2. In special cases, when the first element in any column or array row is supposed to be zero, we examine the stability check of the given transfer function as follows:

$$T(s) = 10/s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3 \tag{7}$$

From this problem, the characteristic equation is the denominator of $T(s)$, which is $s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3$, and it satisfies the rules. Therefore, we need to proceed with the Routh array to obtain the result for the first row as follows:

$$\begin{array}{l|lll} s^5 & 1 & 3 & 5 \\ s^4 & 2 & 6 & 3 \\ s^3 & 0 \rightarrow \varepsilon & 7/2 & 0 \\ s^2 & b_{n-1} & b_{n-3} & b_{n-5} \\ s^1 & c_{n-1} & c_{n-3} & c_{n-5} \\ s & d_{n-1} & d_{n-3} & d_{n-5} \end{array}$$

The sign changes occur in both instances, so it does not matter which one is chosen; both indicate that the brain computer interface system is not supposed to be stable with two poles in the right half plane. The resulting Routh array is as follows:

$$\begin{array}{l|lll} s^5 & & 1 & 3 & 5 \\ s^4 & & 2 & 6 & 3 \\ s^3 & & 0 \rightarrow \varepsilon & 7/2 & 0 \\ s^2 & & 6\varepsilon - 7/\varepsilon & 3 & 0 \\ s^1 & 42\varepsilon - 49 - 6\varepsilon^2/12\varepsilon - 14 & 0 & 0 & 0 \\ s & & 3 & 0 & 0 \end{array}$$

3. In special cases, when all elements of any row of the array are zero, i.e. row of zeros, we need to obtain the right half plane poles in the calculated function above.

$$T(s) = \frac{10}{s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56} \tag{8}$$

As the $T(s)$ satisfies the rules, we form an array and calculate the coefficients:

$$\begin{array}{l|lll} s^5 & 1 & 6 & 8 \\ s^4 & 7 & 42 & 56 \\ s^3 & 0 & 0 & 0 \\ s^2 & \vdots & \vdots & \vdots \end{array}$$

The row with all zeros or row of zeros in the Routh table is replaced with the coefficients obtained by differentiating the auxiliary equation to continue the calculation.

$$\begin{array}{l|lll} s^5 & & 1 & 6 & 8 \\ s^4 & & 7 \rightarrow 1 & 42 \rightarrow 6 & 56 \rightarrow 8 \\ s^3 & & 0 \rightarrow 4 \rightarrow 1 & 0 \rightarrow 12 \rightarrow 3 & 0 \rightarrow 0 \\ s^2 & & 3 & 8 & 0 \\ s^1 & & 1/3 & 0 & 0 \\ s & & 8 & 0 & 0 \end{array}$$

The solution of the linear differential equation or the poles of the transfer function in brain computer interface systems can be represented by the linear combination of the eigenvalues which are the poles or roots of the transfer function. Being stable means the poles or eigenvalues should reside in the left-half plane.

For higher-order linear brain computer interface systems, there are many poles in the transfer function [11]. By reducing dominant poles depending on customer requirement, we will obtain the lower order

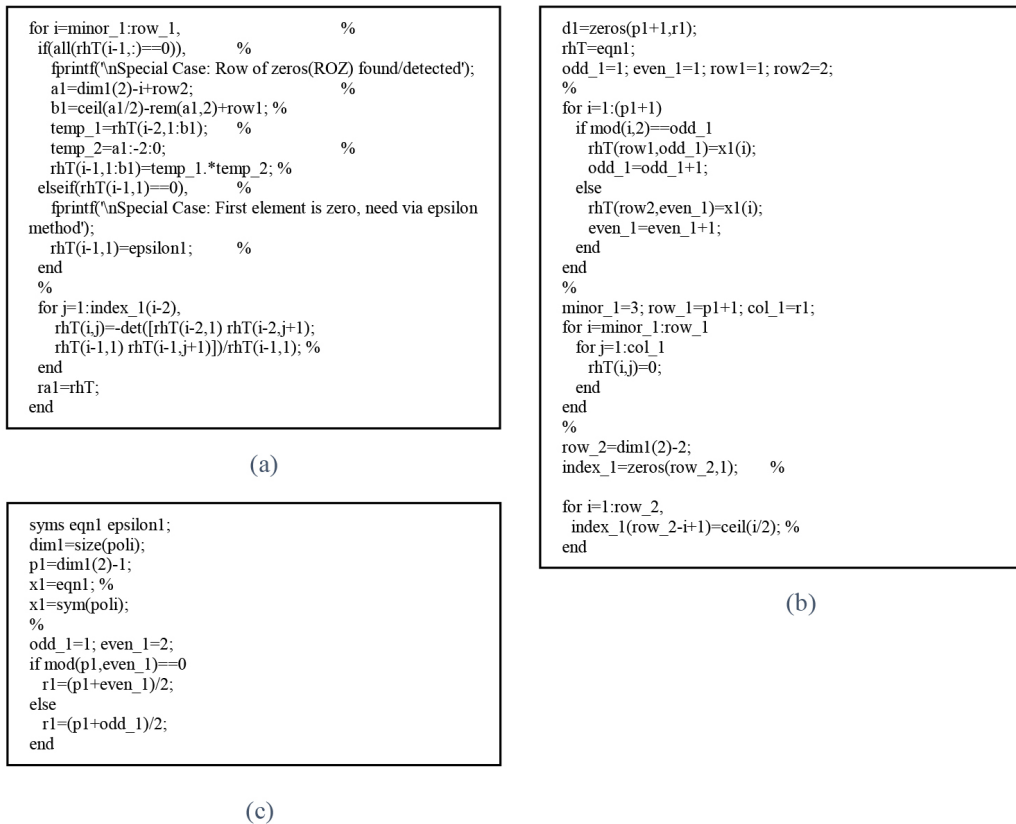


Fig. 2. (a) MATLAB program codes for giving input into simulation program, (b) step to calculate and give Routh array process, and (c) step to check sign changes for the result.

linear brain computer interface system and corresponding tangible transfer function which is applicable for stability analysis. Therefore, a simpler and standard transfer function could be derived.

The following figures are part of the MATLAB programmed code used for creating the Routh table and providing a stable conclusion by inputting the highest s rank in the characteristic equation and its coefficient value (a). The program then calculates and generates the Routh table using the same procedure as in the previous section. Based on the procedure, the program will comment on whether the system is stable by determining the sign changes. Figure 2a shows the code for input with the highest rank or power of s . Afterward, the MATLAB program calculates and gives the Routh array. The process of the program code is written in Fig. 2b, while Fig. 2c indicates that the sign changes for the conclusion. When the simulation is performed, it reads inputs from the user and gives the user the result, as shown in Fig. 3a and b.

As mentioned above in the results section, we showed the procedures and examples of how to make the Routh-Hurwitz theory using the MATLAB software package. After analyzing the mathematical theory, we could run the example to effectively verify the newly proposed Routh-Hurwitz standard condition for brain computer interface systems.

In around 10–20 brain computer interface component systems, the input stage is the electrode on the scalp and the output stage is the filter for electrical waveforms or signals [12,13]. The brain computer interface systems are a multidisciplinary field of technologies embracing biomedical, neuroscience, statistical signal processing, embedded systems, robotics, and artificial intelligence [14,15,16].

<pre>>> format rat; >> ra=routh_s4([1 2 3 5 2 3]) % simple Special Case: First element is zero, need via epsilon method ra = [[1, 3, 5] [2, 6, 3] [epsilon1, 7/2, 0] [(6*epsilon1 - 7)/epsilon1, 3, 0] [-(6*epsilon1^2 - 42*epsilon1 + 49)/(2*(6*epsilon1 - 7)), 0, 0] [3, 0, 0]</pre>	<pre>>> format rat; >> ra=routh_s4([1 1 6 6 25 25]) % ROZ Special Case: Row of zeros(ROZ) found/detected ra = [1, 6, 25] [1, 6, 25] [4, 12, 0] [3, 25, 0] [-64/3, 0, 0] [25, 0, 0]</pre>
(a)	(b)

Fig. 3. Simulation results: giving input into the MATLAB simulation program for (a) the first column zero, and (b) row of zeros.

Here, we discussed and compared the results with previous studies related to the stability issues. In the early stages of research stages, when designing a new brain computer interface system or feasibility studies of a highly new scheme, the stability of a tentative system component is evaluated by a commercially available software stability program or a software approach such as MATLAB, OrCAD Pspice (Cadence Design System Inc., San Jose, CA, USA), Multisim (National Instruments Inc., Austin, TX, USA) and PSCAD (Manitoba Hydro International Ltd., Manitoba, Canada) to check the stability of the overall brain computer interface system applications [17,18,19]. In consequence, the developed simulation tool will much more attractive framework to check the overall stability of new brain computer interface hardware and software [20,21,22]. The digital X-ray-based system for medical purposes is very important to obtain stable patients' images so the conventional Routh-Hurwitz standard criterion analysis provides stable information [23,24]. The nonlinear system was analyzed with the fundamental Routh-Hurwitz theory to obtain stable system parameters [25,26]. The current control system was used with basic Routh-Hurwitz standard criteria to determine the stability of the developed system [27,28]. The common source or common gate power amplifiers with resistive or capacitive feedback circuits need to be analyzed with basic Routh-Hurwitz theory in the circuit level analysis with the S-parameter analysis tool [29,30].

Compared with previous studies, our proposed novel Routh-Hurwitz standard criterion method could provide the stability issue with the eigenvalues of the characteristic equation in the transfer function. This equation could be implemented based on a simulation tool such as MATLAB program code, Simulink, or C programming code.

4. Conclusion

We propose a novel Routh-Hurwitz standard criterion method to provide stability information for brain computer interface systems or applications by observing the eigenvalues of the characteristic equation of a transfer function, which is the denominator polynomial of the function. For such brain computer interface systems or applications, precise and accurate voltage or current signal outputs are crucial because stability issues can result in physical injuries to humans or animals especially for brain computer interface systems or applications. Therefore, commercial simulation tools such as MATLAB or OrCAD Pspice programs utilizing simple and effective criteria to analyze and estimate stability issues need to be used in brain computer interface applications. However, such commercial simulation tools showed complex functions for newly designed medical systems such as digital X-ray or photoacoustic systems.

The characteristic equation or denominator polynomial must satisfy the Routh-Hurwitz polynomial to stabilize the brain computer interface system applications. There are two special cases in our proposed

Routh-Hurwitz standard criterion calculation. The first case is when the at least first term in any row of the column is supposed to be zero, and the second one is when all the elements of any row in the array are supposed to be zero. The simulation program could generate a new Routh-Hurwitz standard criterion output result by providing desired inputs of the highest power of s and a coefficients. Therefore, our proposed method could be used as an effective method for testing new brain computer interface system applications.

Conflict of interest

None to report.

Funding

This work was supported by the Korea Institute for Advancement of Technology (KIAT) grant funded by the Korean Government (MOTIE) (P0011930, The Establishment Project of Industry-University Fusion District). This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. 2020R1A2C4001606).

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