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Introduction

Computational theories and techniques in finance

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Futures and options have unleashed the value of future states into our present. They have augmented the supply and the liquidity of money, the availability of credit and have contributed immensely to financial exchanges and the development of financial markets. We owe therefore great thanks to the extraordinary contributions of economists such as Kenneth Arrow, Gerard Debreu, Lucas, Fisher Black, Myron Scholes and Robert Merton. The underlying state preference theories and their engineered pricing models have raised numerous challenges to computational theories and techniques seeking to bridge economic science with its practical use – even though its application might be less than perfect. While financial modeling requires structured stochastic models with specific mathematical properties, theoretically justifiable within the Arrow–Debreu state preference theory, its application to real problems can only be assessed ex-post – whether it works or it does not.

We are all aware that many underlying assumptions of financial models may be wrong and at times leading to contradictions to finance's fundamental theory. For example, questions arise regarding:

- The predictability of future states and "the normality of randomness".
- The inherent complexity arising from reconciling the future and the present merely based on what appears to happen at a particular instant. This includes the effects of a multiplicity of factors that are not easily accounted for in simple low-dimensional models.
- The simplifying behavioral rationality of markets based theories that leads to equilibrium concepts (fixed point economic equilibrium), etc.

While models are indeed only a partial definition of an unfolding and mostly unknown reality, seeking to bridge what we know with what we do not know, computational techniques are meant to provide as close an approximation as possible to unfolding future events and their prices and characterize the robustness (or fragility) of "framed" computational predictions. Computational technologies that recognize the lacunae of option pricing models are thus essential *to make best do with what we have*. Being for-warned of their implications is as important as their use. Their danger based on our "seeking comfort in numbers" as Paul Samuelson once said. The power of approximation and computational technologies are therefore only means to the ends of often ill defined models.

For example, if the unlikely is likely, as some Greek Philosophers claim, then option prices that imply (or are implied) by future states are anchored in a belief or sentiments we construct for our comfort. Their future usefulness is then no better than a random guess, resulting from aggregations of an infinite (or finite-agents) beliefs. Further, assuming that all possible future states can be enumerated – namely that financial systems are "extensive systems" (in a physical sense, when in fact they are non-extensive systems and thereby uncertain in the sense of Knight's definition of uncertainty), the power of computation of option prices ought to qualify their "robustness or the fragility" of such prices due to non-extensive financial systems. Techniques based on Malliavan calculus as well as robust computational techniques (based on systems sensitivities to presumed parameter) are thus also needed to complement the power of computational techniques.

If a financial system is extensive, namely, that all its future states are enumerable and known, then implied future distributions tend to be of the exponential (or normal) families type. In fact asset prices may indicate kurtosis, fat tails, long run and short run memory, dependence across many variables and other deviations from the fundamental pricing models. These require both theoretical constructs as well as approximate computational techniques that can account for such mathematical–statistical behaviors.

Computational techniques are intimately related to underlying theories and have an effect which cannot be neglected. Explicitly, a computational technology based on "fundamental and complete financial market models" can introduce (and probably introduces) an incompleteness by altering the underlying model for computational purposes. For example, discretizing the underlying Brownian motion in a Black–Scholes option pricing model by say a quadrinomial model induces incompleteness since prices will no longer be unique as would be the case if it were a binomial approximation. Now say that we use a binomial approximation that maintains the underlying process complete. Since there are other binomial approximations that will lead to other prices then to what extent is the approximation valid? In other words, the price is technique sensitive (which is in contradiction with the unique price assumption of fundamental models of financial markets).

Technically, computational power is efficient only within a true model. Since all market models are statistical, these models are in fact a hypothesis and not a certainty. If this is the case, common option prices do not treat such models as hypothesis but as mostly one or two-variables stochastic processes while everything we know about financial systems points to their uncertainty and to the many intervening variables in determining their prices.

Computational approaches are essentially anchored in past information. While prices are anchored on future states and the complex inter-temporal games that participants are engaged in exchanges that define option prices. Such games are notoriously difficult to analyze and their solutions are also defined by broadly varying rationalities. For example, is the Nash conjecture a proper rationality? Are all financial agents truly maximizing their well being without taking into account others well being? Simple assumptions prevalent in most models can lead to an equilibrium theory for prices – in fact it is a fixed point theory. What if equilibrium was dynamic? What would then be an appropriate model? Is this an economic rather than a computational problem?

Nonetheless, being warned is being forwarned. Banks routinely use complex and extensive models to price their assets, their portfolios and their options. Analytical results to these models are mostly impossible. And if they were, they would still be subject to models default and may still be constructed to lead to different results.

For example, when reducing an underlying financial stochastic model to one where prices can be calculated, computational techniques are still required to obtain specific solutions. A number of approaches may then be used:

- A probabilistic approach consisting in reducing the underlying process to one that meets the stringent requirements of a complete financial market probability model (namely, a Martingale and more precisely the Martingale which is consistent with market prices). At the same time, discrete state probabilistic models such as trinomial, quadrinomial and Markov chains may lead to prices that are not the same.
- Time and/or state discretization of underlying models (such as difference schemes) may also lead to broadly varying prices. For example, discretization of partial differential equations can be reached in a number of ways, be consistent and yet lead to differing prices.
- A reduction approach based on an iterative solution of a pricing model based on a specific model we know how to solve analytically is also used. In particular such an approach is used by Olivier Pironneau paper's in this issue. Perturbation techniques approaches applied in stochastic control may also be used to provide subsequent and converging computational techniques when calculating a price.
- In many cases, computational techniques falter due to their complexity in which case Monte Carlo simulation techniques are used. However, applications of such techniques are often misleading and inefficient. They usually neglect techniques and algorithms of stochastic approximations as well as other schemes. Nevertheless, when all else fail (analytical and numerical techniques), Monte Carlo simulation remains the essential technique we can use.

In most cases, there are available software which are adapted to the techniques one may use, most of which are non-transparent and therefore can be assessed only ex-post, once future prices unfold to confirm a given computational price.

This Special Issue consists of a number of important contributions of both theoretical and practical interest for computational finance.

Olivier Pironneau's paper, "Reduced basis for vanilla and basket options" outlines an approach and numerical–empirical results that provide a solution to potentially complex models of option prices based on the analytical solution of the Black–Scholes' option pricing model. This technique provide an alternative to the finite difference/element methods commonly used in banks by constructing an appropriate basis, smaller in size of the basis, and larger support. Instead of using an orthogonal decomposition, a set of rescaled calls with constant volatilities are used to form a better and more treatable Proper Orthogonal Decomposition (POD). The results are based however on certain properties of the solution and are shown to lead to a practically useful and computationally efficient method that can compute very fast option prices. Examples and numerical results are used to confirm the computational technique's efficiency.

Bally and De Marco's paper, "Some estimates in extended stochastic volatility models" demonstrates that for log-normal like stochastic volatility models with additional volatility functions, the tails (both the right and the left tails) of the cumulative distribution of log-returns is decreasing as an exponential function with a parameter which is a function of the underlying model. The technique used in this paper is to construct a solution about a deterministic model and therefore is similar in spirit to perturbation techniques. Subsequently, bounds for the cumulative distribution function and moments are calculated and their minimization leads to sharp bounds on estimates of option prices. The practicality of these results, assuming a given set of parameters provide then a fast computation of prices range. Further, the approach is used is shown to be implementable to non-fixed time marginals as well as to barrier options and exotic models.

Caramellino and Zanette's paper, "Monte Carlo methods for pricing and hedging American options in high dimension", addresses an important practical problem in Financial Engineering, the pricing of American options, for which no formula like Black–Scholes exists. The major difficulty comes from the curse of dimensionality. Analytically, the problem is to solve a parabolic variational inequality. Then, standard techniques like finite differences fail. This paper considers the possibilities offered by Malliavin calculus. Pure Malliavin calculus is not the best choice. The authors show that combining Malliavin calculus with other approaches like the Barraquand–Martineau algorithm, one can get satisfactory results for computing the price and the delta, with sufficient precision and affordable computational cost. A very detailed numerical analysis supports the analysis.

Ma's paper, "w-MPS risk aversion and continuous time MV analysis in presence of Levy jumps". In this article the Author solves a continuous time Markowitz problem with underlying stochastic processes being Levy processes. The efficiency is expressed in maintaining an expected growth rate throughout the horizon, while minimizing the portfolio risk at maturity. The Author shows that the Dynamic programming approach holds and closed form solutions can be obtained. Although some results are reminiscent of well known results for the static Markowitz problem, significant differences in the concept of efficient frontier as well as in the Mutual Fund theorem are mentioned. The concept of weak form mean-preserving-spread is also discussed in connection with the MV analysis.

Ndiaye's paper, "Non-Gaussian optimization mode for systematic portfolio allocation: How to take advantage of market turbulence", shows how to build a systematic quantitative portfolio allocation strategy using non-Gaussian risk metrics and market turbulence detection.

The algorithmic approach proposed in this article has been built step by step from practice. It will therefore be of particular interest to the Academic Community in order to develop its theoretical aspects.

The Author discusses ways to improve the reactivity of the estimates of the covariance matrix in a Markowitz portfolio optimization problem. Moreover, he shows how the risk can be taken into account better by combining variance and skewness in the objective. One of the key novelties lies in the use of non-convex optimization theory. In this context, the author illustrates the interest of this approach to the 2008 financial crisis.

Finally, the paper by Zengjing Chen and Agnes Sulem, is more of a theoretical nature. It provides an important insight in connecting various tools and concepts in probability and analysis. Also it connects theories which have developed at different times. The important Choquet capacity theory has been developed in the fifties in the context of potential theory. The concept of g-expectation has been introduced by Shi Ge Peng in the last decade. The authors study the relation between these two concepts, and deduce useful considerations for PDE and financial mathematics.