## Introduction

## Special Issue on Stochastic Control

Stochastic control has been and remains a work horse of risk analysis and decision making under uncertainty. Contributions to stochastic control span a broad variety of fundamental mathematical results, techniques and numerous applications. The origins of SC are embedded in the fundamental discovery of dynamic programming, introduced by Bellman, followed by so many works, establishing the foundations of the theory and its many applications.

This Special Issue on Stochastic Control consists of 4 papers emphasizing stochastic games, risk aversion and specific jump stochastic control models with applications to important financial problems. The first paper by Andrew Ledvina and Ronnie Sircar is on "Dynamic Bertrand and Cournot competition", studying product differentiation in continuous time oligopolies (in which a small number of firms producing similar goods compete with one another by setting prices or quantities). This paper expands well-known Cournot and Bertrand models of trade by using first a deterministic version of the problem using an asymptotic expansion of the HJB partial differential equation. Their analysis indicates that for firms with highly differentiated goods, the type of competition matters little. For less differentiated goods, they find that the Cournot type market produces a greater value to firms than expected in a Bertrand type market. A stochastic version

for the two games are then considered using numerical techniques.

The second paper by Shanjian Tang and Wenning Wei on the "Representation of dynamic time-consistent convex risk measures with jumps" discusses under a general filtration of Brownian motion and Poisson random measure, the relation between dynamic convex and coherent risk measures and *g*-expectations. A representation of the generator of backward stochastic differential equations with jumps and a converse comparison theorem are proved. An integral representation for the minimal penalty term of a dynamic convex risk measure is discussed.

Hideo Nagai's paper on "Down-side risk minimization under prescribed consumption level" studies the portfolio choice of an investor in several types of securities under a prescribed consumption level. The investor is assumed to minimize the down-side risk probability for his (her) total wealth of falling in the long run below a given target growth rate.

Finally, Hidehiro Kaise and Jun Sekine's paper on "Optimal portfolio for a highly risk averse investor" considers a risk-sensitive portfolio optimization which is treated with a linear-Gaussian-factor differential game model. The main interest of this paper is how a highly risk-averse investor controls his/her interest rate risk