Population-adjusted national rankings in the Olympics

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Abstract. Ranking countries in the Olympic Games by medal counts clearly favors large-population countries over small ones, while ranking by medals-per-capita produces national rankings with very small population countries on top. We discuss why this happens, and propose a new national ranking system for the Olympics, also based upon medals won, which is *inclusive* in the sense that countries of widely-varying population can achieve high rankings. This *population-adjusted probability ranking* ranks countries by how much evidence they show for high capability at Olympic sports. In particular, it ranks countries according to how improbable their medal counts would be in an idealized reference model of the Games which posits that all medal-winning nations have equal propensity per capita for winning medals. The ranking index U is defined using a simple binomial sum. Here we explain the method, and we present population-adjusted national rankings for the last three summer Olympics (London 2012, Rio 2016 and Tokyo 2020, held in 2021). If the advantages of this ranking method come to be understood by sports media covering the Olympics and by the interested public, it could be widely reported alongside raw medal counts, thus adding excitement and interest to the Olympics.

Keywords: Olympics, ranking system, medal counts, medals per capita, national ranks

1. The need for inclusive Olympic national rankings

Since the Olympic Games began in 1896, national rankings have been widely reported by news media covering the Olympics. Participating countries are traditionally ranked by the number of gold medals that they have won (with silver counted only for ties) or alternatively, by the total number of medals: *gold + silver + bronze.* With the help of modern communications (television, websites, newspapers, streaming services, etc.) such rankings are followed by sports fans around the world. The top ten or twenty countries are typically reported in daily updates during the Games. We believe that this has a positive effect on the Olympics. All sporting events depend upon the spirit of friendly competition to generate interest, so it is natural to quantify relative national achievement. This has sociological benefits since it promotes the spirit of national pride (e.g. Hoffman et al. 2002; van Hilvoorde et al. 2010; Flegl & Andrade 2018; Humphreys et al. 2018) and it certainly helps to raise public interest and excitement about the Olympics.

But there is an obvious problem with raw medal-count rankings. Because larger-population countries draw from a larger pool of athletes and tend to have greater resources, ranking by medal counts systematically favors larger-population countries over smaller ones. Indeed, in all the past four Summer Olympics, the final rankings have placed the U.S.A. and China—two of the three most populous countries in the world—in the top two spots, in both gold-only and total-medal counts.

In order to correct for this bias, sports media sometimes show national rankings by *medals-per-capita,*

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Table 1
2020 Tokyo Olympics: National ranking by medals per capita

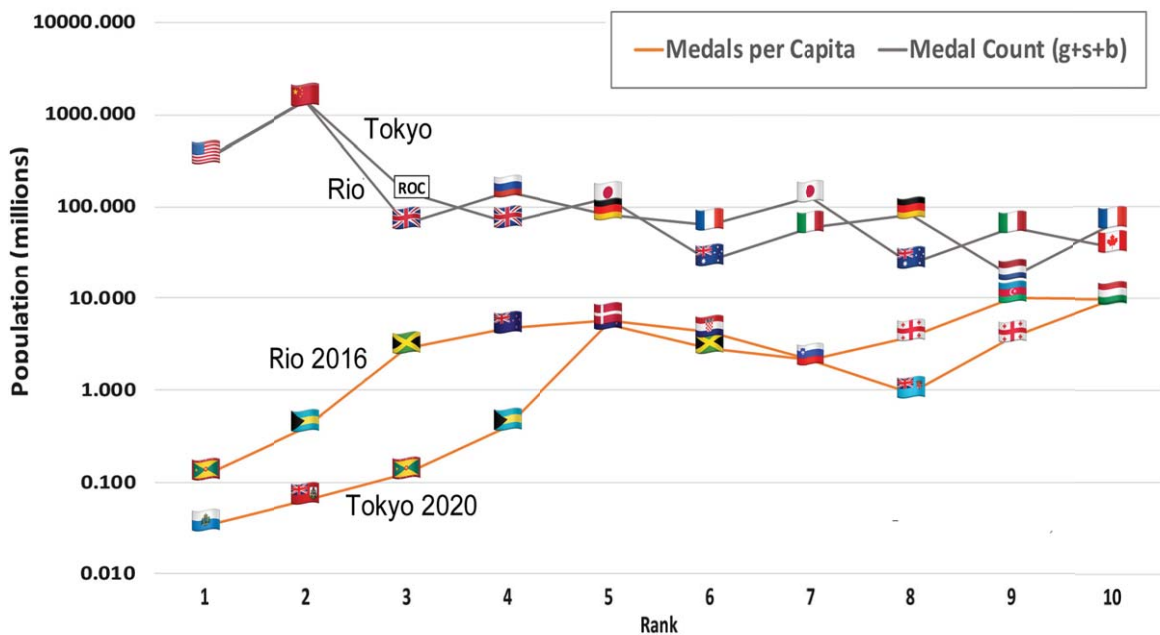| Rank	Country	Population (millions)	Medals Won	Medals/million
1	San Marino	0.034	3	88.90
2	Bermuda	0.064	1	15.58
3	Grenada	0.125	1	8.03
4	Bahamas	0.408	2	4.90
5	New Zealand	5.130	20	3.90
6	Jamaica	2.828	9	3.18
7	Slovenia	2.119	5	2.36
8	Fiji	0.925	2	2.16
9	Georgia	3.758	8	2.13
10	Hungary	9.709	20	2.06

Fig. 1. National population (plotted on a log scale) as a function of rank-number for the past two Summer Olympic Games, using the two accepted, popular ranking methods.

a ranking-index with no intrinsic bias. But such rankings are not often displayed or emphasized because they have an obvious disadvantage in practice, as demonstrated by the final results from the most recent Summer Olympics, shown in Table 1.

The top four spots in this list are taken by *the four smallest-population countries that won medals in the Tokyo Games,* ranked in order of population. Previous Summer Olympic Games have had similar results (see Fig. 1). This is an inevitable consequence of the peculiar data emerging from the Olympics, as we explain in Section 2 below. But the key point is simply that the two most widely-used methods

of ranking produce lists that are, in practice, predictably topped either by the largest-population or smallest-population medal-winning countries.

This is a significant problem in sports communications. It limits public interest in daily ranking updates during the Olympics, thus limiting the excitement and drama of the Games. It also limits public understanding and appreciation for the athletic talents emerging from many different regions worldwide, and for the hard work done by athletes in many countries to achieve sporting excellence. Thus we seek a more informative way to adjust medal counts for national populations, a method that can rank mid-range coun-

tries, with populations between 1 and 100 million, near the top along with countries with extreme (large or small) populations.

When seeking such a system, practical considerations are important. To be widely accepted and used, it is advantageous for the ranking system to satisfy—as nearly as possible—several constraints:

(1) *It should have a simple motivating idea and clear meaning.* Even people who may not completely understand the mathematical details of the ranking method should be able to understand why it is a logical way to rank.

(2) *It must be easily implemented.* For widespread use, the ranking index should be easy to calculate, preferably based on a plug-in formula—a simple function of medal counts and population—rather than a complex mathematical procedure.

(3) *It produces useful, accurate results throughout the Games, even before many medals are awarded.* Showing how national rankings develop and change during the Games can add excitement and interest, so this is important.

(4) *The method for calculating the ranking index must involve no undetermined (and hence potentially controversial) decisions or features, and no free parameters.* This is necessary because no ruling body will officially decide upon the precise method or parameters used for ranking.

You might think that the International Olympic Committee (I.O.C.) could authorize an official national ranking index. But this would be tantamount to specifying which country "won" the Olympics, which would be inconsistent with the Olympic Charter of 1908. To be specific, section 1.6 of the Charter says: *The Olympic Games are competitions between athletes in individual and team events, and not between countries.* (Olympics.com 2023). Thus no official ranking method can be authorized; but the I.O.C. has never discouraged the unofficial ranking of countries by the interested public. Indeed, the I.O.C.'s own online archive offers the option to rank countries in past Games by medal counts, either by gold-only or by total (g + s+b) medals (Olympics.com 2024).

The four conditions given above strongly constrain any ranking system. Nevertheless, we have found a system which satisfies these conditions, and produces interesting, population-inclusive results (as shown in Section 5 below). The key is to rank countries according to which ones show the strongest evidence for high capability at Olympic sports, per person.

Before we explain our method, we want to emphasize that *there exists no perfect and absolutely 'correct' way to do Olympic national ranking*. But the "probability ranking" that we favor has many advantages. Of course, raw medal counts are such an obvious way to measure national achievement that they will always be of interest too. Thus we propose that the probability rankings for the top ten or top twenty countries be posted during the Olympic Games alongside medal-count rankings. This would help to make the Games more exciting and interesting, while raising public understanding of athletic achievements worldwide. Medals-per-capita rankings could also be posted when time and space limitations allow, in order to highlight the achievements of the smallest-population medal-winning countries.

The plan of this paper is as follows. Section 2 discusses medals-per-capita rankings, explaining why—in real Olympic practice—this method invariably ranks the smallest-population medal-winning countries on top. Section 3 then explains probability ranking, and tells how to calculate the relevant ranking index. In Section 4 we consider the relative value of gold, silver and bronze medals, and discuss how this affects national ranking. This leads, in Section 5, to our presentation of results from the past three Summer Olympic Games. We also note some interesting trends in these national rankings. In Section 6 we discuss how economic factors and other considerations could be taken into account in Olympic national rankings. We ultimately conclude that these factors add complexities that make rankings based upon them impractical for wide use. In Section 7 we give our summary, and conclusions.

2. National ranking by medals-per-capita

To understand the practical limitations of medals-per-capita ranking in the Summer Olympics, we need to consider the range of national populations involved. In the 2020 Tokyo Games, the most populous medal-winning country was China, with a summed population $s = 1.444$ billion, and the least populous medal-winner was San Marino with $s = 34.0$ thousand. Thus national populations ranged over a factor $\Delta s = 42{,}500$. This means that if San Marino wins one medal, China would need 42,500 medals to surpass it, in medals-per-capita. Another key number

characterizing the Games is *M,* the maximum number of medals that any country could possibly win. This is the number of gold, silver and bronze medals that would be won if a country had athletes (or teams) that qualified in every event up to the Olympic quota for that event, and then all of these athletes/teams won medals. For the Tokyo games $M = 579$, although the total number of medals awarded was $T = 1080$.[1] Finally, we note that many small-population countries competed in the Tokyo Olympics. Indeed, 53 countries with less than 1 million people had won at least one medal in previous Summer Olympics or in Tokyo, and 23 additional sub-million-population countries had not.

Taken together, these facts explain why Olympic medals-per-capita rankings are led by the smallest medal-winning countries in the Games. This can be understood as a collective phenomenon. Since many small-population countries participate in the Olympics, the aggregate of very small nations constitutes a significant population, with 17.21 million people in medal-winning countries with $s < 1$ million. This sum is comparable to the population of Netherlands, and more than three times the population of New Zealand. Even if the people in these countries have only average sporting abilities—or below average abilities—within this significant aggregate population it is likely that a medal, or a few medals, will be won somewhere, and in countries where this happens, the *medals per capita* will be extremely high—higher than any large-population countries can possibly match because the number of medals that can be won is only $M = 0.0136 \, \Delta \, s$.

This does not imply that there is any *systematic* bias favoring small countries, since the great majority of very small countries finish at the bottom of the medals-per-capita rankings, with zero medals. Nor does this imply that medals-per-capita is a useless index. It has clear meaning which can be useful for comparing the performances of countries, especially when they have mid-range populations. All that we are saying is that the medals-per-capita *top ten list* is predictably led by all the smallest-population medal-winners, often with just one medal each, so this list is not optimal for giving the public interesting information about relative national performance in the highest levels of the Olympic competition.

Fig. 2. Kirani James of Grenada (right) finishes 0.02 seconds ahead of Michael Cherry (U.S.A.) and 0.12 seconds ahead of Michael Norman (U.S.A.) to win bronze in the men's 400 meter run. Meanwhile, Anthony Zambrano (Columbia) wins silver, and Steven Gardiner (Bahamas) wins gold—thus elevating the Bahamas to 4th place in *medals-per-capita*, although if Gardiner had run 0.36 seconds slower he would not have won a medal. (Photo by Bob Strong, UPI.)

Moreover, there is a sense in which the top medals-per-capita rankings are not very reliable. Consider, for example, the case of Grenada in the 2020 Tokyo Games. This small Caribbean island, with about one-fifth the population of Staten Island, NY, is ranked third in medals-per-capita because the Granada-born Kirani James —a former track star at the University of Alabama—won a bronze medal in the 400-meter run. James is clearly a great athlete, but if he had run *two one-hundredths of a second* (0.02 s) slower he would have missed his medal, and Grenada's medal count would have been zero (Fig. 2).

Given the razor-thin margins between finishers (0.02, 0.12, 0.36 s), if you ran the 2020 Tokyo Olympics men's 400 meter final race multiple times, leaving plenty of time for the runners to recover between races, the order of finishing would not uniformly hold true, as any 400 meter runner can attest. So in some "runnings of the Olympics," the rankings of Grenada and the Bahamas would drop dramatically, with Granada falling from third to last place, with zero medals-per-capita. Any ranking system subject to such profound variability is not highly reliable, which is another reason for seeking a more informative ranking system for public communications.

3. Probability ranking

3.1. Background discussion and motivation

To rank data with one independent variable according to the disparity of the dependent variable, a common approach is based upon regression analysis.

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[1] No single country can win all the medals awarded in an Olympic Games, because each country is only allowed one entrant in team sports (such as football/soccer, basketball, and rowing) and entrance quotas less than 3 also exist in many other sports (such the 2-entrant quota in gymnastics).

This involves finding patterns in the data and then identifying the most extreme outliers. In the case of Olympic national medal-wins N as a function of population s, this would involve fitting the data to the line $\tilde{N} = R\,s$, where R is the rate of wins per capita, as determined by the fit, and then ranking countries by their positive deviations from this fit. For this method to work, the data residuals must show a clear pattern of deviation, such as Gaussian distribution of known variance as a function of s. But the Olympic data is extremely heteroskedastic, and shows no predictable patterns of residuals as $\tilde{N} = R\,s$ varies over a factor of 42,500 within the data set, so reliable regression-based ranking is not possible.

Thus we must use a different approach. Instead of using patterns in the data, we propose to rank based upon an idea: an obvious idealization that everyone can understand. To be specific, we propose to rank by comparing real Olympic results to an idealized reference model in which the populations within all medal-winning countries worldwide have equal capacity and motivation, per person, for winning medals.

Note that the reference model does not idealize that *all people* have equal propensity for winning medals. In any country, only a tiny fraction ϵ of people have both the ability and motivation to become top Olympic athletes. We simply idealize that this fraction ϵ is constant throughout medal-winning countries worldwide, so that medal-winning achievement scales in proportion to national population.

To do the ranking, we compare the real medal-winnings of participant countries to this idealized model, with medal-winning idealized as a stochastic process. The way to do this is simple and uncontroversial, so it needs no official authorization. We simply ask: how probable is it, in this idealization, for a given high-performing country (with a given population) to have won as many medals as it actually did, or more medals? The lower this probability, i.e. the more improbable the number of medals won, the more the revealed evidence for higher-than-average capability. This improbability can thus be used as the national ranking index, with smaller probabilities ranked higher.

Note that our reference model is highly idealized, in the sense that the real Olympic results do not conform to this model very well. But our ranking method does not depend upon accurately fitting the data; instead it is a based upon a conceptually-obvious idealization. As long as we compare national medal-wins to this idealization uniformly when calculating

the ranking index for all participating countries, the ranking procedure is mathematically valid, and its fairness is shown in Section 3.4. We shall also see that probability ranking yields plausible, interesting, population-inclusive results.

3.2. The equal-capability reference model

We now make these ideas mathematically precise. In our idealized reference model, we will treat Olympic medal-winning as a stochastic process, with each country having a probability p_i of winning each medal (with the subscript i identifying the country), as it competes for every medal in the Games, limited only by entry quotas.

Of course, in the real Olympics, only top athletes from a subset of countries actually compete in each event, because quotas are not the main restriction on entry. Only a limited group of athletes meet Olympic qualifying standards, and even fewer are granted wild-card entry, although many more athletes try to qualify worldwide. This is consistent with our idealized reference model if the lead-up to the Games, including all national Olympic Trials, are considered as part of the competition. Thus every country is assumed to initially have a chance at every medal that it is possible for it to win (given entry quotas), so the baseline probability p_i is nonzero for every medal; but as the Games come together, even before the first Olympic event begins, some countries fail to find and develop qualified entrants in some sports, effectively losing their chance to win some medals.

Thus, since each country gets M chances to win an Olympic medal (taking into account Olympic quotas; see footnote 1), and since each chance has probability p_i for success, the probability that a country i will win exactly N medals is given by the binomial formula (see Appendix A for a review):

$$P_i\,(N,\ p_i) = \frac{M!}{N!\,(M-N)!}\ p_i^N\ (1-p_i)^{M-N} \quad (1)$$

We emphasize that this model probability is an idealization used as a theoretical tool for ranking only. We never use it to predict medal-winning results; we only use it to find national ranking indices.

Now, if all people in medal-winning countries worldwide have equal medal-winning capacity in all events, then the number of medals that each country is expected to win, $\langle N \rangle_i$ is simply equal to the fraction of the whole international population that resides within that country, times the total number of medals awarded in the Games:

$$\langle N \rangle_i = (s_i/S)T, \tag{2}$$

where s_i is the population of country i, S is the summed population of all competitive countries (which have won at least one medal in this or previous summer Games) and T is the total number of medals awarded in the current Games. *Equation (2) is the basic defining condition for our equal-capacity, idealized reference model, along with the binomial probability Equation (1).* To relate this to p_i, note that a well-known consequence of the binomial distribution is [cf. Appendix A, Equation (17)]

$$\langle N_i \rangle = \sum_{N=0}^{M} N P_i (N, p_i) = p_i M. \tag{3}$$

Thus from Equations (2) and (3),

$$p_i = \frac{T}{M} \frac{s_i}{S} \tag{4}$$

in our idealized model. This probability is easily understood. When the number of medals that any given country is able to win is only a fraction (M/T) of the total, then on average the number of people in countries competing for *any given medal*, S_m, is also a fraction (M/T) of the total: $S_m = (M/T)S$. Assuming that people have equal capacity for winning worldwide, the national probability is $p_i = s_i/S_m$, which yields Equation (4).

To make these ideas clear, let's apply them to the 2020 Tokyo Olympics, in which $T = 1080$ medals were awarded, but only $M = 579$ medals could be won by any single country due to quotas. The total population of all countries that had won medals in a summer Olympics (current or previous) was, at that time, $S = 7.229$ billion. Thus the U.S.A., with a population of $s = 337.0$ million, was expected to win $\langle N \rangle = 50.35$ medals, assuming equal medal-winning capability worldwide. Instead it won 113, more than twice the expected value—a highly improbable result by chance: $P = 3.0 \times 10^{-16}$ according to Equations (1) and (4). This means that the U.S.A. should place high in probability rankings for the Tokyo Olympics.

3.3. The probability ranking index U

So how, exactly, do we do the ranking? Among high-performing countries, all with medal-tallies satisfying $N_i > \langle N \rangle_i = (s_i/S) T$ (i.e., the number of medals won exceeds the number expected for uniform worldwide capability per person) smaller values of $P_i(N_i, p_i)$, as calculated from Equation (1), mean that

the national medal-count is *more improbably high,* indicating superior performance. However, among countries which have $N_i < \langle N \rangle_i$, a smaller P_i implies that the medal-count is more improbably low. Hence, for consistent ranking, it is useful to define a cumulative "ranking probability," P_r which is the probability for a country to win as many medals as it actually did, or more medals:

$$P_r (N) = \sum_{n=N}^{M} P (n, p). \tag{5}$$

(Here, and in what follows, we omit the i subscript on all variables for simplicity.) This has the advantage that countries with smaller P_r can be ranked higher (at lower rank-numbers) regardless of whether $N > \langle N \rangle$ or $N < \langle N \rangle$. By the Binomial Theorem (Equation [16]), this is equivalent to:

$$P_r (N) = 1 - \sum_{n=0}^{N-1} P (n, p) \tag{6}$$

which is often more convenient for calculation.

For very highly-performing countries, the ranking probabilities P_r can be extremely small numbers. For example, the U.S.A. with $N = 113$ has $P_r = 4.89 \times 10^{-16}$. Such tiny numbers are inconvenient to work with, so it is useful to define the "Probability Ranking Index" U as the minus logarithm (base 10) of the ranking probability:

$$U = -\log P_r(N). \tag{7}$$

This means that as P_r gets smaller, U gets larger, i.e., if $P_r = 0.1, 0.01$ or 0.001, then $U = -\log P_r = 1, 2$ or 3. Thus we rank countries by the value of U, with *larger positive values ranked higher on the national list* (i.e., at lower rank-numbers). This preserves the rank-order of countries found from directly ranking by P_r (with smaller- P_r ranked higher), but using a more intuitive, positive ranking index.

We can express our proposed ranking index concisely in the formula:

$$U (s, N) =$$
$$-\log_{10} \sum_{n=N}^{M} \frac{M!}{n! (M - n)!} p^n (1 - p)^{M-n} , \tag{8}$$

with $p = (T/M) \cdot (s/S)$. This is the formula used for computing U values in "National Rankings" tables for the Olympic Games (as given in Section 5, below). For the U.S.A. in the 2020 Tokyo Games, $U = 15.31$.

Note that Olympic entry quotas do not significantly affect the rank-ordering of countries in practice. This makes sense, because no countries actually come close to winning either M or T medals. Thus a simpler, approximate version of probability ranking, which works very well, adopts the idealization that every country has a chance to win every medal in the Games (i.e., the number of binomial trials is T), with probability for winning $p_i = (s_i/S)$, which makes intuitive sense. The resulting ranking index is

$$U_T\,(\text{s},\,N) =$$
$$-\log_{10} \sum_{n=N}^{T} \frac{T!}{n!\,(T-n)!}\,(s/S)^n\,(1-s/S)^{T-n}\,. \quad (9)$$

This can be used instead of Equation (8) when calculational simplicity is needed. The binomial distributions in the two ranking methods have exactly the same $\langle N \rangle$, and nearly the same variance $\sigma_i^2 = \langle N \rangle (1-p_i)$, since $p_i \ll 1$ for all countries. We have verified that Equations (8) and (9) produce exactly the same rank-order for the top 30 countries in the Tokyo Olympics.

The formulas of Equations (8) and (9) may look daunting, but binomial sums are easily done by computer. Online high-precision calculators can do the sum reliably for any given values of M, T, N, and p (Burton 2022). In *Microsoft Excel*, the popular spreadsheet calculator, such sums can be done with the BINOM.DIST.RANGE function. Since the number of binomial trials is large ($M \gg 1$ and $T \gg 1$), for all but the smallest-population countries, the ranking probability P_r can also be well-approximated by an integral over a Gaussian (normal) distribution.

Probability ranking could have other applications. To use it for the Winter Olympics requires just one simple update: the summed population S of all medal-winning countries is much smaller in Winter than in Summer Games. Probability ranking is only reliable if S is accurate, and because countries from tropical zones usually do not compete in sports like skiing and ice skating, their populations should not be included. The natural criterion for inclusion in S is that a country has won at least one medal in the Winter Games, either in previous years or in the present Olympics—the same criterion which is used, independently, for the Summer Games. Probability ranking could also add interest and excitement to many other international competitions with multiple events, such as the Paralympic Games, the World Athletics Championships, and the World Aquatics Championships.

3.4. A test for statistical fairness

To verify that U-ranking is fair, we need to make sure that when countries win their expected number of medals $N = \langle N \rangle$ they will all be ranked essentially the same, regardless of population. So consider a set of imaginary countries with populations that are integral multiples of (S/T), so that their expected numbers of medals $\langle N \rangle$, in the idealization of uniform capacity per person worldwide, are integers. Now imagine entering these countries in the 2020 Tokyo Olympics, for which (S/T)= 6.69 million. Then as $\langle N \rangle$ goes from 1 to 210, s goes from 6.69 million to 1,405 million, from a little more populous than Denmark to a little less than India or China. Now we ask, if each of these countries wins precisely $N = \langle N \rangle$ medals, what will be their ranking index U? The answer is that it is remarkably uniform, varying only from 0.20 to 0.29 over the whole population range. This difference is tiny compared to the range of values of U in the real Olympics (33.0 to 0), giving evidence that probability ranking is not significantly biased toward either large or small-population countries.

4. The relative value of gold, silver, and bronze

The probability ranking method described above is based upon *total* medal counts (*gold + silver + bronze*), so it does not take into account the higher value of gold medals relative to silver, or silver relative to bronze. However, if the relative weighting-factors for different medal-colors (or medal-metals) can be agreed upon, the probability ranking index U can be updated to take these weights into account. In this section, we will show that as long as medal-color weighting factors are neither generally agreed upon, nor officially set by the I.O.C., the most reasonable and accurate way to do probability ranking is to base the U-index calculations upon total medal counts.

We begin by summarizing how medal-color weighting has traditionally been done in the Olympics. For more than a century, news and sports information sources worldwide have ranked countries by raw medal counts. Some do the ranking by the number of gold medals alone, counting silver only when there are gold ties, and bronze only for gold-and-silver ties, a procedure known as *lexicographic*

ranking. Others rank by the total medal count, adding the numbers of gold, silver and bronze medals to find the ranking index.

Alternatively, some information sources assign specific color weight-factors. This is equivalent to assigning differing numbers of 'points' to gold, silver, and bronze wins, and ranking by point totals. For example, the official report of the 1908 London Games used (*gold, silver, bronze*) weights of (5, 3, 1) (Cook 1908). Other commonly-used schemes include (3, 2, 1) and (4, 2, 1) (Wood 2010).

Sitarz (2013) argues for weight factors (6.3, 2.4, 1) based upon the "incenter" criterion: the condition that the medal-weight factors lie in the geometric center of the space of all mathematical possibilities (with weight-factor triads treated as Cartesian coordinates), subject to the constraints that gold medals must be weighed greater than silver by a factor exceeding the factor by which silver is weighed above bronze. Why this incenter criterion is the best choice for medal-weights is not clear to us, but it is a plausible choice.

Lexicographic ranking has been used in much of Europe and Asia during recent Olympics. Could lexicographic weights also be used in probability ranking? Since the formula for U (Equation 5) never produces rank-index ties, this would simply be using probability ranking based on gold medals alone, rather than total medals. The gold-only probability index U_G can be found from Equations (8) or (9) by using each country's gold medal count N_G as the lower-limit of the binomial sum, plus appropriate numbers of gold medals (e.g., $M_G = T_G = 340$ for Tokyo 2020). However, we do not favor doing this, because it gives zero weight to both silver and bronze medals, so it fails to use a lot of information relevant to national sporting achievement, thus significantly reducing the accuracy and inclusiveness of the ranking.

This information-based argument was quantified by Ogwang & Cho (2021), who found an "objective weighting scheme" for the Olympics. They began by calculating the correlation matrix for gold, silver and bronze national winnings for all countries in all Winter Olympics throughout history, finding (G-S, G-B, S-B) correlations of (0.931, 0.859, 0.948). This shows that different-color winnings are highly correlated, so different-color medal counts seem to be measuring the same property of countries (i.e. intrinsic sporting capability). For the last three summer Olympic Games we found (G-S, G-B, S-B) correlations of (0.9100, 0.8713, 0.8823), corroborating this result.

To find optimal, objective weighting factors, Ogwang & Cho used a well-known tool from multivariate statistics known as Principal Component Analysis (Dunteman 1989), which can reduce a set of correlated variables to a reduced number of principal components (PCs) — linear combinations accounting for much of the variation in the original variables. In this case, the color-color correlations were so strong that a single PC accounted for 94% of the variations in the historical Winter Olympics data and has nearly equal weights for G/S/B:

$$\text{First PC} = 0.3291N_G + 0.3398N_S + 0.3311N_B. \tag{10}$$

Since all the coefficients here are very close to 1/3, Ogwang & Cho concluded that equal weights should be assigned to all three medal colors, in order to reflect optimal ranking by the revealed information from medal-winning. We corroborated this with a PCA analysis of the London, Rio and Tokyo Games taken together, finding that the first principal component accounts for 92.5% of the variation:

$$\text{First PC} = 0.3342N_G + 0.3353N_S + 0.3305N_B. \tag{11}$$

This conclusion is consistent with the fact that the average performance variations between first, second, and third position in elite Olympic competitions are fractionally very small, suggesting that highly discrepant color-weightings are not justified as measures of capability (Bian 2005).

However it is important to note that different national ranking methods may be needed to realize different objectives. Ranking countries by the status or prestige gained from Olympic wins (however that quantity may be defined) would depend highly upon the relative prestige of gold, silver and bronze medals—which are socially-determined quantities (or, in the present-day situation, socially undetermined). If, on the other hand, you seek to rank countries by the degree of sporting capability that is revealed by their medal-winning, adjusted for population, then it is important to use the information from the different medal colors optimally in order to produce the most accurate ranking.

5. Olympic ranking results

To show the utility of probability ranking, we now give population-adjusted national rankings for the three most recent summer Olympic Games. All the

Table 2
2020 Tokyo Olympics: National ranking by probability index

Rank	Country	Population (millions)	Expected Medals	Medals Won	U Index
1	Australia	25.92	3.87	46	33.008
2	Great Britain	67.28	10.05	64	30.381
3	Netherlands	17.50	2.61	36	28.063
4	New Zealand	5.13	0.77	20	21.147
5	Hungary	9.71	1.45	20	15.878
6	USA	337.00	50.35	113	15.310
7	Italy	59.24	8.85	40	14.139
8	Japan	124.61	18.62	58	13.244
9	Cuba	11.26	1.68	15	9.475
10	Jamaica	2.83	0.42	9	9.117
11	Switzerland	8.69	1.30	13	8.889
12	France	64.53	9.64	33	8.712
13	Denmark	5.85	0.87	11	8.623
14	Canada	38.16	5.70	24	8.138
15	Germany	83.41	12.46	37	8.073
16	San Marino	0.03	0.01	3	7.674
17	Georgia	3.76	0.56	8	6.845
18	Croatia	4.06	0.61	8	6.594
19	Czechia	10.51	1.57	11	6.098
20	Norway	5.40	0.81	8	5.677
21	Serbia	7.30	1.09	9	5.667
22	Slovenia	2.12	0.32	5	4.697
23	Sweden	10.47	1.56	9	4.437
24	Ukraine	43.53	6.50	19	4.350
25	Republic of Korea	51.83	7.74	20	3.832

data used for these rankings can be found in online archives of national medal counts (Olympics.com 2024) and of national populations (United Nations 2022).

5.1. The 2020 Tokyo Olympics

The 2020 Tokyo Games were held in the summer of 2021 because of the COVID-19 pandemic. During 16 days of competition, 92 different countries won medals. The total population of all medal-winning countries (i.e. all countries that won medals in Tokyo or in any previous summer Olympics) was $S = 7.229$ billion, and the maximum number of medals that any country could have won was $M = 579$ out of the total $T = 1080$ that were awarded. (Here team-sports wins are counted as single medals, even though all team members were honored.) Table 2 lists the top 25 countries as ranked by the index U, where the ranking probability is $P_r(N) = 10^{-U}$.

Note that within the top ten, national populations ranged over a factor of 119, from 337 million to 2.83 million, USA to Jamaica. Moreover, within the top 20, the populations ranged over a factor of *ten-thousand* (USA to San Marino, which was ranked 16th) so this ranking is verified as population-inclusive. The top twenty also spanned

a large geographical range, with Europe (Great Britain, Netherlands, Italy, Hungary, France, etc.), the 'down-under' Pacific (Australia, New Zealand), the Caribbean (Jamaica, Cuba), East Asia (Japan) and North America (USA, Canada) all represented.

Where is Russia in the list? Numerous athletes of the Russian Olympic Committee (ROC) were allowed to compete in Tokyo, but the International Olympic Committee made it clear that, due to Russia's recent history of state-sponsored doping, the ROC athletes did not represent Russia. Thus using a national ranking statistic based upon Russia's population s is not appropriate, and the ROC cannot be ranked. However, if the ROC could be identified with Russia's national population, they would have done well. They won 71 medals, with an expected number of 21.68, giving $U = 17.296$ which would have been good for fifth place.

5.2. The 2016 Rio de Janeiro Olympics

In the 2016 Rio Games, athletes from 85 countries won medals, with 2 more medals won by Independent Olympic Athletes (IOA). The summed population of all Summer Olympics medal-winning countries at the end of the Rio Games was 6.872 billion. The total number of medals (gold + silver + bronze) that any

Table 3
2016 Rio de Janeiro Olympics: National ranking by probability index

Rank	Country	Population (millions)	Expected Medals	Medals Won	U Index
1	Great Britain	65.66	9.29	67	35.135
2	USA	327.21	46.28	121	21.972
3	New Zealand	4.67	0.66	18	19.443
4	Australia	24.20	3.42	29	17.155
5	France	63.99	9.05	42	15.256
6	Denmark	5.71	0.81	15	13.920
7	Azerbaijan	9.98	1.41	18	13.804
8	Jamaica	2.80	0.40	11	12.222
9	Germany	82.33	11.64	42	11.684
10	Netherlands	17.12	2.42	19	10.892
11	Russia	145.11	20.52	56	10.639
12	Hungary	9.82	1.39	15	10.616
13	Croatia	4.22	0.60	10	9.066
14	Kazakhstan	18.08	2.56	17	8.747
15	Canada	36.11	5.11	22	7.696
16	Italy	60.12	8.50	28	7.159
17	Sweden	9.95	1.41	11	6.560
18	Cuba	11.34	1.60	11	6.012
19	Japan	126.99	17.96	41	5.889
20	Georgia	3.77	0.53	7	5.830
21	Czechia	10.53	1.49	10	5.443
22	Belarus	9.71	1.37	9	4.875
23	Serbia	7.49	1.06	8	4.828
24	Republic of Korea	51.31	7.26	21	4.692
25	Switzerland	8.37	1.18	7	3.647

single country could possibly have won was $M = 521$ out of $T = 972$. Table 3 gives the top 25 countries in the final ranking. Note that national populations within the top three ranged over a factor of 70 (USA to New Zealand).

5.3. The 2012 London Olympics

In the 2012 London Games, 86 different countries won medals. The final summed population of all prior Summer Olympics medal-winning countries was $S = 6.562$ billion, with $M = 513$ and $T = 961$. Table 4 gives the final London top 25.

5.4. Trends in Olympic national rankings

Table 5 compares probability rankings for the past three summer Olympic Games. It is significant that many countries achieve similar rank-numbers in the three games, often showing gradual evolution. This is evidence for the reliability of the ranking system. In particular, Great Britain was clearly an outstanding performer, ranked [1, 1, 2] in the [London, Rio, Tokyo] Olympic Games. The 'down under' Pacific countries also did extremely well, with Australia ranked [2, 4, 1], and New Zealand [7, 3, 4]. Other top performers in the past three Olympics were the

U.S.A. [5, 2, 6], Hungary [6, 12, 5] and Jamaica, which showed a downward trend [4, 8, 10] due in part to the retirement of the 8-time gold medalist sprinter Usain Bolt. Other countries showing modest downward trends were Germany [8, 9. 15], Belarus [16, 22, 29] and the Republic of (South) Korea [11, 24, 25]. On the other hand, significant improvement over the three Games was attained by the Netherlands [9, 10, 3], Canada [17, 15, 14], Switzerland [39, 25, 11] and Japan [21, 19, 8]. Indeed, Japan may have benefited from the "home advantage" of hosting the Tokyo Games.

5.5. The Medal-winning Population and the 2024 Paris Games

We now discuss how probability ranking is done *during* the Olympic Games, which may be useful for future applications. Consider the Tokyo Olympics. On July 23, 2021, the first official day of the Games, the total population of all medal-winning countries was $S = 7.200$ billion. This did not include 63 countries that had failed to win any medals in 31 previous summer Olympic Games. Such countries are very unlikely to win medals, thus they do not significantly affect other country's medal-winning prospects (as measured by p_i), so their populations are not included

Table 4
2012 London Olympics: National ranking by probability index

Rank	Country	Population (millions)	Expected Medals	Medals Won	U Index
1	Great Britain	63.81	9.34	65	33.189
2	Australia	22.73	3.33	35	23.563
3	Russia	143.63	21.03	66	15.305
4	Jamaica	2.76	0.40	13	15.135
5	USA	316.65	46.37	104	14.216
6	Hungary	9.92	1.45	18	13.593
7	New Zealand	4.41	0.65	13	12.583
8	Germany	81.55	11.94	44	12.533
9	Netherlands	16.79	2.46	20	11.714
10	France	63.07	9.24	35	10.389
11	Republic of Korea	49.63	7.27	30	9.840
12	Cuba	11.31	1.66	15	9.573
13	Italy	60.19	8.81	28	6.848
14	Denmark	5.60	0.82	9	6.679
15	Czechia	10.51	1.54	11	6.185
16	Belarus	9.69	1.42	10	5.624
17	Canada	34.69	5.08	18	5.242
18	Azerbaijan	9.49	1.39	9	4.836
19	Georgia	3.80	0.56	6	4.598
20	Spain	46.76	6.85	20	4.554
21	Japan	127.85	18.72	38	4.387
22	Croatia	4.33	0.63	6	4.288
23	Ukraine	45.41	6.65	19	4.232
24	Kazakhstan	17.10	2.50	11	4.231
25	Mongolia	2.79	0.41	5	4.175

in S. However, during the Tokyo Games, three of these countries (Burkina Faso, San Marino and Turkmenistan) won their first-ever medals, so S was immediately updated. The final value of S was thus slightly increased by 0.0285 billion (or 0.395%) to $S = 7.229$ billion. (See Table 6 for a full list of countries whose populations were *not* included in calculating S.) Note that the R.O.C. definitely reduced the medal-winning probability p_i for other countries by taking 71 medals in Tokyo, so the population of Russia was included in S, even though Russia itself was not eligible for U-ranking due to its national disqualification.

There will be 329 medal events in Paris, with $T = 1041$ total medals awarded. (Note that two bronze medals are awarded in all wrestling, boxing, judo and taekwondo events, with 30 men's events and 23 women's events and one mixed event, yielding a Games total of 383 bronze medals.) The summer Olympics in Paris 2024 will be the first summer Games in which the number of events and medals awarded will decrease in comparison to previous Games. Due to Olympic entry quotas the total number of medals that any single country could win will be $M = 559$. (See Table 7 for the sport-by-sport breakdown.) This number is not affected by "universality" or "host country" special invitations for entry with-

out qualifying. Note that the I.O.C. has banned Russia from the Paris Games due to the invasion of Ukraine, and the Russian Olympic Committee (R.O.C.) will not be recognized either, although some Russian athletes will be allowed to compete as Individual Neutral Athletes (AINs) (reference: International Olympic Committee 2023).

Table 8 shows final values of S for the past three summer Olympic Games, along with the projected S for the start of the 2024 summer Olympic Games in Paris.

Throughout the Paris Games, and during future Olympics, probability rankings will be posted online at the following website: https://www.olympicnationalrankings.com/. These rankings will be updated daily at the end of competitions (i.e., in the evening in Paris).

5.6. Age-grouped probability ranking

Finally, we mention an alternative way to do population-adjusted ranking by taking age demographics into account. As before, the probability ranking index is given by Equation (8), but now all national populations s (and the total population S) are counted only within the limited youthful age-range of highly competitive athletes. A plausi-

Table 5
National ranking by probability index for three Olympic Games

London 2012		Rio 2016		Tokyo 2020	
Rank		Rank		Rank	
1	Great Britain	1	Great Britain	1	Australia
2	Australia	2	USA	2	Great Britain
3	Russia	3	New Zealand	3	Netherlands
4	Jamaica	4	Australia	4	New Zealand
5	USA	5	France	5	Hungary
6	Hungary	6	Denmark	6	USA
7	New Zealand	7	Azerbaijan	7	Italy
8	Germany	8	Jamaica	8	Japan
9	Netherlands	9	Germany	9	Cuba
10	France	10	Netherlands	10	Jamaica
11	Republic of Korea	11	Russia	11	Switzerland
12	Cuba	12	Hungary	12	France
13	Italy	13	Croatia	13	Denmark
14	Denmark	14	Kazakhstan	14	Canada
15	Czechia	15	Canada	15	Germany
16	Belarus	16	Italy	16	San Marino
17	Canada	17	Sweden	17	Georgia
18	Azerbaijan	18	Cuba	18	Croatia
19	Georgia	19	Japan	19	Czechia
20	Spain	20	Georgia	20	Norway
21	Japan	21	Czechia	21	Serbia
22	Croatia	22	Belarus	22	Slovenia
23	Ukraine	23	Serbia	23	Sweden
24	Kazakhstan	24	Republic of Korea	24	Ukraine
25	Mongolia	25	Switzerland	25	Republic of Korea
26	Trinidad and Tobago	26	Slovenia	26	Chinese Taipei
27	Ireland	27	Spain	27	Austria
28	Sweden	28	Uzbekistan	28	Bulgaria
29	Lithuania	29	Armenia	29	Belarus
30	Slovenia	30	Lithuania	30	Armenia

Table 6
Countries that have not won any summer Olympics medals prior to 2024

Albania	Bosnia/Herzegovina	El Salvador	Liberia	Myanmar	Sao Tome/Principe
Andorra	Brunei	Equatorial Guinea	Libya	Nauru	Seychelles
Angola	Cambodia	Micronesia	Liechtenstein	Nicaragua	Sierra Leone
Antigua/Barbud	Cape Verde	Gambia	Madagascar	Oman	Somalia
Aruba	Cent. African Rep.	Guinea	Malawi	Palau	South Sudan
Bangladesh	Chad	Guinea-Bissau	Maldives	Palestine	Swaziland
Belize	Comoros	Honduras	Mali	Papua New Guinea	Timor-Leste
Benin	Congo	Kiribati	Malta	Rwanda	Tuvalu
Bhutan	Dem. Rep. Congo	Laos	Mauritania	Saint Lucia	Vanuatu
Bolivia	Dominica	Lesotho	Monaco	St Vincent/Grenadines	Yemen

ble age-range is 13–39, since the great majority of Olympic medal-winners have ages within this range. (But note that the minimum age for competition exceeds 13 in some sports.) We implemented this ranking for ages 13–39 in recent Olympic Games, using the best available demographic data. We found only minor changes from the total-population *U*-ranking, but these changes distinctly favored wealthy countries, with high-GDP(PPP) per capita, relative to less economically-advantaged countries.

This result is easy to understand. Countries with stronger economies tend to have better and more widely-available medical care, with more of their citizens surviving to advanced age, thus the *fraction* of their population within the competitive, youthful age-group is smaller. This tends to lower their probabilities p and expected medals $\langle N \rangle$, driving up U, thus favoring wealthier countries in the age-grouped rankings. Indeed, age-grouped ranking effectively penalizes economically-disadvantaged countries in

Table 7
Numbers of Medals in the Paris 2024 Olympic Games With Entry Quota Adjustments

Sport	Events	Total Medals	Maximum Medals (per country)
Aquatics	49	147	86
Archery	5	15	11
Athletics	48	144	132
Badminton	5	15	5
Basketball	4	12	4
*Boxing	13	52	13
Breaking	2	6	4
Canoeing	16	48	26
Cycling	22	66	40
Equestrian	6	18	12
Fencing	12	36	24
Field hockey	2	6	2
Football	2	6	2
Golf	2	6	4
Gymnastics	18	54	30
Handball	2	6	2
*Judo	15	60	15
Modern pentathlon	2	6	4
Rowing	14	42	14
Rugby sevens	2	6	2
Sailing	10	30	10
Shooting	15	45	27
Skateboarding	4	12	12
Sport climbing	4	12	8
Surfing	2	6	4
Table tennis	5	15	7
*Taekwondo	8	32	8
Tennis	5	15	10
Triathlon	3	9	7
Volleyball	4	12	6
Weightlifting	10	30	10
*Wrestling	18	72	18
Total	329	$T = 1041$	$M = 559$

*Combat events with 2 bronze medals.

Table 8
Population and medal amounts in four summer Olympics

	2012 London	2016 Rio	2021 Tokyo	2024 Paris
Total world population (millions)	7,162	7,514	7,909	8,119
Number of countries with no medals in all prior summer Olympic Games	66	63	60	60 (as Games begin)
S, Total population (millions) of Olympic medal-winning countries	6,562	6,872	7,229	7,397
T, Number of medals awarded in Games	961	972	1080	1041
M, Maximum possible medals that a country could win	513	521	579	559

two ways: (1) for not having many older citizens, and—in a separate but related effect—(2) by failing to recognize the advantages gained by wealthy countries due to its older citizens (aged above 40) who contribute to Olympic efforts, through coaching, recruiting, administration, and financial support. Many older parents also support their progeny's Olympic aspirations.

Thus adopting specific, youthful age-groups for U-ranking does not seem optimal for public com-

munications. If wealth-dependent adjustments are to be made, it would seem more appropriate to give a compensatory boost to less wealthy countries, as discussed in the next section. There are also practical reasons to reject age-group ranking. It is more complicated to implement, and the relevant age range is somewhat subjective and controversial. Should it be 13–40, 14–30, 15–35, 16–45 etc.? Without official I.O.C. authorization, this is unclear. Moreover, age-distribution data for many parts of the world are

somewhat unreliable, due to infrequent and perfunctory demographic surveys.

6. Economically-adjusted probability rankings

Countries with stronger economies are at an advantage for achieving sporting success, because the people in wealthier countries, with higher average incomes, tend to have more time and resources available for the pursuit of sports. Government support of Olympic athletes also trends higher in wealthier countries. Thus it could make sense to adjust for economic prosperity as well as for population in Olympic national rankings, although this is a matter of choice, since it clearly changes the meaning of the rankings.

To adjust for national economic strength alone, probability ranking can be done with *gross domestic product based on purchasing power parity*, or GDP(PPP) as the basis variable, rather than population. GDP(PPP) is a well-documented measure of economic productivity, with local currency-values adjusted to quantify the *local, domestic* costs of goods and services (rather than being based on international exchange rates) so that the GDP(PPP) accurately reflects the summed economic means of citizens. The economically-adjusted probability ranking index is then the probability that a country with a given GDP(PPP) would win as many medals as it did, or more medals, under the idealization that all countries *have equal sporting capability per dollar GDP(PPP)*. So if a country's GDP(PPP) is d, and if the sum of d's for all medal-winning countries is D (estimated to be 183.95 trillion dollars in 2024 [International Monetary Fund 2023]), the probability for a country to win any given medal is $p = (T/M) \cdot (d/D)$; and the GDP-adjusted probability index V is

$$V(N, d) = -\log_{10} \sum_{n=N}^{M} \frac{M!}{n! (M-n)!} p^n (1-p)^{t-n}. \tag{12}$$

Using this formula, GDP-adjusted national probability rankings can easily be found. But note that two countries with the same GDP(PPP) would not be expected to win the same number of medals if they had significantly different populations, so *it makes little sense to adjust for GDP without also adjusting for population.*

One plausible way to adjust for the size of the population and economic resources together is to use

$$p = W_s \left(\frac{Ts}{MS} \right) + W_d \left(\frac{Td}{MD} \right), \tag{13}$$

in Equation (12) for the ranking index, where W_S and W_d are weighting factors less than unity that satisfy the normalization condition $W_S + W_d = 1$. However, there is no agreement on the values of W_S and W_d, so it is unclear how to implement these rankings usefully.

Given this situation, unless the I.O.C. specifies official values for W_S and W_d (an unlikely prospect), we favor population-adjusted *U*-ranking without economic adjustment as used in Section 5. Population is clearly the most relevant single number measuring national differences, and the population-adjusted *U*-rankings have no uncertain, disputable free parameters.

Note that various cultural factors also play a significant role in determining Olympic achievement. Some countries simply have less interest in Olympic sports than do others, due to divergent cultural influences within their populations. For example, nearly half of the highly-ranked countries in recent Olympics have been European—a result that is probably no coincidence, since Europe was the birthplace of both the ancient and modern Olympics. This is a complex issue that is difficult to quantify. Trying to adjust for this would be even more controversial than adjusting for economic differences.

An alternative approach to Olympic ranking that takes economic factors into account has been developed by Operations Research experts, using computational modelling methods known as Data Envelopment Analysis (DEA). These methods were originally developed for economic planning, to optimize the production of factory goods in competitive markets. Some key papers that use DEA for Olympic ranking are: Ball 1972; Lozano et al. 2002; Lins et al. 2003; Bernard and Busse 2004; Churilov and Flitman 2006; Li et al. 2008; Saaty 2008; Wu et al. 2009; Flegl and Andrade 2016. These papers rank countries according to their "technical efficiency scores" for producing Olympic medals. They use linear programming to determine how *efficiently* countries achieve medal winnings, taking into account the relevant 'inputs' to medal production, including population, economic resources as measured by GDP (PPP), and in some cases, other factors like host country benefits, culture, politics, team size, etc. DEA rankings

can provide useful insights into how countries achieve Olympic excellence, but such rankings are impractical for wide promulgation by sports media because: (1) they require complex software, (2) some inputs and features of the computational models are controversial, with no official I.O.C. authorization to decide the details; (3) the meaning of the ranking may not be obvious to the general public, and (4) the models can produce *ties* at the top of the ranked list, with more than one country rated at 100% efficiency. For example, in their groundbreaking DEA-based ranking of the 2000 Sydney Olympics, Lins et al. (2003), ranked nine countries together in first place, including the USA with 97 medals and Barbados with one bronze medal.

7. Summary and conclusions

National rankings by medal counts, using either [gold+silver+bronze] totals or gold-alone counts, are widely reported in Olympic media coverage, influencing public perception of the Olympics Games worldwide. Unfortunately, this kind of national ranking systematically favors larger-population countries. Another commonly-used ranking index is *medals-per-capita*, but rankings by this index are, in practice, invariably topped by the *smallest*-population medal-winners. This happens because the populations of medal-winning countries vary over an enormous range—a factor of 42,500 in the 2020 Tokyo Games, which is much greater than the number of medals which any country could win, $M = 579$— and because many small-population countries compete, constituting a significant aggregate population within which some medals are expected (as explained in section 2).

We have proposed a new method for national ranking in the Olympics, to promote deeper public understanding and appreciation of the relative sporting capabilities of countries with varying populations. This "probability ranking" is based upon the index U as given by Equation (8) [or, in an excellent approximation, Equation (9)]. We suggest that popular media coverage of the Olympics report the top ten or top twenty probability-ranked countries in daily updates, along with medal-count rankings. The two rankings would complement each other well, generating excitement and suspense about the Games at the national level, thus enriching the Olympics. But *the probability index U should never be described as a "score" determining who "won" the Olympics.*

It should always be described accurately as a simple, objective way to quantify the evidence for high sporting ability given by Olympic medal-wins, while adjusting for population. Specifically, it is a measure of the improbability that a country of the given population could win as many medals as it did, or more medals, if all people worldwide had equal capacity for winning medals.

In this paper, we have based Olympics rankings upon *total medal counts N*—the sum of a country's gold, silver and bronze counts. In Section 4 we justified this using principal component analysis, a standard tool of multivariate statistical analysis. Our calculations showed that using total medal counts optimizes ranking accuracy, on average, when no official color weights are specified.

Figure 3 shows probability ranking results for the 2020 Tokyo Games. The top ten *probability-ranked* countries are highlighted in blue (cf. Table 2), and the top ten countries ranked by *medals-per-capita* are highlighted in red (cf. Table 1), with rank-numbers given in the same colors. Three countries (New Zealand, Hungary and Jamaica) are highlighted in purple because they made both top-ten lists. The top countries as ranked by *total medal counts* can also be easily found from Fig. 3, by starting at the top of the graph and moving downward: U.S.A., China, R.O.C., Great Britian, Japan, etc. (Note that the R.O.C. can be ranked by raw medal counts, but not by any population-adjusted index because the I.O.C specified that R.O.C. did not represent Russia in the 2020 Games.)

No ranking system is perfect, and given the vast population range and heteroskedasticity of the Olympic data, no single ranking system can highlight all national achievements comprehensively. Thus when seeking to highlight the achievements of very small-population countries, it may be useful to post medal-per-capita rankings alongside medal-count and probability rankings, at least some of the time. Tiny-population countries are effectively excluded from the top twenty in raw medal counts, and they tend to not reach the top five in probability rankings either, although some small-population countries can be *U*-ranked highly, such as Jamaica which was 3rd, 8th and 10th in the last three Summer Games, or San Marino, which was 16th in Tokyo. Tiny San Marino (population 34,000) would have needed one more medal to reach the U-ranks top ten, or 3 more medals to reach the top five, or, alternatively, 9 more medals to be ranked first. This might seem like a lot to ask for, but it is much less than the 110 more
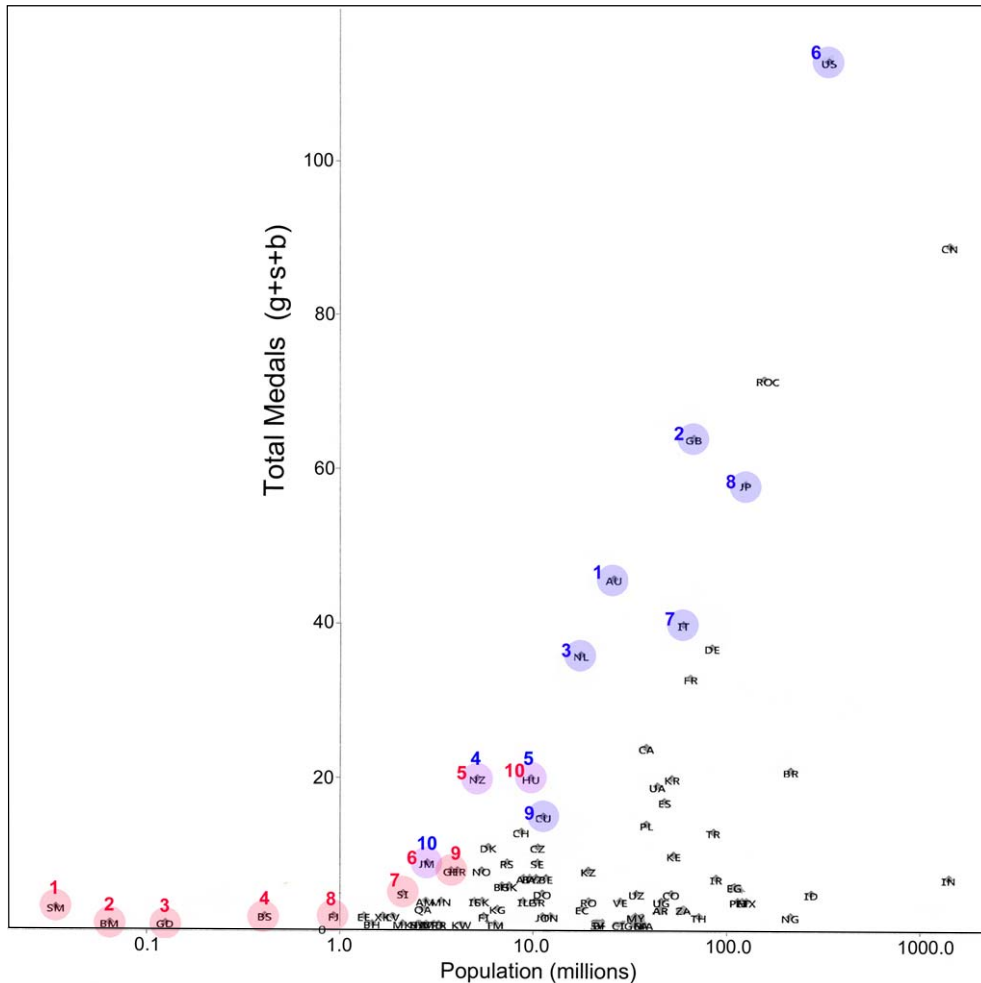
Fig. 3. The numbers of medals won by various countries, plotted as a function of national population (on a log scale) for the 2020 Tokyo Games. Probability rank-numbers are given in blue, and medals-per-capita rank-numbers are in red.

medals that San Marino would need to be first in the total medal-count; or the wildly impossible 126,763 medals that China would need to be ranked first in medals per capita.

In conclusion, we encourage sports media and public information sources to use population-adjusted probability ranking when reporting and assessing national performance in the Olympics. To facilitate this, probability rankings will be posted online at https://www.olympicnationalrankings.com/ and updated daily during future Olympic Games. If these *U*-rankings are presented regularly alongside raw medal-counts, this will deepen the public's understanding and appreciation of achievement at the highest level of sports, worldwide, and add to the excitement and drama of the Olympics. We hope that this will make the Olympics more fun for every-

one, while raising appreciation of the determined efforts made by athletes in many different countries to achieve Olympic excellence.

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Appendix A

The Binomial Distribution

We now give a brief pedagogical review for readers who are unfamiliar with Equation (1) for $P_i(N, s_i)$. This basic theory is very well-known to statisticians, but it may be helpful to summarize the essential ideas here for sports journalists and others seeking to apply probability ranking.

The Binomial Probability Distribution is defined as follows. If p is the probability for success in a single trial event, then the probability for getting exactly N successes in a series of M individual, independent trials is

$$P(N, p, M) = \frac{M!}{N!(M-N)!} p^N (1-p)^{M-N},$$

(14)

where the factorial function is defined (as usual) by: $N! = N \cdot (N-1) \cdot (N-2) \ldots \cdot 3 \cdot 2 \cdot 1$.

For example, if p is the probability for getting "heads" in a coin-flip, which has the value $p = 0.5$, then the probability for getting exactly $N = 8$ heads when you do a trial of $M = 10$ coin-flips is

$$P(8, 0.5, 10) = \frac{10!}{8!2!} (0.5)^{10} = 0.05273. \quad (15)$$

To derive Equation (14), note that the factor $p^N (1-p)^{M-N}$ is just the probability that N *specific independent trials out of t will succeed*, because each success has a probability p and these probabilities multiplied together give a net probability p^N, while the $(M-N)$ non-successes each have probability $(1-p)$. To then find the probability that *any* set of

N tests succeed we must multiply this by the number of ways that you can select N specific events out of a total number M. By simple combinatoric reckoning, it can be shown that the numbers of combinations of M things taken N at a time is $M!/N!(M-N)!$, yielding Equation (14). Now, each time you do 10 coin-flips, the number of heads N must be some value between 0 and 10, so the probabilities for all those outcomes must add up to 1. This is guaranteed by the "normalization condition"

$$1 = \sum_{N=0}^{M} P(N, p, M). \quad (16)$$

Using Equation (14), this can be verified as a mathematical identity, which is known as the "Binomial Theorem."

If you do many series of trials, each with M trial-events, then the *mean* number of successes that you will find, averaged over all the trial series, is

$$\langle N \rangle = \sum_{N=0}^{M} N P(N, p, M) = pM. \quad (17)$$

which also can be proven mathematically using Equation (14), but is obvious from the definition of the probability p. For 10 coin-flips, the mean number of heads observed is $\langle N \rangle = 5$. Note that whenever $\langle N \rangle$ is significantly greater than 1, the probability distribution $P(N)$ given by Equation (14) is strongly peaked around $N = \langle N \rangle$.