

Identifying Pacing Profiles in 2000 Metre World Championship Rowing

Dani Chu*, Ming-Chang Tsai, Ryan Sheehan, Jack Davis and Renny Doig

Department of Statistics and Actuarial Science, Simon Fraser University, University Drive, Burnaby BC, Canada

Pre-press 4 March 2023

Abstract. The pacing strategy adopted by athletes is a major determinant of success during timed competition. Various pacing profiles are reported in the literature and its importance depends on the mode of sport. However, in 2000 metre rowing, the definition of these pacing profiles has been limited by the minimal availability of data. **Purpose:** Our aim is to objectively identify pacing profiles used in World Championship 2000 metre rowing races using reproducible methods. **Methods:** We use the average speed for each 50 metre split for each available boat in every race of the Rowing World Championships from 2010–2017. This data was scraped from www.worldrowing.com. This data set is publicly available (https://github.com/danichusfu/rowing_pacing_profiles) to help the field of rowing research. Pacing profiles are determined by using k-shape clustering, a time series clustering method. A multinomial logistic regression is then fit to test whether variables such as boat size, gender, round, or rank are associated with pacing profiles. **Results:** Four pacing strategies (Even, Positive, Reverse J-Shaped, and U-Shaped) are identified from the clustering process. Boat size, round (Heat vs Finals), rank, gender, and weight class are all found to affect pacing profiles. **Conclusion:** We use an objective methodology with more granular data to identify four pacing strategies. We identify important associations between these pacing profiles and race factors. Finally, we make the full data set public to further rowing research and to replicate our results.

Keywords: Pacing profiles, rowing, time series clustering

1. Introduction

Across “closed-loop” design sports, competitions where athlete(s) attempt to complete a set distance in the shortest time (Abbiss and Laursen, 2008), different pacing strategies have been identified. Most of these pacing strategies have been defined in running and cycling races and attempts have been made to define these strategies in 2000m rowing (Garland (2005); Kennedy and Bell (2003); Muehlbauer and Melges (2011); Muehlbauer and Melges (2011)). However, these attempts approach the problems in a different manner and come to different conclusions. We attempt to standardize the definition of pacing profiles in rowing by using more granular data than

other studies. The more granular data provides the opportunity to more accurately and objectively classify similar pacing profiles

Determining optimal pacing profiles can be done using ergometric data (Kennedy and Bell, 2003) or by using observational data from actual competitions (Garland (2005); Muehlbauer and Melges (2011); Muehlbauer et al. (2010)).

Kennedy and Bell (2003) used simulated rowing and training results to suggest that there were different optimal race profiles for different genders. They found that a constant pacing profile was optimal for men and an all-out profile was optimal for women. Garland (2005) used observational data from the 2000 Olympics, 2001 World Championship, and 2001 & 2002 British indoor Rowing Championship competitions. The analysis found that when using four time splits measured every 500 metres that men and women show no difference in their observed

*Corresponding author: Dani Chu, Department of Statistics and Actuarial Science, Simon Fraser University, 8888 University Drive, Burnaby BC, Canada V5A1S6, 778-873-8582, E-mail: danic@sfu.ca.

53 pacing strategies. Garland (2005) eliminated races
 54 that showed signs of slowdowns from the analysis.
 55 They did so because they wanted to only include
 56 boats that finished their race in the fastest time pos-
 57 sible. Muehlbauer et al. (2010) and Muehlbauer and
 58 Melges (2011) used the same type of split time data
 59 to model pacing profiles. In 2010 they found that
 60 gender, round of race (whether race was in quali-
 61 fying heat or the final race for the category), size of
 62 boat, coxed, and scull did not affect pacing strate-
 63 gies for the 2008 Olympics. In 2011 they had a
 64 different finding that indicated that round of race
 65 affected pacing profiles in World Championship races
 66 between 2001 and 2009. They performed these anal-
 67 yses by fitting linear quadratic models to the four time
 68 splits.

69 1.1. Types of pacing profiles

70 In other fixed distance cycling and running
 71 races, six pacing profiles have been defined
 72 (Abbiss and Laursen, 2008). The six profiles are
 73 “negative”, “all-out”, “positive”,
 74 “even”, “parabolic-shaped”, and “-
 75 variable pacing”.

76 A negative-split pacing profile is defined by an
 77 increase in speed across splits (which result in smaller
 78 relative split times as the race progresses) and is often
 79 used in middle-distance events (20km cycling for
 80 example). An all-out profile is used when it is believed
 81 that energy reserves are best distributed at the start of
 82 the race. This is commonly found in shorter events
 83 like the 100 metre sprint. A positive pacing profile is
 84 one where the athletes’ speed decreases through each
 85 split in the event. This is often found in swimming
 86 (100-m and 200-m), where the diving start allows
 87 athletes to reach their maximum speed quickly. Even
 88 pacing profiles are categorized by a relatively small
 89 portion of the race spent in the acceleration phase and
 90 the majority of the race at a constant pace.

91 According to Abbiss and Laursen (2008), there
 92 are three pacing sub-strategies for Parabolic-Shaped
 93 pacing profiles. J-Shaped, Reverse J-shaped, and U-
 94 shaped. In general these strategies follow a parabolic
 95 shape where the middle of the race sees the lowest re-
 96 lative speeds. In the U-shaped strategy, the start and
 97 end of the race see the same relative speed. The J-
 98 Shaped strategy has a greater relative speed at the end
 99 of the race while the Reverse J-Shaped profiles have
 100 a greater relative speed at the start of the race. The
 101 last profile mentioned is “Variable Pacing”. It
 102 is a strategy that is used to adapt to changing condi-

103 tions in the race course, like uphill and downhill in
 104 cycling.

105 The classification of pacing profiles has histori-
 106 cally been approached by fitting linear models to
 107 split times (Garland (2005); Muehlbauer and Melges
 108 (2011); Muehlbauer et al. (2010)). We believe that
 109 using more granular data describing a boat’s
 110 speed throughout the race will be able to paint a better
 111 picture of how the boat is performing throughout the
 112 race. We also believe that using a clustering technique
 113 to classify similarly shaped speed curves together will
 114 provide a novel approach to defining pacing profiles.

115 There is a large body of literature in clustering
 116 and the area of longitudinal clustering is growing. In
 117 sports specifically, model-based clustering has been
 118 used to cluster player trajectories in basketball (Miller
 119 and Bornn, 2017), football (Chu et al., 2019), and soc-
 120 cer (Gregory, 2019). The previous works leverage the
 121 flexibility of model-based clustering to work across
 122 multiple dimensions to group similar shapes across
 123 time together. Their probabilistic framework is con-
 124 venient for handling outliers. They also do not require
 125 a fixed specification of shape types allowing the data
 126 to speak for itself. This approach would be novel for
 127 rowing pacing profiles as previous works imposed
 128 structure on the pacing profiles. The previous works
 129 demonstrate that clustering with sufficiently granular
 130 data can help discover the underlying structures of a
 131 given dataset.

132 Longitudinal clustering is an emerging area of
 133 research and has been applied across fields for
 134 shape based clustering problems. McNicholas et
 135 al. (2012) used a model-based clustering approach
 136 that uses mixtures of multivariate t-distributions
 137 with a linear model for the mean and a modified
 138 Cholesky-decomposed covariance structure to clus-
 139 ter gene expressions over time. Additionally, Kumar
 140 and Futschik (2007) used a soft clustering technique
 141 to cluster the shapes of microarray data. Finally,
 142 using UCR time-series datasets (Chen et al., 2015),
 143 to test clustering techniques and improve the clus-
 144 tering techniques that are published, Paparrizos and
 145 Gravano (2016) developed the k -shape clustering
 146 technique for time series data. Good performance on
 147 the UCR time-series datasets is the gold standard for
 148 applied time-series techniques.

149 The objectives of this paper are twofold. Firstly,
 150 objectively determine the different types of racing
 151 strategies that are most frequently employed in row-
 152 ing using more granular data. Secondly, investigate
 153 how the strategies were used in different race scenar-
 154 ios.

Table 1
Number of Boats from each World Championship

Year	Men	Women	Total
2010	498	250	748
2011	911	467	1378
2012	765	424	1189
2013	759	387	1146
2014	776	466	1242
2015	1112	632	1744
2016	235	112	347
2017	772	383	1155

2. Methods

2.1. Athletes and event

We gathered GPS data from www.worldrowing.com for the average speed and stroke rate (SR) at each 50 metre split for each boat in every race of the Rowing World Championships from 2010-2017 (the years which were available when we collected data). This includes both lightweight and open races, men, women, and mixed-gender races, boat category, and all other race descriptors. Additionally, data was extracted that described the boats. We also collected finishing place, and lane data. For example, the discipline of the race is important as it is a different type of rowing style. Sculling describes a boat where rowers use two oars and Sweep describes boats where rowers have only one oar each. In Table 1 we present the number of boats by year. Note that in 2012 and 2016 only non-Olympic events were held since these were Olympic years.

2.2. Data Analysis

Data was initially filtered to eliminate races with GPS errors where the reported average speed is lower than the true average speed, with an unreported average speed at any of the split measurements (at every 50 metres), average speed less than two metres per second, with boats that received ‘‘Did not Starts’’ or ‘‘Did not Finishes’’ or ‘‘Exclusions’’. We did not consider data from para-rowing races in the analysis. This reduced the number of boat-sâ™ races from 9264 to 8054. To determine pacing profiles raw speeds at each split are often compared to the mean speed of a boat throughout the race Garland (2005). So we define $x_{i,j}$, as the speed at split i

for boat j and normalize to get $y_{i,j}$, where

$$y_{i,j} = \frac{x_{i,j} - \bar{x}_j}{\sigma_j} \quad (1)$$

By normalizing the speed we can compare the pacing profile of different boats while accounting for the difference in speeds. Clustering was used to group speed curves of similar shape together. In k -shape clustering a new distance method, called ‘‘Shape-based distance (SBD)’’, and a new method for computing centroids are used. When SBD is evaluated against other distance metrics such as Dynamic Time Warping, it reaches similar error rates on the UCR datasets but with shorter computation times. The k -shape algorithm is implemented in the dtwclust package (Sarda-Espinosa, 2018). In its implementation it normalizes the columns to the same scale. So it takes $y_{i,j}$ defined in Equation (1) and transforms it into $z_{i,j}$ defined as

$$z_{i,j} = \frac{y_{i,j} - \bar{y}_j}{\sigma_j}$$

A k -Shape algorithm therefore functions very similarly to the k -means algorithm (Lloyd, 1982) in that the method uses iteratively defined clusters to minimize within-cluster distance.

We fit a multinomial logistic regression with pacing profile as a dependent variable on the boat size, race placement in a heat or final, discipline, gender, and weight class variables. We reported the odds ratio for each variable in the model. An effect is determined to be a statistically significant if the p-value from the Wald z-test is smaller than 0.05 divided by 39 (accounting for multiple comparisons via a Bonferroni correction) (Dunn, 1961).

3. Results

3.1. Comparison of pacing profiles

We performed k -Shape clustering for $k = 3, 4, 5, 6$, and 7 . We found that $k = 4$ gave us the most distinct shapes, created the largest decreases in within group distances from each cluster center and corresponds to the elbow of the often used elbow method heuristic (Thorndike, 1953). The k -Shape clustering algorithm converges, which means there is an iteration of the algorithm where cluster memberships do not change, for our given seed.

To understand the shape of the clusters, we plot the centroids for each cluster in Figure 1. The centroids

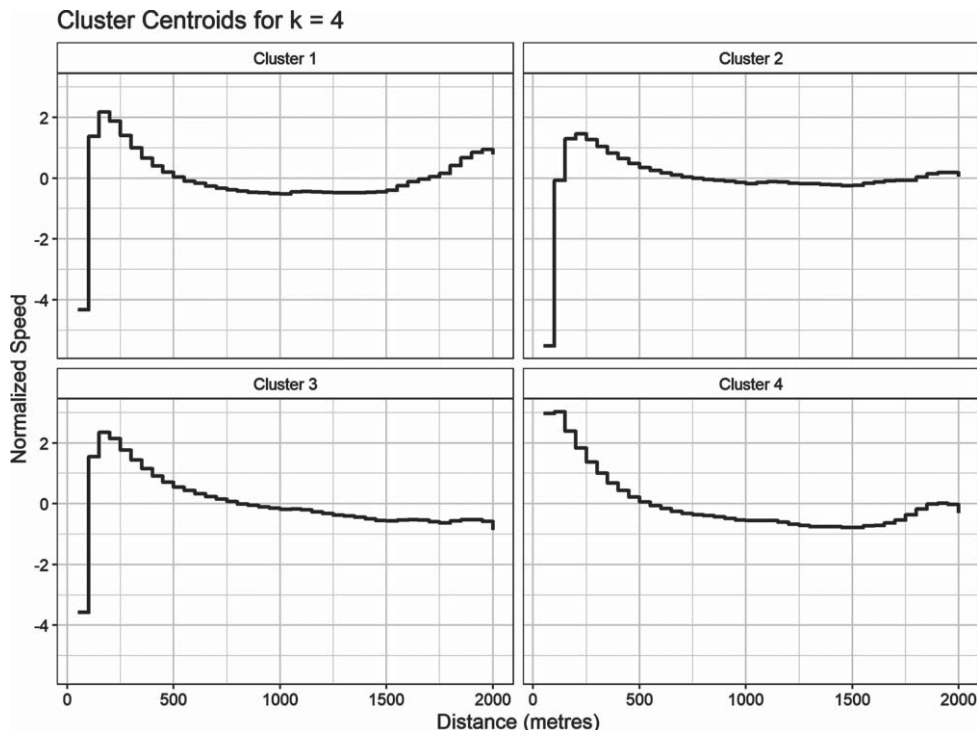


Fig. 1. Cluster Centroids for k -Shape Clustering with 4 Clusters.

201 are similar, as expected in an all-race average; how- 226
 202 ever, there are distinct features that separate them. 227
 203 The centroids are plotted with respect to the normal-
 204 ized speed by race ($y_{i,j}$), in order to identify the shape
 205 of the pacing curve without the effect of magnitude
 206 that size of boat, weight class, and other variables
 207 would affect.

208 We will now name the clusters based on the defini-
 209 tions given by Abbiss and Laursen (2008).

210 **Cluster 1, $n = 1951$** is defined by a slow accel-
 211 eration to a moderate peak velocity, a slow middle
 212 section, and a final sprint that almost reaches peak
 213 velocity. This agrees with the definition of the U-
 214 Shaped pacing profile.

215 **Cluster 2, $n = 2277$** is defined by a slower accel-
 216 eration, a smaller peak velocity, and a low variance
 217 in speed throughout the rest of the race. This agrees
 218 with the definition of the Even pacing profile.

219 **Cluster 3, $n = 2548$** is defined by an acceleration
 220 to top speed in the first 150 metres and a decline in
 221 speed for every proceeding split. This agrees with the
 222 definition of the Positive pacing profile.

223 **Cluster 4, $n = 1444$** is defined by a quick accel-
 224 eration to a higher peak velocity, a slower middle
 225 portion of the event, and finally a faster push to the

finish. This agrees with the definition of the Reverse
 J-Shaped pacing profile.

3.2. Pacing profiles and race factors

228 The results of the multinomial logistic regression
 229 can now help us unpack how race variables impact the
 230 use of each pacing profile discovered during our clus-
 231 tering process. The results of the multinomial logistic
 232 regression are reported in Table 3.

233 To explain how to interpret the table we will use
 234 the boat size variable as an example. There are no
 235 results reported for the “Even” pacing profile as it
 236 is used as our baseline level. Additionally, we used
 237 single sculling boats as the baseline for the boat size
 238 variable, hence the estimates are relative to these cat-
 239 egories.

240 The odds that eights would follow a “Positive” pac-
 241 ing profile over a “Even” pacing profile is 0.03 times
 242 as large as those of a single sculling boats holding all
 243 other variables constant (p -value $< 1e-16$). We can
 244 see that all odds ratios for the eights are less than 1
 245 indicating that eights are more likely to exhibit an
 246 “Even” pacing profile than singles are (all p -values
 247 $< 1e-16$). 248

Table 2

Odds ratio changed by each variable holding all others constant. Statistically significant entries are bolded

	Positive	Reverse J-Shaped	U-Shaped
Intercept	0.8972	0.7654	1.4110
Size: One-person (baseline)	–	–	–
Size: Two-person	0.4795	0.3802	0.6796
Size: Four-person	0.1272	0.1360	0.1607
Size: Eight-person	0.0356	0.0757	0.0354
Round of Race: Final (baseline)	–	–	–
Round of Race: Heat	1.8120	1.037	0.5397
Race Placement: 1st Place (baseline)	–	–	–
Race Placement: 2nd Place	0.8631	1.020	1.2070
Race Placement: 3rd Place	1.0780	1.3260	1.4980
Race Placement: 4th Place	1.3580	1.6040	1.6000
Race Placement: 5th Place	1.7600	1.9280	1.2420
Race Placement: 6th Place	3.1620	3.1600	1.2050
Discipline: Sculling (baseline)	–	–	–
Discipline: Sweep	1.8140	1.2010	1.9660
Gender: Men (baseline)	–	–	–
Gender: Women	1.8830	1.6830	1.6630
Weight Class: Lightweight (baseline)	–	–	–
Weight Class: Open	1.4320	1.5230	1.2840

Holding all other variables constant, the “Positive” pacing profile is nearly 2 times more likely to be used than an “Even” pacing profile in a heat than a final (p-value $2e-16$). The U-shaped profile is nearly 2 times less likely to be used than an “Even” profile in a heat than a final (p-value $<1e-16$).

The pacing profile seems to have an effect on a given boat’s placement in the race. The baseline in this case is boats that came in first place. The question is whether this would affect how the boats would pace themselves. There is no significant difference between pacing profiles chosen by first and second place boats (p-values, Positive: 0.14, Reverse J-Shaped: 0.87, U-Shaped: 0.06). Third place boats have a similar distribution but are more likely to have a “U-Shaped” pacing profile (p-values, Positive: 0.46, Reverse J-Shaped: 0.02, U-Shaped: 0.0001). 4th place boats were more likely to follow both the Reverse J-Shaped and U-Shaped profiles (p-values, Positive: 0.0029, Reverse J-Shaped: 0.000096, U-Shaped: 0.00001). 5th and 6th place boats are significantly more likely to follow the Positive (largest p-value: $7e-8$) and Reverse J-Shaped profiles (largest p-value: $1e-7$).

Rowing is classified into two disciplines, Sculling and Sweep. We see that “Positive” and “U-Shaped” pacing profiles are more likely in Sweep boats than Sculling boats (p-value $9e-14$ and $2e-14$ respectively).

Women were statistically less likely to follow “Even” pacing profiles when accounting for all other variables included in the model. “Positive” pacing

profiles were seen relatively most often for women when compared to men (p-value $<1e-16$).

The “Open” weight class also saw a different distribution of pacing profiles compared to the “Lightweight” class after accounting for the other variables. Holding the other variables constant the “Positive” (p-value: $3e-8$) and “Reverse J-Shaped” (p-value: $3e-8$) pacing profiles were more likely to be used.

4. Discussion

4.1. Type of pacing profiles

The bigger the boat the more likely one was to observe an “Even” pacing profile. This is most likely because it takes a lot of inertia for the bigger boats to get moving. In order for the boat to increase its speed the rowers would need to exert power proportional to the cube of the drag force. Therefore, it is harder for larger boats with more people, such as an eight, to adjust speed mid-race. Put simply, once at a high speed it’s harder for an eight to speed up.

It was noted above that the “Positive” pacing profile is nearly two times more likely to be used in a heat than a final. This would make sense as boats that are in heats are more likely to want to conserve their energy for their future races. Anecdotally, boats will often race the first half of the race as planned and then reassess their effort if they should back off to conserve energy for the next round based on their placing at that

309 moment (Garland, 2005). This behaviour propagates
310 from the slowest boats to the fastest. Once the fastest
311 boats are ahead (usually by some margin) they will
312 react based on the slower boats’s strategy; hence
313 contributing to higher odds of using “Positive”
314 racing strategy. However, slowing down to conserve
315 energy is against the FISA rules (FIS, 2019) so pub-
316 licly speaking about this strategy or overtly slowing
317 downs risks disqualification.

318 We also found that placing in the last three places to
319 be statistically significant to the pacing profile used.
320 One explanation could be that in most races the first
321 three boats are the ones to qualify for the next race. As
322 discussed above, once the placing is secured (espe-
323 cially in heats) you begin to conserve energy. Third
324 and fourth place boats are usually battling for a quali-
325 fying spot. So, the top 2 and bottom 2 boats displayed
326 similar strategies. Unfortunately, looking for an inter-
327 action between the race placement and the round of
328 the race would require more data than we have avail-
329 able, so we leave this for further investigation.

330 Sweep boats were more likely than sculling boats
331 to exhibit “Positive” and “U-Shaped” pacing profiles.
332 This aligns with what we see in the raw race data.
333 Sculling boats are more consistent (smaller average
334 standard deviation of speeds through 500m to 1500m)
335 than their sweep counterparts when comparing boats
336 of the same size (2 sculls against 2 sweeps and 4 sculls
337 against 4 sweeps). The reason for this difference in
338 consistency could be due to the competitiveness of the
339 different disciplines. Sculling races are often thought
340 to have deeper more competitive fields (Good, 2004).
341 If, when going into a race, a boat believes that the field
342 is relatively even they may opt for a more conserva-
343 tive and balanced start (and may exhibit an “Even”
344 or “Reverse J-Shaped” profile). Whereas, if a boat
345 believes that it is outmatched by its competition, it
346 may be more likely to attempt a faster start with
347 a higher chance of fatiguing later in the race, thus
348 exhibiting a “Positive” or “U-Shaped” profile. Both
349 men and boats from the light weight class were asso-
350 ciated with a greater chance of following the “Even”
351 pacing profiles. This conflicts slightly with the find-
352 ings of Garland that there were no differences in
353 pacing profile between men and women (Garland,
354 2005); although, it is important to note that we used a
355 higher resolution of data and different approaches.
356 We are hesitant to conjecture why there are these
357 effects and believe a more in-depth study is needed
358 to determine why we found this association.

359 Finally, in Figure 2 we illustrate some of the strong
360 associations we see between the size of the boats,

361 the round of the race and the pacing profile used.
362 When compared proportionally it is striking to see
363 how much more often the “U-shaped” profile is used
364 in Finals compared to Heats (1.8 times more likely
365 than an “Even profile”, p-value < 1e-16) and how
366 drastically the Positive profile usage drops in Finals
367 compared to Heats (0.5 times as likely as an “Even”
368 profile, p-value 2e-16). It is also easy to see the large
369 difference in frequency for “Even” profiles in large
370 boats. To further show the impact of each variable we
371 plot the expected number of boats using each pacing
372 profile holding size or round constant.

373 For example holding round constant we’d expected
374 to see 500 boats (indicated by the red line) using the
375 “Positive” pacing profiles in single heats. However,
376 we observe nearly 800 of them. A more complex
377 example considers the blue lines which adjusts both
378 size and round to their baseline categories. For exam-
379 ple, if the heats for Eights were the same as finals for
380 Singles we’d expect to see nearly 110 “U-Shaped”
381 pacing profiles in the heats for Eights. However,
382 we’ve estimated that “U-Shaped” usage is nearly half
383 as likely as “Even” usage in heats than finals (p-value
384 < 1e-16) and 0.03 times for Eights compared to Sin-
385 gles (p-value < 1e-16). In reality, we observe 13 boats
386 rather than the 110 expected in that category.

387 It is important to note that we are not inferring
388 any causal relationships between the variables as
389 we are studying observational data. We are at risk
390 to have unmeasured confounders and sampling bias
391 due to variable interactions. Additionally, we do not
392 currently account for the interaction between boats
393 during the race. We are only able to measure the
394 exhibited pacing profile not the desired or intended
395 pacing profile. These are all areas for improvement
396 and future research.

397 5. Conclusion

398 Our approach makes an important contribution
399 to the current literature. We provide an objective,
400 data-driven approach to quantifying racing strategy.
401 Previous analyses have been done through a sub-
402 jective quantification. This approach uses a complex
403 time series clustering method to characterize racing
404 strategies. With these clusters, we developed a model
405 which allows inference to be made about these racing
406 strategies in relation to other factors present during a
407 race. The granularity of the data we provide is what
408 allows the methods we have presented to make accu-
409 rate classifications. Furthermore, the granular data

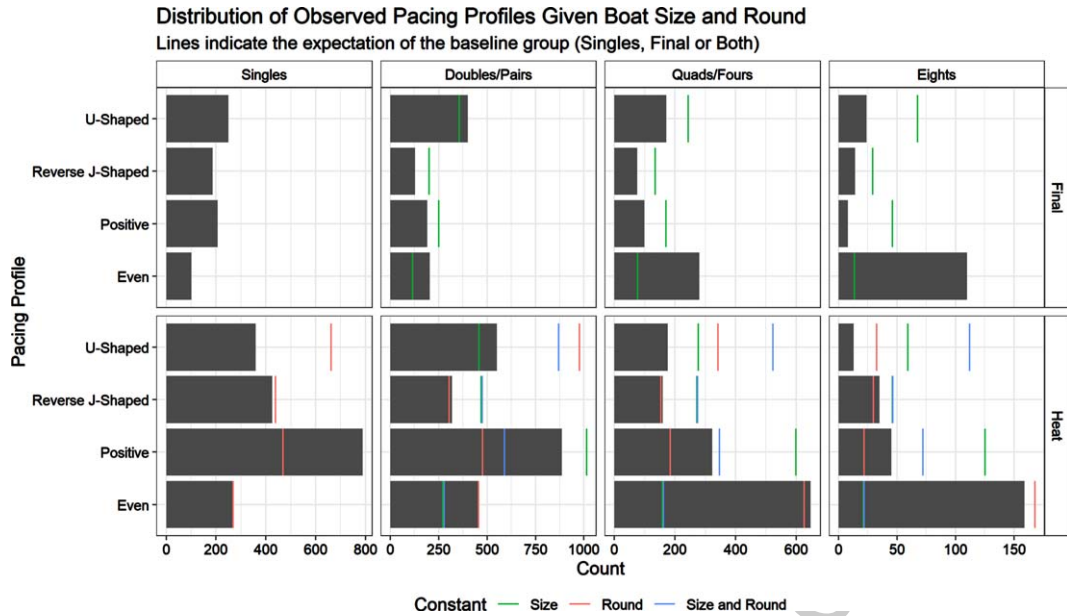


Fig. 2. Distribution of Observed Pacing Profiles Given Boat Size and Round. Coloured lines indicate the expectation of the baseline group.

410 collected has been made available to the public so
 411 that future analyses may be performed with similar
 412 accuracy.

413 **Acknowledgments**

414 We would like to thank Dr. Dave Clarke for
 415 organizing this partnership between Simon Fraser
 416 University and Canadian Sport Institute Pacific,
 417 Chuck Rai for his incredible help with scraping data,
 418 and Lucas Wu and Kevin Floyd for their consulta-
 419 tions on statistics and rowing strategies. We would
 420 also like to thank Ron Yurko, Sam Ventura, Rebecca
 421 Nugent, and the Carnegie Mellon Sports Analytics
 422 Club for hosting the reproducible research competi-
 423 tion that pushed us to make our work reproducible
 424 and available to the public.

425 **References**

426 (2019). *FISA RULE BOOK*. Federation Internationale des Societes
 427 d'Aviron.
 428 Abbiss, C. R. and Laursen, P. B., 2008, Describing and under-
 429 standing pacing strategies during athletic competition, *Sports*
 430 *Medicine (Auckland, N.Z.)*, 38, 239–52.
 431 Chen, Y., Keogh, E., Hu, B., Begum, N., Bagnall, A., Mueen, A.,
 432 and Batista, G. 2015, *The UCR Time Series Classification*
 433 *Archive*.

434 Chu, D., Reyers, M., Thomson, J., and Wu, L. Y., 2019, Route
 435 identification in the national football league, *Journal of Quan-*
 436 *titative Analysis in Sports*, 0(0).
 437 Dunn, O. J., 1961, Multiple comparisons among means, *Journal*
 438 *of the American Statistical Association*, 56(293), 52–64.
 439 Garland, S. W., 2005, An analysis of the pacing strategy adopted by
 440 elite competitors in 2000 m rowing, *British Journal of Sports*
 441 *Medicine*, 39(1), 39–42.
 442 Good, M., 2004, The sculling crisis, *Rowing News*, 11(5), 4251.
 443 Gregory, S., 2019, Ready player run: Off ball run identification
 444 and classification. In *Proceedings of the 2019 Barca Sports*
 445 *Analytics Summit*.
 446 Kennedy, M. D. and Bell, G. J., 2003, Development of race pro-
 447 files for the performance of a simulated 2000-m rowing race,
 448 *Canadian Journal of Applied Physiology*, 28(4), 536–546.
 449 Kumar, L. and Futschik, M., 2007, Kumar l, futschik e.. mfuzz:
 450 a software package for soft clustering of microarray data.
 451 *bioinformatics* 2:5-7. *Bioinformatics*, 2, 5–7.
 452 Lloyd, S. P., 1982, Least squares quantization in pcm, *IEEE Trans-*
 453 *actions on Information Theory*, 28, 129–137.
 454 McNicholas, P., Sanjeena, and Subedi, 2012, Clustering gene
 455 expression time course data using mixtures of multivariate
 456 t-distributions, *Journal of Statistical Planning and Inference*,
 457 142, 1114–1127.
 458 Miller, A. C. and Bornn, L., 2017, Possession sketches : Mapping
 459 nba strategies. In *Proceedings of the 2017 MIT Sloan Sports*
 460 *Analytics Conference*.
 461 Muehlbauer, T. and Melges, T. (2011), Pacing patterns in competi-
 462 tive rowing adopted in different race categories, *The Journal*
 463 *of Strength & Conditioning Research*, 25.
 464 Muehlbauer, T., Schindler, C., and Widmer, A., 2010, Pacing pat-
 465 tern and performance during the 2008 olympic rowing regatta,
 466 *European Journal of Sport Science*, 10(5), 291–296.

- 467 Paparrizos, J. and Gravano, L., 2016, k-shape: Efficient and accurate clustering of time series, *SIGMOD Record*, 45(1), 69–76. 472
- 468 473
- 469 Sarda-Espinosa, A., 2018, *dtwclust: Time Series Clustering Along* 472
- 470 *with Optimizations for the Dynamic Time Warping Distance*, 473
- 471 R package version 5.5.0.

Uncorrected Author Proof