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Book Review

Abstract. "Numerical Methods for Special Functions" by Gil, Seguar and Temme is a collection of methods and approaches for finding the most efficient and accurate ways of computing values of functions that have no general definition.

"Numerical Methods for Special Functions" might appear at first glance to be a boring and perhaps dry subject for non-mathematicians, and indeed it is a little difficult to approach without being an expert in the area, but if you do find yourself in need of some hints for the best implementations available, this book might be a life saver.

Numerical Methods for Special Functions, by Amparo Gil, Javier Segura and Nico M. Temme, 430 pp., 2008, Cambridge Univ. Press, UK, ISBN 978-0-898-71634-4.

1. Outline of the book

The book is split into four parts, 'Basic Methods', 'Further Tools and Methods', 'Related Topics and Examples' and 'Software'. Within each part there are 1–5 chapters, for a total of 12 chapters.

The 'Basic Methods' part consists of five chapters, on *Convergent and Divergent Series*, *Chebyshev Expansions*, *Linear Recurrence Relations and Associated Continued Fractions* and *Quadrature Methods*. Each of these five chapters discusses related methods that can be used for finding the values of Special Functions. The emphasis in these chapters is on accuracy/validity of the approaches over certain intervals, and mention is made of what makes each algorithm valuable.

The 'Further Tools and Methods' part is very similar to the 'Basic Methods' part, however what distinguishes these chapters is that they do not describe a method for solving a specific Special Function, but rather, they give additional related information. For example, the main topics of the chapters are *Numerical Aspects of Continued Fractions*, *Computation of Zeros of Special Functions*, *Uniform Asymptotic Expansions* and a grab bag of *Other Methods*, including Pade Approximants, Sequence Transformations, Taylor Expansions for solving ODEs and other quadrature methods.

The third part of the book, 'Related Topics' deals with inversion of cumulative distribution functions and another grab bag chapter (Chapter 11) addresses the Euler Summation Formula, Approximations of Stirling numbers, Symmetric elliptic integrals and inversion of Laplace transforms.

The fourth and final part is 'Software', and contains six sections addressing the Fortran software supplied online.

2. Online material

The book's authors have constructed a website on Javier Segura's University web server to supply Fortran 90 implementations of the software introduced in the final chapter of the book (functions.unican.es). A blog is attached to the website and provides errata for the book.

The Fortran 90 programs are provided as text files, with some testing data and some compilation suggestions. I did not compile all the programs, however they seemed to be Fortran 77 compatible. Software for generating Airy and Scorer functions (based on a 2002 paper), toroidal harmonics, Modified Bessel Functions of purely imaginary order and parabolic cylinder functions are available. Four codes seem to require a subscription to *Computer Physics Communications* and *Elsevier* journal. I would suggest these be supplied as text files in the future.

3. Approach of the book

The book takes the approach of discussing each method of approximating a special function, rather than devoting consecutive space to special functions of a certain type. This definitely an economical decision, and keeps the volume slim (at just over 400 pages) however it does make the task of a user of the book who has a particular special function at hand and just wants to understand the limits of certain implementations that much harder – they unfortunately have to read large tracts of the book to ensure they understand the pitfalls of each application, and this may force them to go through a lot of unnecessary text.

The book is somewhat elusive on defining what a Special Function is, stating on p. 1 "A special function should be useful for applications and should satisfy certain special properties which allow analytical treatment". This book uses as an example throughout the book of the Airy function, but makes it clear that hypergeometric functions (Bessel functions, Legendre functions, Kummer functions) are also to be recognized as special functions. The approach of the authors is to discuss 'corner cases' that have not been fully explored in other texts, and groups the common methods of solving these corner cases together.

One review quoted on the back of the book says it will 'completely satisfy the needs of a scientist or engineer seeking to numerically compute special functions'. While I agree the book is very complete from a mathematical point of view, the scientist of engineer will have to bring along their own motivation for use of the Special Functions because there is only perfunctory 1–2 sentence mentions of the fields in which a special function might be used.

4. Readability

This book is written in excellent English, however right from the introduction, the book introduces recessive solutions to Airy functions and assumes the reader has encountered these before. Therefore, it is not a basic introductory book and is not suited for self study if the reader is not already familiar with numerical methods for special functions.

The book provides 246 references and uses a numbered referencing system and a single alphabetically sorted bibliography.

The book does not have a polemical approach – which in some ways is good since it presents all the techniques for solutions of special functions evenhandedly. However, it does also lack an overarching goal as a result – the reader does not feel like they are going on a journey of great significance because there is no building cadence to a great result at the end.

The book has excellent production values, nice quotes at the start of every chapter and is typeset extremely nicely, and has wistful little cartoons at the start of each chapter, and this successfully serves to draw the reader in. However, this may also serve as a false promise to the non-mathematically inclined and may prove frustrating to a reader looking for a self-study text with '*ab initio*' explanations of Special Functions. The introductory material is restricted to the first paragraph of each chapter, and is soon replaced by Definitions, Theorems, Corollaries and the like.

This approach is ideal for the mathematician who comes pre-motivated and just wants to get to the crux of the problem.

5. Suggestions for second edition

A welcome addition to the book would be a table indexing types of functions and the availability of a method to compute them in the general areas of power series, Chebyshev expansions, linear recurrence relation or quadrature methods. As it is, one has to read the entire chapter to find out the relevant details. The index is pretty standard and is designed to assist in navigating to where the text is relevant, however, it would be appreciated by the occasional reader who will only pick the volume up on an annual basis if functions were tabulated in a codex.

Online software should be made available without the requirement for a subscription to an *Elsevier* journal.

6. Who will this book appeal to and why?

Who will this book appeal to? This book would be expected to be most valuable as a recipe book for mathematicians or programmers who are writing a software library to compute values for special functions and are looking for well-thought-out explanations of the limits of certain implementations of numerical algorithms they intend to employ. Additionally, this book offers alternative approaches and treats them in sufficient depth that one would be able to choose an alternative for computing the result of a special function (choosing between power series, Chebyshev expansion, linear recurrence relations or quadrature methods).

7. Conclusions

The book takes the approach of being mathematically concise, and that is how the authors have managed to fit so much into the book's 417 pages. This inevitably means that if the reader is not used to reading proofs and theorems and definitions and if they are looking for physical or qualitative insight rather than mathematical insight, this may not be the book to reach for. Although the authors say some of the mater-

ial was drawn from their teaching classes, no problems are provided, perhaps acknowledging that the book is not suitable for uses in a undergraduate teaching environment. A small class of graduate students might tackle the material together.

If you are in the position of having to implement an algorithm or two in the future and think you may employ Special Functions, this book may be an occasional godsend.

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