# New scoring system to reduce unfairness in men's doubles 

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#### Abstract

Tennis uses essentially the same scoring system in doubles as in singles. In this paper it is shown that the scoring system commonly used in men's doubles has the potential to be unfair. Aspects of this potential unfairness are identified and methods for reducing it are outlined. The statistical characteristics of the present scoring system are compared with those of a proposed new system.


Keywords: Men's doubles, unfairness in men's doubles scoring, mean and variance of the number of points played in a tennis match, a new men's doubles scoring system, first to 9 points tiebreak game, 'sudden death at second deuce' games, '0-15 games'

## 1. Introduction

In the present scoring system commonly used in men's doubles, a match comprises the best-of- 3 tiebreak sets, where the winner of a tiebreak set is the first pair to win (at least) 6 games leading by 2 games, and if 6 games all is reached a tiebreak game is played to determine the set winner.

- Service games are 'deuce' games in which to win the game a doubles pair must win 4 points leading by 2 points; if 3 points all (deuce) is reached, the game continues until one pair achieves a lead of 2 points.
- Tiebreak games are TB7 games in which to win the game (and hence the set) a doubles pair must win 7 points leading by 2 points; if 6 points all is reached, the tiebreak game continues until one pair achieves a lead of 2 points. Sometimes the TB 10 tiebreak (using an obvious notation) is used in the third set.

[^0]Tennis has a 3-nested scoring system. Points are played within games; games are played within a set and the match is typically the best of three sets. It is very important that a scoring system is as close to fair as is possible (where fairness is defined as two equal players or pairs each having a $50 \%$ chance of winning the match). Given that the parameters relevant for doubles are typically somewhat different to those for singles, a scoring system that is fair for singles may not be fair for doubles. Further, there are criteria other than fairness that are relevant when comparing two or more scoring systems.

The criteria that are used to compare tennis scoring systems are:

1. Fairness which is an essential characteristic of any scoring system. For example, two equal pairs should have as close to an equal chance of winning as possible.
2. The expected value and standard deviation of the total number of points played. Exact methods for calculating these values have been outlined previously (Pollard, 1983). These exact methods have been used to obtain all the results in Tables 1 to 7, whilst very accurate large sim-

Table 1
The magnification of 'deuce games' for two equal pairs when $\mathrm{pA} 1=0.7, \mathrm{pA} 2=0.6$,

$$
\mathrm{pB} 1=0.66, \mathrm{pB} 2=0.64
$$

| Percentage of Service Games won | 'deuce' game |
| :--- | :---: |
| $\%$ of service games won by A1 when pA1 $=0.7$ | $90.08 \%$ |
| \% of service games won by A2 when pA2 $=0.6$ | $73.57 \%$ |
| Average \% pair A | $81.83 \%$ |
| \% of service games won by B1 when pB1 $=0.66$ | $84.57 \%$ |
| \% of service games won by B2 when pB2 $=0.64$ | $81.26 \%$ |
| Average \% pair B | $82.92 \%$ |

Table 2
The magnification of the ' $0-15$, SD at deuce 2 ' game for two equal pairs when

$$
\mathrm{pA} 1=0.7, \mathrm{pA} 2=0.6, \mathrm{pB} 1=0.66, \mathrm{pB} 2=0.64
$$

| Percentage of Service Games won | '0-15, SD at <br> deuce 2' game |
| :--- | :---: |
| \% of service games won by A1 when pA1 $=0.7$ | $77.02 \%$ |
| \% of service games won by A2 when pA2 $=0.6$ | $56.09 \%$ |
| Average \% pair A | $66.56 \%$ |
| \% of service games won by B1 when pB1 $=0.66$ | $69.10 \%$ |
| \% of service games won by B2 when pB2 $=0.64$ | $64.87 \%$ |
| Average \% pair B | $66.99 \%$ |

Table 3
Set results using 'deuce' service games for two equal pairs, when $\mathrm{pA} 1=0.70, \mathrm{pA} 2=0.60$ for pair A , and $\mathrm{pB} 1=0.66, \mathrm{pB} 2=0.64$ for pair B

| Set results using 'deuce' <br> service games | TB9 game, <br> A1 serves <br> first in set | TB9 game, <br> B1 serves <br> first in set | Average |
| :--- | :---: | :---: | :---: |

Table 4
Set results using ' $0-15, \mathrm{SD}$ at deuce 2 ' service games when $\mathrm{pA} 1=0.70$ and $\mathrm{pA} 2=0.60$ for pair A , and $\mathrm{pB} 1=0.66$ and $\mathrm{pB}=0.64$ for pair B

| Set results using '0-15, <br> SD at deuce 2' | TB9 game, <br> A1 serves first <br> in set | TB9 game, <br> B1 serves first <br> in set | Average |
| :--- | :---: | :---: | :---: |

Table 5
Some characteristics of single set scoring systems when pair $A$ is better than pair $B, \mathrm{pA} 1=0.71, \mathrm{pA} 2=0.61, \mathrm{pB} 1=0.65$ and $\mathrm{pB} 2=0.63$

| Single set scoring | Col 1 | Col 2 | Col 3 | Col 4 | Col 5 | Col 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| System | Present | $0-15$ start, | Present | $0-15$ start, | Average | Average |
|  | Deuce | SD at deuce2 | Deuce | SD at deuce2 | Col 1, 3 | Col 2, 4 |
|  | TB7, A1 | TB9, A1 | TB7, B1 | TB9, B1 |  |  |
|  | serves first | serves first | serves first | serves first |  |  |
| P(A wins before TB | 0.4317 | 0.4944 | 0.4317 | 0.4944 | 0.4317 | 0.4944 |
| P(B wins before TB) | 0.3415 | 0.3673 | 0.3415 | 0.3673 | 0.3415 | 0.3673 |
| P(TB needed) | 0.2268 | 0.1383 | 0.2268 | 0.1383 | 0.2268 | 0.1383 |
| P(A wins TB game) | 0.5278 | 0.5377 | 0.5504 | 0.5431 | 0.5391 | 0.5404 |
| P(B wins TB game) | 0.4723 | 0.4623 | 0.4496 | 0.4569 | 0.4609 | 0.4596 |
| P(A wins set) | 0.5514 | 0.5688 | 0.5565 | 0.5695 | 0.5540 | 0.5691 |
| P(B wins set) | 0.4486 | 0.4312 | 0.4435 | 0.4305 | 0.4460 | 0.4309 |
| E(\# Points) | 64.86 | 54.85 | 65.18 | 55.02 | 65.02 | 54.94 |

Table 6
Some characteristics of single set scoring systems when pair $B$ is better than pair $A, \mathrm{pA} 1=0.69, \mathrm{pA} 2=0.59, \mathrm{pB} 1=0.67$ and $\mathrm{pB} 2=0.65$

| Single set | Col 1 | Col 2 | Col 3 | Col 4 | Col 5 | Col 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| scoring system | Present | $0-15$ start, | Present | $0-15$ start, | Average | Average |
|  | Deuce | SD at deuce2 | Deuce | SD at deuce2 | Col 1,3 | Col 2, 4 |
|  | TB7, A1 | TB9, A1 | TB7, B1 | TB9, B1 |  |  |
|  | serves first | serves first | serves first | serves first |  |  |
| P(A wins before TB) | 0.3188 | 0.3744 | 0.3188 | 0.3744 | 0.3188 | 0.3744 |
| P(B wins before TB) | 0.4572 | 0.4880 | 0.4572 | 0.4880 | 0.4572 | 0.4880 |
| P(TB needed) | 0.2240 | 0.1377 | 0.2240 | 0.1377 | 0.2240 | 0.1377 |
| P(A wins TB game) | 0.4607 | 0.4628 | 0.4832 | 0.4681 | 0.4720 | 0.4655 |
| P(B wins TB game) | 0.5393 | 0.5372 | 0.5168 | 0.5319 | 0.5280 | 0.5345 |
| P(A wins set) | 0.4220 | 0.4381 | 0.4271 | 0.4388 | 0.4245 | 0.4385 |
| P(B wins set) | 0.5780 | 0.5619 | 0.5729 | 0.5612 | 0.5755 | 0.5615 |
| E(\# Points) | 65.14 | 55.06 | 64.78 | 54.93 | 64.96 | 55.00 |

ulation methods have been used to obtain the results in Table 8.
3. Measures of the 'upper tail' of the distribution of match duration (i.e. number of points played). This is an important criterion as very long matches in tennis are undesirable. We use CD to represent cumulative distribution, and CD90 to represent the duration such that $90 \%$ of matches have a duration less than CD90. The median of the match duration is thus CD50.
4. The probability that the better pair wins the match.

Given two scoring systems with the same expected duration, the one in which the better pair has a higher probability of winning is preferable with respect to criteria 4 above. Correspondingly, given two scoring systems with the same likelihood of the better pair winning, the one which has the smaller expected duration is preferable as it is the more efficient at identifying the better pair.

There can be a need for a compromise or balance between these four criteria when comparing tennis scoring systems. A 'good' scoring system for a set or
match of tennis has an appropriate number of points played on average, an appropriate value for the probability that the better pair wins, a relatively small standard deviation of the number of points played, and a relatively small upper tail of the distribution of duration.

When considering how a scoring system might be modified to better meet the above criteria, there are several things that might be considered: modify the structure of a game, modify the structure of a set, and/or modify the structure of the TB game. It was considered that the 'first to 6 games, leading by 2 games' component of a set was unchangeable. At present there are two game scoring systems in use: the advantage game and the 'no ad' game in which a single point is played when deuce is first reached. Miles (1984) studied the efficiency of tennis scoring systems, and concluded that, when players had strong services, starting every game at $0-15$ (rather than at $0-0$ ) increased the efficiency of the tennis scoring system. However, despite this modification, some sets would still have more points than is desirable. That is, the set variance of duration using ' $0-15$ games' was large. It would seem that by making use of this

Table 7
A summary of some results in Tables 5 and 6

| Case | Prob Better <br> Pair wins set <br> (Present) | Prob Better <br> Pair wins set <br> $(0-15$, SD <br> at deuce 2) | E(\# Points) <br> (Present) | E(\# Points) <br> (0-15, SD <br> at deuce 2) |
| :--- | :--- | :--- | :--- | :--- |
|  |  | 0.5691 | 65.02 | 54.94 |
| Case 1 (Table 5) | 0.5540 | 0.5615 | 64.96 | 55.00 |
| Case 2 (Table 6) | 0.5755 | 0.5653 | 64.99 | 54.97 |
| Average | 0.5647 |  |  |  |

Table 8
Some characteristics of two best of 3 sets scoring systems when pair A is better than pair B, and (pA1-pA2) equals (pB1-pB2)

| Comparison of two best of 3 sets scoring systems | Present <br> Best of 3 <br> sets, deuce <br> games, TB7 | Best of 3 sets, ' $0-15$, SD at deuce 2' games, TB9 | Present <br> Best of 3 <br> sets, deuce <br> games, TB7 | Best of 3 sets, ' $0-15$, SD at deuce 2' games, TB9 |
| :---: | :---: | :---: | :---: | :---: |
| pA1 | 0.6800 | 0.6800 | 0.7000 | 0.7000 |
| pA2 | 0.6400 | 0.6400 | 0.6200 | 0.6200 |
| pB1 | 0.6600 | 0.6600 | 0.6800 | 0.6800 |
| pB2 | 0.6200 | 0.6200 | 0.6000 | 0.6000 |
| Prob. 2 set match | 0.5092 | 0.5086 | 0.5097 | 0.5086 |
| Prob. 3 set match | 0.4908 | 0.4914 | 0.4903 | 0.4914 |
| Prob. A wins match | 0.5971 | 0.5979 | 0.5964 | 0.5974 |
| Prob. B wins match | 0.4029 | 0.4022 | 0.4036 | 0.4026 |
| Mean No. Points | 162.5 | 137.1 | 161.6 | 137.0 |
| Standard Deviation | 40.47 | 35.30 | 40.12 | 35.10 |
| Efficiency | 0.5338 | 0.6431 | 0.5279 | 0.6365 |
| Median=CD50 | 160 | 136 | 159 | 135 |
| CD90 | 218 | 185 | 217 | 185 |
| CD95 | 231 | 197 | 230 | 197 |
| CD99 | 254 | 220 | 252 | 219 |
| CD99.5 | 262 | 228 | 260 | 227 |

' $0-15$ construct' but playing a single point on the second deuce (a variation of the 'no ad' game), a scoring system with good efficiency and fewer 'long matches' might be produced. This is done in this paper.

A close analysis of the TB7 game reveals that the TB7 game can be unfair in doubles, and that a TB9 game might resolve much of that unfairness. Thus, this modification was incorporated into this study.

Exact mathematical methods have been used to calculate almost all of the results in this paper. However, simulations were used to estimate, for example, the median number of points played.

This paper proposes two changes to the present scoring system with the objective of reducing the potential unfairness in the scoring system and reducing the length of 'long' matches:

- Replace the current 'deuce' service game by one where the service game starts at $0-15$, with a sudden death (SD) point played after the second deuce. In the remainder of this paper, we often abbreviate and refer to this service game as a ' $\mathbf{0} \mathbf{- 1 5}$, SD at deuce 2 ' game.
- Replace the current TB7 tiebreak game with a TB9 tiebreak game, in which to win the game (and hence the set) a doubles pair must win 9 points leading by 2 points; if 8 points all is reached, the tiebreak game continues until one pair achieves a lead of 2 points.

Together these changes reduce the unfairness, reduce the length of matches, and maintain the probability that the better pair wins.

## 2. Characteristics of the 'start at $0-15$, sudden death at the second deuce' service game

Suppose pair A consists of players A1 and A2, and pair B consists of players B1 and B2. Suppose player A1 has a probability of winning a point on service of pA 1 and player A2 has a probability of winning a point on service of pA2. Also, suppose player B1 has a probability of winning a point on service of pB 1 and player B 2 has a probability of winning a
point on service of pB 2 . Then pairs (A1, A2) and ( $\mathrm{B} 1, \mathrm{~B} 2$ ) are equal pairs if $\mathrm{pA} 1+\mathrm{pA} 2=\mathrm{pB} 1+\mathrm{pB} 2$. In a fair scoring system, equal pairs should each have an equal probability of winning a set.

In men's doubles the server wins on average about $65 \%$ of the points he initiates. Where one player A1 in an 'average pair' is the more effective server, he might win (say) $70 \%$ of points on his service (pA1 $=0.7$ ), whilst his partner A2 wins only $60 \%$ on service ( $\mathrm{pA} 2=0.6$ ). The game scoring rules used in tennis 'magnify' these percentages. Thus, it can be shown mathematically that when 'deuce' games are played the 'stronger server' A1 with a point probability of 0.7 wins $90.08 \%$ of his service games whilst the 'weaker server' A2 with a point probability of 0.6 wins $73.57 \%$ of his service games. Thus, on average the pair (A1, A2) wins $81.83 \%$ of games on service. For an 'equal pair' (B1, B2) winning $66 \%$ and $64 \%$ of points on service ( $\mathrm{pB} 1=0.66$, $\mathrm{pB} 2=0.64$ ), the percentages of service games won are $84.57 \%$ and $81.26 \%$ respectively, for an average of $82.92 \%$ of games on service. This can be seen in Table 1. Thus, the (equal) pair of players with a greater difference between their effectiveness on service is disadvantaged when 'deuce' games are used. This magnification inherent in the 'deuce' game structure leads to a potential unfairness in doubles.

This unfairness in doubles can be reduced by using the new game scoring system ' $0-15, \mathrm{SD}$ at deuce 2 ' introduced above, where the server starts the game at $0-15$, and a single point ('sudden death', SD) is played if the second deuce is reached to determine the winner of the game. This new game scoring system can be considered to be a variant of the 'no ad' game system, in which if deuce is reached, a single point is played to determine the winner of the game. Relative to the 'no ad' game, when using the ' $0-15$, SD at deuce 2 ' game the server has the advantage of there being up to 2 deuces whilst he has the disadvantage of starting from $0-15$. This game has a maximum of 8 points, 4 points to the first court and 4 points to the second court. With the present 'no ad' game there is a maximum of 7 points, and this creates the issue of whether the $7^{\text {th }}$ point should be to the $1^{\text {st }}$ or $2^{\text {nd }}$ court. This is resolved by giving the receiving pair the choice. Some tennis fans may see this as a rather awkward solution to the issue.

Returning to the example above of two equal pairs, with (pA1, pA2) $=(0.7,0.6)$ and $(\mathrm{pB} 1, \mathrm{pB} 2)=(0.66$, 0.64 ), the difference in the percentage of games won on service by these two equal pairs is reduced from $1.09 \%$ ( $=82.92 \%-81.83 \%$ in Table 1) when
using 'deuce' games to $0.43 \%$ ( $=66.99 \%-66.56 \%$ in Table 2 ) when using ' $0-15$, SD at deuce 2 ' games. Thus, it might be anticipated that the ' $0-15$, SD at deuce 2' game can be used to produce a 'fairer' set scoring system than at present.

## 3. An outline of the unfairness of the TB7 tiebreak game in doubles

The first to 7 points (leading by 2 points) tiebreak game (denoted by TB7) can also be somewhat unfair in doubles. This tiebreak game is initially the best of 12 points, with additional rules if the score reaches 6 points all. The player who serves on the second point of this tiebreak game in fact serves on 4 of these 12 points, whilst his partner serves on only 2 . For this reason, the doubles pair who wins the toss sometimes elects to receive in the first game of the set so that their first server will get to serve on 4 of the first 12 points if the TB7 game is played. The extent of the unfairness generated by this characteristic can be considerably reduced by using (a correspondingly defined) TB9 (or 'best of 16 points') where each player has a potential 4 serves. Further details are given in Section 5 below.

## 4. Parameter values for the analysis

The key input parameters in modelling a doubles tennis match between pair (A1, A2) and pair (B1, B2) are $\mathrm{pA} 1, \mathrm{pA} 2$ and $\mathrm{pB} 1, \mathrm{pB} 2$ as defined earlier.
Recently Pollard, Noble and Pollard (2021) studied the potential for reducing the length of very long singles matches. After considering recently available data on values for ( $\mathrm{pA}, \mathrm{pB}$ ) for men's singles matches, the values they used to represent a typical close match in men's singles between two players A and B with 'average services' were ( $\mathrm{pA}, \mathrm{pB}$ ) $=(0.65,0.63)$. They noted that the corresponding values for an average close match in men's doubles should average 0.01 more than in singles. Thus, the corresponding values for a typical close men's doubles match should average 0.66 for pair (A1, A2) and 0.64 for pair (B1, B2).
Thus, the parameters used in this study for an average 'close' men's doubles matches are pA 1 and pA 2 averaging 0.66 and pB 1 and pB 2 averaging 0.64 , and for matches between 'equal' pairs an average of 0.65 for each pair.

It is assumed throughout this paper that the more effective player within a pair serves in their first service game of each set.

## 5. The unfairness of the TB7 tiebreak game for two equal (but not identical) doubles pairs

We consider the unfairness in the present TB7 game as well as the unfairness in the proposed TB9 game using the parameters $\mathrm{pA} 1=0.70$ and $\mathrm{pA} 2=0.60$ for pair A , and $\mathrm{pB} 1=0.66$ and $\mathrm{pB} 2=0.64$ for pair B. Note that the parameters for players A1 and A2 have the greater difference.

First we assume that the players' service order is A1, B1, A2, B2. This order applies to service games and if a TB game is needed is also the order in which the players serve (for a TB game A1 will serve the $1^{\text {st }}$ point, then B1 will serve the $2^{\text {nd }}$ and $3^{\text {rd }}$ points, then A2 will serve the $4^{\text {th }}$ and $5^{\text {th }}$ points, then B 2 will serve the $6^{\text {th }}$ and $7^{\text {th }}$ points, etc). It can be shown that $\mathrm{P}(\mathrm{A}$ wins TB7) $=0.4942$, whereas $\mathrm{P}(\mathrm{A}$ wins TB9 $)=0.5002$, a value closer to 0.5 .

Likewise if the players' service order is B1, A1, B2, A2 with B1 serving first in the set and also first in the TB game if one is needed, it can be shown that $\mathrm{P}(\mathrm{A}$ wins TB7 $)=0.5168, \mathrm{P}(\mathrm{A}$ wins TB9) $=0.5057$, a value closer to 0.5 . Thus, TB9 is fairer than TB7 no matter whether A1 or B1 serves first.

## 6. Assessing the fairness of the 'deuce' game and the ' $\mathbf{0}-15$, SD at deuce 2 ' game within a single set

The fairness of the 'deuce' game and the ' $0-15$, SD at deuce 2' game in doubles within the present tiebreak set structure (modified by using TB9) are now considered. The parameters used here for assessing fairness are $\mathrm{pA} 1=0.70, \mathrm{pA} 2=0.60$ for pair A and $\mathrm{pB} 1=0.66, \mathrm{pB} 2=0.64$ for pair B.

The values in Tables 3 and 4 are exact values obtained using recurrence methods.

1. Set results using 'deuce’ service games

It can be seen in Table 3 that P (A wins set) averages 0.4885 . These results indicate how pair A , the (equal) pair with the greater difference in point probability parameters is disadvantaged under the present deuce game structure.
2. Set results using ' $0-15$, SD at deuce 2 ' service games
It can be seen that using ' $0-15, \mathrm{SD}$ at deuce 2 ' service games (in conjunction with a TB9 game) produces a set outcome that is considerably fairer than
when 'deuce' service games are used ( 0.5039 is closer to 0.5 than 0.4885 ).

## 7. The single set analysis when pairs are not equal

In this section the present set scoring system, using 'deuce' service games and TB7 tiebreak game (if needed) is compared with one using ' $0-15$ start, SD at deuce 2' service games and TB9 tiebreak game (if needed). This comparison is carried out for two different cases:

- Case 1: Single set analysis when pair A is the better pair, for $\mathrm{pA} 1=0.71, \mathrm{pA} 2=0.61, \mathrm{pB} 1=0.65$ and $\mathrm{pB} 2=0.63$, where results are given in Table 5.
- Case 2: Single set analysis when pair $B$ is the better pair, for $\mathrm{pA} 1=0.69, \mathrm{pA} 2=0.59, \mathrm{pB} 1=0.67$ and $\mathrm{pB} 2=0.65$, where results are given in Table 6.

For each of Case 1 and Case 2, single set analysis is carried out for both A1 serves first and B1 serves first, with results for A1 (B1) serves first occurring in Col 1 and $\mathrm{Col} 2(\mathrm{Col} 3$ and Col 4$)$ of each of Tables 5 and 6 respectively.

- Case 1: We consider a single set where pair A is the better pair with an average $p$-value of 0.66 , and pair B has an average $p$-value of 0.64 . In this example pair A has the 'more variable' $p$-values.

In Table 5, it can be seen that at present P (TB7 is needed $)=0.2268$, while Col 5 shows that P (A wins the TB7 game) averages 0.5391 and P (A wins the set) averages 0.5540 .

Correspondingly, for the scoring system using ' 0 15 start, SD at deuce 2' games and TB9, $\mathrm{P}($ TB9 is needed $)=0.1383$, while Col 6 shows that $\mathrm{P}(\mathrm{A}$ wins the TB9 game) averages 0.5404 and P (A wins the set) averages 0.5691 .

It is noted that the probability that the TB game needs to be played and the expected number of points in a set are considerably reduced when ' $0-15$ start, SD at deuce 2' games are used.

- Case 2: We consider a single set where pair B is the better pair with an average $p$-value of 0.66 , and pair A has an average $p$-value of 0.64 . Again, pair A has the 'more variable' $p$-values.
It can be seen in Table 6 that at present $\mathrm{P}(\mathrm{TB} 7$ is needed $)=0.2240$, while Col 5 shows that $\mathrm{P}(\mathrm{B}$ wins
the TB7 game) averages 0.5280 , and $\mathrm{P}(\mathrm{B}$ wins the set) averages 0.5755 .

For the scoring system using ' $0-15$ start, SD at deuce $2^{\prime}$ games and TB9, $\mathrm{P}($ TB 9 is needed $)=0.1377$, while Col 6 shows that $\mathrm{P}(\mathrm{B}$ wins the TB9 game) averages 0.5345 , and $\mathrm{P}(\mathrm{B}$ wins the set) averages 0.5615 .

Again, it is noted that the probability that the TB game needs to be played and the expected number of points in a set are considerably reduced when ' $0-15$ start, SD at deuce 2' games are used.

In Tables 5 and 6 the 'better' pair averages a pointwinning probability on service of 0.66 , whilst the other pair averages a point-winning probability of 0.64 on service. In Table 5 the better pair has the more variable $p$-values on service, whilst in Table 6 it is the weaker pair (pair A) that has the more variable $p$-values. There appears to be no particular reason why in general the better or the weaker pair would be the more variable on service. Thus, Tables 5 and 6 might be summarized as in Table 7 .

It can be seen in Table 7 that the average probability that the better team wins using ' $0-15$, SD at deuce 2 ' games is very similar to its value under the present system, whilst the expected number of points played in a set is about 10 fewer. Thus, it can be concluded that the system using ' $0-15, \mathrm{SD}$ at deuce 2 ' games is considerably more efficient than the present system [approximately $((10 / 65) * 100=15 \%)$ ].

It is noted that all the values in Tables 1 to 7 have been derived using exact mathematical methods. However, as a check, large simulation methods produced results that were highly consistent with the tabled values.

## 8. Best of $\mathbf{3}$ sets analysis

Two best of 3 sets examples are given in Table 8, each with Pair A better than pair B, and with (pA1pA 2 ) equal to ( $\mathrm{pB} 1-\mathrm{pB} 2$ ). Simulation methods were used to generate these results ( $100,000,000$ matches for each column in Table 8). We alternated the pair that serves first in the match and used the identical rules that presently exist for deciding the pair to serve first in the $2^{\text {nd }}$ and $3^{\text {rd }}$ sets. Either player A1 or B1 (not A2 or B2) serves in the first game of every set.

In the first two columns of Table 8 we consider as an example the case where ( $\mathrm{pA} 1, \mathrm{pA} 2$ ) $=(0.68$, $0.64)$ and $(\mathrm{pB} 1, \mathrm{pB} 2)=(0.66,0.62)$, noting that $\mathrm{pA} 1-$ $\mathrm{pA} 2=\mathrm{pB} 1-\mathrm{pB} 2=0.4$. In the last two columns we consider the case where (pA1, pA2) $=(0.70,0.62)$ and
$(\mathrm{pB} 1, \mathrm{pB} 2)=(0.68,0.60)$, noting that $\mathrm{pA} 1-\mathrm{pA} 2=\mathrm{pB} 1$ $-\mathrm{pB} 2=0.8$.

It can be seen that the two best of 3 sets scoring systems using standard games and TB7 (first and third columns), and the other using ' $0-15, S D$ at deuce 2' games and TB9 (second and fourth columns) have very similar values for the probability that the better pair $A$ wins the match. However, for the proposed new scoring system (in columns 2 and 4), the mean number of points played is reduced by about 25 points, the standard deviation of points played is reduced by about 5 points, and the CD 90 values are reduced by about 32 points. Given that the doubles pairs have about the same likelihood of winning the best of 3 sets matches, this saving in match duration and reduction in variability of duration are very attractive properties of the proposed scoring system.

Table 8 includes the efficiency measure, as described in a very elegant paper by Miles (ibid) and extended for doubles by Pollard and Pollard (2008). [It is noted that scoring systems with efficiency close to 1 have very large variances of duration and hence are simply quite unsuitable for tennis matches]. The two scoring systems that have been considered in this paper have a similar value for $P$, the probability that the better pair A wins the match. However, the scoring system using ' $0-15, \mathrm{SD}$ at deuce 2 games' is the more efficient as it has a smaller value for the expected number of points played. The increased efficiency of this system is related to the fact that, with point $p$ values near 0.65 , points within ' $0-15$, SD at deuce 2 ' games have a greater average importance and a greater average excitement. The reader who is interested in this relationship is referred to two papers by Pollard (2017a, 2017b).

## 9. Conclusions

Fairness in sport is considered an essential characteristic. The scoring system presently used in men's doubles can be unfair when the two players in a pair are not equally effective on service. This unfairness can be reduced by modifying the present system whilst maintaining many aspects of it.

The modifications recommended in this paper for men's doubles are:

- Change the rules for a service game so that instead of the present 'deuce' game, the game starts at $0-15$, with a sudden death point played
after the second deuce. Such a service game has a maximum of 8 points and is fairer than the present 'deuce' game.
- Change the rules for a tiebreak game so that instead of the present TB7 game (first to 7 points leading by 2 points), it becomes a TB9 game (first to 9 points leading by 2 points). Such a tiebreak game is fairer than the present TB7 game.

No other changes are recommended. Thus, a match remains the best-of-3 tiebreak sets, where the winner of a tiebreak set is the first pair to win 6 games leading by 2 games, and if 6 games all is reached a tiebreak game is played.

Together the two recommended changes maintain the probability that the better pair wins, while reducing the unfairness, and considerably reducing the length of matches, the variance in the number of points played in a match and the likelihood of 'long matches', thus increasing the set and match efficiency.

While commencing a service game at $0-15$ may be challenging for some players to accept initially, it simply counterbalances the server's gains from a sudden death point being played after the second deuce (rather than the first), and produces positive results in increasing fairness and decreasing the number of points played.

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