

When to call on an advantageous restart option

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Abstract. In an NCAA wrestling match, a coin toss occurs at the end of the first period. The coin toss winner can elect to begin the second period in a position he views as advantageous, in which case his opponent will determine the third period's starting position, or the coin toss winner can defer his choice of starting position until the third period. This study examines two questions: What is the impact of the coin toss on the ultimate match outcome? Having won the coin toss, is it preferable to select the restart position in the second or the third period? The analysis finds that winning the coin toss does not, in itself, have a significant effect on the ultimate match outcome. However, the decision to defer has a considerable negative effect on the ultimate match outcome for the coin toss winner, with this effect being strongest when the match score is close at the end of the first period. This result is particularly surprising, given that wrestlers do not appear to demonstrate a uniform preference for deferring in close matches.

Keywords: decision making in sports, coin toss effects, wrestling

1. Introduction

In the United States, the National Collegiate Athletic Association (NCAA) sanctions intercollegiate wrestling championships for men. This variety of wrestling (referred to in the United States as *folkstyle*) is similar in many respects to the *freestyle* wrestling discipline of the Olympic Games. Each wrestler attempts to pin his opponent by holding his opponent's back to the wrestling mat, in which case the match is terminated immediately, with the pinning wrestler victorious.

A wrestler scores points throughout the match by performing a prescribed set of techniques (such as taking an opponent down from a standing position and controlling him on the mat). Should a pin not occur within the match's predefined duration (typically, seven minutes of action, divided into three

periods), the wrestler who scores the greater number of points wins the match. If the score is tied at the end of the third period, overtime periods are wrestled until a winner is determined.

Each opponent in the match wears a colored ankle band, either red or green; these notional opponents will hereafter be referred to as the *red wrestler* and *green wrestler*. The referee wears a red wristband on one arm and a green wristband on the other to indicate point scoring, rules violations, etc.

Upon match stoppages, which may occur, for example, at the end of a period, the action resumes in one of two positions:

1. *neutral*, in which both wrestlers are standing facing each other in the center of the wrestling mat,
2. *referee's position*, in which:
 - (a) one wrestler is kneeling in the center of the mat with his hands lying flat on the mat in front of his knees (this wrestler is referred as being on *bottom*); and
 - (b) the other (*top*) wrestler is kneeling behind the bottom wrestler, with the top wrestler

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placing one hand on the elbow of the bottom wrestler, and the top wrestler placing his other arm around the waist of the bottom wrestler.

One key difference between folkstyle and freestyle wrestling is that over the course of a folkstyle wrestling match, each wrestler has an opportunity to select a match restart position that he views as advantageous. The first period of an NCAA Championship wrestling match begins with each wrestler standing in the neutral position. At the end of the first period, the referee flips a disc, which is colored red on one face and green on the other face.

To illustrate this concept, assume that the disc lands red-side up. By virtue of winning the coin toss, the red wrestler must now make a choice among two alternatives. He can elect to begin the second period in a position which he views as advantageous (either *top*, *bottom*, or *neutral*), in which case the green wrestler will determine the starting position for the third period. Alternatively, the red wrestler can give the green wrestler the choice for the second period's starting position, in which case the red wrestler will determine the starting position for the third period (in which case we say that the red wrestler *defers* his choice).

The questions to be addressed in this study are: (1) *What is the impact of the coin toss on the ultimate match outcome?*; and (2) *Having won the coin toss, should a wrestler elect to begin the second period in what he views as an advantageous restart position, or should he defer this advantage for the start of the third period?* We will examine these questions using data from the 2014 NCAA Division I Wrestling Championships (640 total matches).

This study will not examine the implications of specific position choices for the match restart at the beginning of the second and third periods. Such research would be complicated by the fact that, in a given match, each wrestler might view himself as having a different relative advantage versus his opponent; for example, the red wrestler might view himself as being greatly superior to his opponent from the neutral position in one match, but might view himself as having a particular advantage from the top position versus a different opponent. Instead, our focus is on the decision to either take an advantage early and leave a future advantage to an adversary, or to allow an opponent an early advantage and retain a future advantage for oneself.

1.1. Literature review

A number of other sports also present situations in which a coin toss winner is given the option to either select an advantageous position earlier, or defer selection until a later point in the match. In the National Football League (NFL), a coin toss is performed at the beginning of the game. The coin toss winner can choose between (a) kicking off or receiving a kick at the start of the game (in which case the opponent will have the same choice at the beginning of the second half); or (b) kicking off or receiving a kick at the start of the second half (in which case the opponent has the same choice at the beginning of the game). A review of the literature was unable to find any scholarly publications examining the impact of this coin toss, although numerous articles in the popular press [Brustein (2011), Trotter (2012), Lisk (2012), Matuszewski (2014)] have discussed this topic, with Matuszewski noting that, in 2014, the team winning the coin toss chose to defer its choice 65 percent of the time. Trotter reported that, over the interval September 2010 - November 2012, the team winning the coin toss won 325 games and lost 324 games; moreover, coin-toss winning teams electing to defer won 55% of their games, while coin-toss winning teams that did not defer won 47% of their games.

In the event that an American football game ends regulation time with a tied score, a coin toss is again utilized. The structure of the overtime differs between the NFL and NCAA. Prior to the 2010 season, in the NFL, the coin toss winner could elect to either kickoff or receive a kick, with the first team to score winning the game. The rule was slightly changed beginning with the 2010 playoffs, such that if the first possession ends in a field goal, the other team is given possession and an opportunity to match (with a field goal, thus extending the game) or better (with a touchdown, thus ending the game) the field goal. These modified overtime rules were then applied to regular-season games beginning with the 2012 season. Jones (2004) found that, over the period 1974-2002, teams winning the overtime coin toss won 52% of games, with coin-toss losing teams winning 43% of the time, and 5% of games ending in a tie (note that this analysis considered only games played under the rules in which the first team to score in overtime won the game). In NCAA football, each team is given an opportunity to begin with possession of the ball on the opponent's 25-yard line. The coin toss winner can elect to either begin the first overtime on offense or on defense. After each team has had one such possession

opportunity, the overtime period ends, with the team scoring more points in this overtime period the winner. Should the score be tied at the end of this overtime period, a second overtime period is utilized, now the team that began the first overtime period on defense begins the second overtime period on offense; the process continues until a winner is determined. Rosen and Wilson (2007) found that there is some advantage for the coin toss winner to elect to begin the first overtime period on defense, although there are situations in which this strategy is not preferred. Rothhoff (2012) contrasted the impact of these differing overtime structures on NFL and NCAA teams' decision-making late in regulation time, and found that NFL teams are more likely to perform risky plays (ostensibly, in an effort to avoid overtime), whereas NCAA teams do not appear to make significantly different decisions late in a game, when contrasted with those made earlier in the game.

Soccer also uses a coin toss as part of the procedure for determining a winner in the event of a tie game, for matches in which it is necessary to determine a winner, as in tournament play. In such a situation, should the game end with a tie score, a coin toss is utilized to determine the order in which teams will kick during a penalty shoot-out. When penalty shoot-outs were first instituted in 1970, the winner of the coin toss was required to kick first; this rule was changed in June 2003 to give the winner of the coin toss the choice whether to take the first or second shoot-out kick. The teams then continue to alternate penalty kicks in the same order, until each team has had the opportunity for five kicks (or until one team gains an insurmountable advantage); the team with more successful penalty kicks is the match winner. Should the teams have an equal number of successful penalty kicks after five attempts each, they continue to alternate penalty kicks in the same order, until one team has more successful kicks in an equal number of opportunities. Apesteguia and Palacios-Huerta (2010) examined a set of 129 penalty shoot-outs from 1970 to June 2003, and found a statistically significant ($p = 0.017$) advantage for the team kicking first. This study also observed that, for a small sample of post-June 2003 matches, the team winning the coin toss elected to kick first in every case but one. A later study by Kocher, Lenz, and Sutter (2012) reexamined this issue, considering the same time period (1970-2003), but including a broader sample of 540 matches (including all of the matches considered by Apesteguia and Palacios-Huerta (2010)); this paper found a small, but not statistically

significant ($p = 0.15$) advantage for the team kicking first. Kocher et al. (2012) attributed this difference in results to differences in the sampling processes used by the two studies.

Despite wrestling being a very widely-practiced sport, with 180 national wrestling federations represented at United World Wrestling (the sport's international governing body) and inclusion in the International Olympic Games program, the author is not aware of any published econometric analysis addressing decision-making in wrestling.

2. Analyzing binary outcomes: win or lose the match

Our data set consists of all 640 matches competed at the 2014 NCAA Division I Wrestling Championship tournament. Because this study is concerned with the coin toss occurring at the end of the first period, we eliminated from further consideration the 42 matches that were concluded before the end of the first period. In 33 of these 42 matches, a wrestler won by pin in the first period; nine other matches were determined by forfeit, typically due to one of the competitors having been injured in a previous tournament match. We conduct our analysis on the remaining 598 matches.

2.1. *Impact of winning coin toss*

We begin by examining the impact of winning the coin toss on the match outcome. For this question, this study considers a randomized natural experiment. As discussed in Manski (1995), such a situation guarantees internal validity, because the conditions for causal inference are satisfied. Thus, we are confident that these results allow us to attribute differences in the match outcome to the treatment under consideration, namely the results of the coin toss.

The wrestler who won the coin toss ended up winning in 290 matches and losing in 308 matches. However, these simple counts do not account for a particularly important piece of information: the relative scoring differential in which the wrestler who wins the coin toss finds himself at the end of the first period.

Consider the group of wrestlers who were wearing the green ankle band during the match. To examine the impact of the coin toss on the match outcome, we performed a logistic regression using a binary response *Did Green win match?* (=1 if yes, =0 if no); and two predictor variables:

Table 1
Logistic regression results examining impact on binary match outcome “Did Green win match?”

	Coef	SE Coef	Z	P	Odds Ratio	95% CI Lower	95% CI Upper
Green wins coin toss?	-0.261	0.202	-1.29	0.196	0.77	0.52	1.14
Green score differential	0.854	0.080	10.73	0.000	2.35	2.01	2.74
Constant	0.135	0.141	0.95	0.340			

- Binary factor *Did Green win coin toss?* (=1 if yes, =0 if no);
- Match score difference (Green’s score minus Red’s score) at end of first period.

The results from this analysis appear in Table 1. This analysis suggests that the score differential at the end of the first period is extremely unlikely to have no effect on the ultimate match outcome ($p = 0.000$), but that winning the coin toss does not, in itself, have a significant effect on the ultimate match outcome ($p = 0.196$). Moreover, the odds ratio of 0.77 suggests that, holding the Green score differential at a fixed value, the odds of Green winning the match when Green wins the coin toss are 23% lower than the odds when Green loses the coin toss, although at the 95% confidence level this is statistically indistinguishable from the coin toss being associated with no change in the odds of Green winning. The odds ratio of 2.35 suggests that, holding Green’s coin toss result at a fixed value, a one-unit increase in Green’s score differential increases Green’s odds of winning the match by 135%. One may conclude that, having controlled for the match score at the end of the first period, winning the coin toss does not significantly impact a wrestler’s likelihood of winning the match.

2.2. Impact of deferring choice of restart position

It is possible, however, that this lack of an effect for the coin toss outcome may be due to wrestlers making a sub-optimal choice after winning the coin toss. We observed in our data that the coin toss winner elected to *defer* in 283 matches (47.3% of occasions), and to begin the second period in the position of his choice in 315 matches. This suggests, at a minimum, that

wrestlers do not have a uniform preference for deferring. Contrast this with the experience in soccer and the NFL, each of which demonstrate a strong preference (not deferring in the case of soccer, and deferring in the NFL). Wrestlers who did not defer ultimately won in 172 matches and lost in 143 matches, while wrestlers electing to defer ultimately won on 118 occasions and lost in 165 matches. A two-way contingency table analysis reveals an odds ratio of 0.59 for wrestlers who defer, suggesting that the odds of the coin toss winner winning the match when the coin toss winner defers are 41% lower than the odds when the coin toss winner does not defer. But again, these simple counts do not account for the relative scoring differential in which the wrestler who won the coin toss finds himself at the end of the first period.

We examined the impact of the defer/do not defer choice by performing another logistic regression, this time using a binary response *Did coin toss winner win match?* (=1 if yes, =0 if no); and two predictor variables:

- Binary factor *Did coin toss winner defer?* (=1 if yes, =0 if no);
- Match score difference (coin toss winner’s score minus his opponent’s score) at end of first period.

The results from this analysis appear in Table 2. Again, we observe that the score differential at the end of the first period is extremely unlikely to have no effect on the ultimate match outcome ($p = 0.000$). However, we now observe that the decision to defer the choice of restart position has a considerable negative effect on the ultimate match outcome for the coin toss winner, with $p = 0.073$. The odds ratio of 0.69 suggests that, holding the coin toss winner’s score

Table 2
Logistic regression results examining impact on binary match outcome “Did coin toss winner win match?”

	Coef	SE Coef	Z	P	Odds Ratio	95% CI Lower	95% CI Upper
Coin toss winner defers?	-0.364	0.203	-1.79	0.073	0.69	0.47	1.03
Coin toss winner score differential	0.845	0.079	10.66	0.000	2.33	1.99	2.72
Constant	0.039	0.138	0.28	0.777			

differential at a fixed value, the odds of the coin toss winner winning the match when the coin toss winner defers are 31% lower than the odds when the coin toss winner does not defer, although at the 95% confidence level this is statistically indistinguishable from the coin toss winner's choice to defer being associated with no change in the odds of the coin toss winner winning the match. The odds ratio of 2.33 suggests that, holding the coin toss winner's decision to defer or not at a fixed value, a one-unit increase in the coin toss winner's score differential increases the coin toss winner's odds of winning the match by 133%. Said differently, having controlled for the match score at the end of the first period, should the coin toss winner elect to defer his choice of restart position until the beginning of the third period, he has significantly (at the $p = 0.1$ level) reduced his likelihood of winning the match. What is most surprising about this finding is that, as discussed above, wrestlers did not demonstrate a clear preference for not deferring.

A collinearity test was performed to determine if these two predictor variables are independent, this test found tolerance value of 0.9854 (or, equivalently, a Variance Inflation Factor of 1.0148), suggesting that these predictor variables are not significantly correlated with one another. However, to perform additional testing examining the relationship between these predictors, we performed two additional analyses: a difference of means t -test, contrasting the average score differential for those wrestlers who deferred with the average score differential for wrestlers who did not defer; along with a logistic regression, with the defer decision as the binary response and the match score difference as the sole predictor. The t -test found a significant difference, at the $p = 0.01$ level, between the mean score differentials at the end of the first period for the wrestlers who did defer (who, on average, were losing by 0.25 points) and those who did not defer (who, on average, were winning by 0.37 points). The logistic regression analysis found an odds ratio of 0.91, suggesting that a one-unit increase in the coin toss winner's score differential decreases the coin toss winner's odds of deferring by 9%. Because the 95% confidence interval for this odds ratio was [0.85, 0.97], which does not include the value 1, this effect is statistically significant at the $p = 0.05$ level. The evidence of dependence between the coin toss winner's choice to defer and the score differential potentially confounds the interpretation of the logistic regression explaining the coin toss winner's odds of winning the match and

the coin toss winner's choice to defer and the score differential.

An analysis of this question is further complicated by the fact that the coin toss winner's decision is potentially confounded with other unmeasured factors that are not captured in the score differential. Said differently, it is possible that there are other, unobserved, factors that differentiate between two wrestlers that, each having won the coin toss with an identical score differential, would make one more likely to win the match when deferring and the other more likely to win the match by not deferring. Examples of such potential confounding factors include the relative stamina and fatigue of the opponents, or perhaps events that occurred in the first period that did not result in points being scored but demonstrated a relative advantage (such as physical strength) for one wrestler.

However, consider the practical implications of such potential confounding factors. We have observed that the coin toss winner's odds of winning the match are 31% lower when he defers, and yet 47% of coin toss winners defer. Were all wrestlers homogeneous, it would be clear that any wrestler who defers is not making a rational choice. However, were there some heterogeneity in the population of wrestlers, as in the case of confounding factors, with some wrestlers truly increasing their likelihood of winning by electing to defer, then the odds of winning must be even worse for all other wrestlers who defer, making their decision even more ill-founded. This suggests that the possibility of confounding factors only strengthens the finding that many wrestlers are making sub-optimal decisions.

It is possible that a wrestler who defers his choice to the beginning of the third period may not even have the opportunity to make this choice because the match could end during the second period. In our data set, 36 of the 598 matches ended during the second period, 31 due to fall and 5 due to *technical fall* (match termination which occurs when one wrestler has earned a 15-point advantage over his opponent). A wrestler could potentially modify his deferral strategy in light of this fact by not deferring, if he thought that he might be able to achieve a victory by either fall or technical fall in the second period, and thus deny his opponent the opportunity to select the match restart position. And in fact, out of this set of 36 matches, there were 4 matches in which the coin toss winner was winning by 5 or more points at the end of the first period, in each case the coin toss winner did not defer (although, somewhat surprisingly, one of these

was an instance in which the coin toss winner lost the match by fall in the second period).

In order to strengthen our confidence in the finding that a decision to defer reduces a wrestler’s likelihood of winning the match, a second analysis was performed. In this analysis, the outcome of a wrestling match was modeled as a 1-step stochastic process, with the single transition occurring between the end of the first period and the end of the match. To simplify the language for the remainder of this paper, we will refer to the wrestler who won the coin toss as the *red wrestler*, irrespective of which color ankle band this wrestler was actually wearing during the match. Define the following states for the match status at the end of the first period:

- *losing big*, denoted Lbig: when the red wrestler is losing by at least 6, but not more than 14, points at the end of the first period; **16 matches were in this state**;
- *losing medium*, denoted Lmed: when the red wrestler is losing by at least 3, but not more than 5, points at the end of the first period; **54 matches were in this state**;
- *close*, denoted Close: when, at the end of the first period, the red wrestler is either losing by no more than 2 points, tied with his opponent, or winning by no more than 2 points; **460 matches were in this state**;
- *winning medium*, denoted Wmed: when the red wrestler is winning by at least 3, but not more than 5, points at the end of the first period; **54 matches were in this state**;
- *winning big*, denoted Wbig: when the red wrestler is winning by at least 6, but not more than 14, points at the end of the first period; **14 matches were in this state**.

Observe that the vast majority of matches were in the Close state at the end of the first period. This is not surprising because the NCAA Championship tournament includes only those wrestlers who were successful at qualifying tournaments or awarded an at-large bid; the intent is that the entrants in this tournament are the best collegiate wrestlers in the United States, and thus we should expect that the contestants in each match are generally of comparable skill. Because a match is terminated when one wrestler gains a 15 point advantage over his opponent, our state definitions do not need to consider any cases in which the score differential is greater than 14 points at the end of the first period (further, there were no such matches in our data set).

We are interested in determining whether the decision to either select the match restart position at the beginning of the second period or to defer this choice until the start of the third period has a significant effect on the match result. Define p_{ij} as the probability that a wrestler in state i at the end of the first period transitions to state j at the end of the match, where j can be one of two alternatives: win the match or lose the match. It is possible that the transition probability between states i and j depends upon the decision of the red wrestler to either select the restart position at start of second period or to defer selection of the restart position until start of third period. Let $p_{ij}(k)$ denote the probability that a red wrestler in state i at the end of the first period transitions to state j at the end of the match, having made decision k , where $k = 2$ denotes selecting restart position at the start of the second period, $k = 3$ denotes deferring selection of restart position until the start of the third period.

We can use our data set to compute estimates of these transition probabilities. Define $n_{ij}(k)$ as the number of transitions between states i and j for the red wrestler’s decision k . The estimates $\hat{p}_{ij}(k)$ of these transition probabilities $p_{ij}(k)$ are:

$$\hat{p}_{ij}(k) = \frac{n_{ij}(k)}{\sum_j n_{ij}(k)} \tag{1}$$

$$\hat{p}_{ij} = \frac{\sum_k n_{ij}(k)}{\sum_{j,k} n_{ij}(k)}. \tag{2}$$

Tables 3 and 4 present these estimates for the winner of the coin toss, as a function of his decision to select the match restart position at the start of the second period (does not defer) or at the start of the third period (defers), dependent on state i (score differential state at the end of the first period).

The test for transition probability matrices provided in Anderson and Goodman (1957) can be used to determine whether the decision to either select the match restart position at the beginning of the second period or to defer this choice until the start of

Table 3

Match result for winner of coin toss, when selecting restart position at start of second period ($k = 2$)

$i \backslash j$	$n_{ij}(k = 2)$		$p_{ij}(k = 2)$	
	Red win	Red loss	Red win	Red loss
Lbig	0	6	0	1
Lmed	1	24	0.040	0.960
Close	135	112	0.547	0.453
Wmed	26	0	1	0
Wbig	10	1	0.909	0.091

Table 4

Match result for winner of coin toss, when deferring selection of restart position until start of third period ($k = 3$)

$i \backslash j$	$n_{ij}(k = 3)$		$p_{ij}(k = 3)$	
	Red win	Red loss	Red win	Red loss
Lbig	0	10	0	1
Lmed	3	26	0.103	0.897
Close	87	126	0.408	0.592
Wmed	25	3	0.893	0.107
Wbig	3	0	1	0

Table 5

Likelihood ratio test statistics for binary “win or lose” outcomes

i	λ_i	probability
Lbig	1	1
Lmed	0.660	0.362
Close	0.012	0.003
Wmed	0.128	0.043
Wbig	0.778	0.478

the third period has a significant effect on the match result. The null hypothesis is that, for each (i, j) pair, $p_{ij}(k) = p_{ij} \quad \forall k$. The likelihood ratio criterion for each initial state i can be computed as:

$$\lambda_i = \prod_{j,k} \left[\frac{\hat{p}_{ij}}{\hat{p}_{ij}(k)} \right]^{n_{ij}(k)} \quad (3)$$

When the null hypothesis is true, the distribution of $-2\ln(\lambda_i)$ is chi-squared with $(J - 1) * (K - 1) = 1$ degree of freedom, where J represents the total number of possible states at the end of the match and K represents the total number of possible decisions.

Table 5 presents these test statistics and corresponding p-values for each state i at the end of the first period, comparing Tables 3 and 4. Observe that for the state Close, the probability associated with this test statistic is less than 0.01, allowing us to reject the null hypothesis at the 1% level and conclude that for this state, the likelihood of match victory is significantly impacted by the decision to defer or not. Given that our estimates for this set of states ($i = \text{Close}$ and $j = \text{Red Win}$) are $p_{ij}(2) = 0.547$ and $p_{ij}(3) = 0.408$, we conclude that the red wrestler enjoys a statistically significant increase in his likelihood of victory in this state by not deferring and selecting the match restart position at the start of the second period. For state Close, an analysis of the two-way contingency table reveals an odds ratio of 0.57 for wrestlers who defer, suggesting that the odds of the red wrestler winning the match when he defers are 43% lower than the odds when the red wrestler does not defer. State Wmed

shows a similar relationship, with statistical significance at the 5% level, of a benefit to the red wrestler by not deferring. Note that there are zero observations in which a wrestler in state Wmed elected to not defer and subsequently lost the match, preventing the calculation of an odds value for this state. For state Lmed, the data show a slight advantage to the red wrestler if he defers his choice until the start of the third period, but this difference is not statistically significant. For state Wbig, only one wrestler lost the match, this being the aforementioned wrestler who lost by fall in the second period. For state Lbig, the red wrestler lost every match, regardless of his choice to defer or not.

Taken together, the logistic regression analysis and transition probability matrix analysis suggest that, having accounted for the match score differential at the end of the first period, the wrestler who wins the coin toss can improve his likelihood of winning the match by not deferring and selecting the match restart position at the start of the second period, with this effect being strongest for Close matches (absolute score differential no greater than two points at the end of the first period).

3. Analyzing margin of victory considerations

In addition to the crowning of individual champions at each of 10 different weight classes (each wrestler competes in an individual tournament in his own weight class), the NCAA Wrestling Championships also identify a team champion, based on the total team points scored by each competing university’s wrestlers. A wrestler earns points for his team by winning a match (which is worth 1 team point for matches in the championship bracket, 0.5 team points for matches in the consolation bracket of wrestlers who have already lost in the championship bracket). The winning wrestler can earn additional bonus points for his team as follows:

- 2 additional points earned for pinning one’s opponent, referred to as winning by *fall*;
- 1.5 additional points earned for winning by *technical fall*, which occurs when one wrestler has earned a 15-point advantage over his opponent;
- 1 additional point earned for winning by *major decision*, which occurs when the winning wrestler finishes the match with an advantage greater than 7 points but less than 15 points over his opponent.

Table 6

Margin of victory match result for winner of coin toss, when selecting restart position at start of second period, $p_{ij}(k = 2)$

$i \backslash j$	LBF	LBTF	LBMD	LBD	WBD	WBMD	WBTF	WBF
Lbig	0.167	0.167	0.500	0.167	0	0	0	0
Lmed	0.120	0.000	0.240	0.600	0.040	0	0	0
Close	0.040	0	0.028	0.385	0.470	0.053	0.004	0.020
Wmed	0	0	0	0	0.308	0.577	0	0.115
Wbig	0.091	0	0	0	0	0.636	0.182	0.091

One might assume that, having gained nearly-certain victory (or defeat), a wrestler would turn his focus to attempting to score (or avoid surrendering) bonus points for his team. In a manner similar to the analyses employed in Section 1, we can test the hypothesis that the red wrestler’s decision to defer or not impacts the expected number of team points he will score in a match.

Define the following states at the end of the match:

- *lose by fall*, denoted LBF, surrendering two bonus points to the opponent;
- *lose by technical fall*, denoted LBTF, surrendering 1.5 bonus points to the opponent;
- *lose by major decision*, denoted LBMD, surrendering one bonus point to the opponent;
- *lose by decision*, denoted LBD, surrendering no bonus points to the opponent;
- *win by decision*, denoted WBD, gaining no bonus points;
- *win by major decision*, denoted WBMD, gaining one bonus point;
- *win by technical fall*, denoted WBTF, gaining 1.5 bonus points;
- *win by fall*, denoted WBF, gaining two bonus points.

Tables 6 and 7 present the state transition probability estimates for the winner of the coin toss, as a function of his decision to select the match restart position at the start of the second period (does not defer) or at the start of the third period (defers), obtained as before using equation (1).

We again employ the test statistic of Anderson and Goodman (1957) to determine whether the decision to either select the match restart position at the

beginning of the second period or to defer this choice until the start of the third period has a significant effect on the match result, now differentiating between results based on their accrual of team bonus points. Table 8 presents these test statistics and corresponding p-values for each state i at the end of the first period, comparing Tables 6 and 7. Observe that for state Close, the probability associated with this test statistic is less than 0.01, allowing us to reject the null hypothesis at the 1% level and conclude that for this state, the likelihood of increasing the earned number of team bonus points is significantly impacted by the decision to defer or not. State Wmed is significant at the 10% level, suggesting that the decision to defer or not is somewhat significant here—much less significant than when in the state Close, but more significant than when in any other state.

The transition probability estimates in Tables 6 and 7 can be used to compute the red wrestler’s expected number of team points scored, given his decision to defer or not at the start of the second period, for each end-of-first period state i , denoted $E(i, k)$. For these calculations, we will assume that the winner of the match scores one team point (as in the championship bracket, as noted above), in addition to any team bonus points earned. Table 9 presents these calculations, along with the percent difference in expected team points scored by not deferring, relative to the expected team points scored by deferring.

Observe that the wrestler winning the coin toss can increase the expected number of team points earned if he elects to not defer, and instead choose the restart position at the beginning of the second period, in each of the states Close and Wmed identified above as statistically significant. When a wrestler

Table 7

Margin of victory match result for winner of coin toss, when deferring selection of restart position until start of third period, $p_{ij}(k = 3)$

$i \backslash j$	LBF	LBTF	LBMD	LBD	WBD	WBMD	WBTF	WBF
Lbig	0.100	0.400	0.200	0.300	0	0	0	0
Lmed	0.103	0.034	0.241	0.517	0.103	0	0	0
Close	0.028	0.005	0.070	0.488	0.343	0.019	0.005	0.042
Wmed	0.036	0	0	0.071	0.393	0.286	0.143	0.071
Wbig	0	0	0	0	0	0.667	0.333	0

Table 8

Likelihood ratio test statistics for margin of victory outcomes

i	λ_i	probability
Lbig	0.339	0.950
Lmed	0.331	0.947
Close	6.41×10^{-5}	0.007
Wmed	0.002	0.086
Wbig	0.550	0.991

Table 9

Expected team points scored, as a function of the coin toss winner's decision at the end of the first period

i	$E(i, k = 2)$	$E(i, k = 3)$	$\frac{E(i, k=2) - E(i, k=3)}{ E(i, k=3) }$
Lbig	-2.083	-2.000	-4.2%
Lmed	-1.400	-1.293	-8.3%
Close	0.083	-0.207	140.2%
Wmed	1.808	1.357	33.2%
Wbig	1.727	2.167	-20.3%

finds himself in a close match, not deferring increases his expected team points by 0.29 (a 140% increase). A wrestler who ends the first period winning by a medium amount (state Wmed), would increase his expected team points by 0.45 (a 33% increase) by not deferring. Moreover, given our previous observation that the team-scoring related transition probabilities for the wrestler winning the coin toss demonstrate a statistically significant difference for the defer or not decision when this wrestler finds himself in either a close match (significant at the 1% level) or when winning by a medium amount (significant at the 10% level), we conclude that not deferring does in fact appear to be the superior decision for the wrestler winning the coin toss when in either of these two states at the end of the first period.

Observe that the red wrestler earns fewer team points if he elects not to defer when in any of the other three states at the end of the first period. However, these percent differences are small, and because we previously observed that there was no statistically significant difference in the team-scoring related transition probabilities for the red wrestler in each of these states, we conclude that our data provide no support for the superiority of either deferring or not in the states Lbig, Lmed and Wbig.

4. Conclusions and further research

This paper presents the first academic study of decision making in wrestling matches. In an American intercollegiate wrestling match, one of the

competitors faces an interesting decision at the end of the first period. Having won a coin toss, should he elect to begin the second period in a position which he views as advantageous, in which case his opponent will determine the starting position for the third period? Or should the wrestler who won the coin toss (referred to here as the red wrestler) give his opponent (the green wrestler) the choice of restart position for the second period, and retain his advantage for the beginning of third period?

We consider two questions in this study. First, whether the outcome of the coin toss has an effect on the match outcome. For this question, this study considers a randomized natural experiment. We examined this question using a logistic regression, and found that, having controlled for the match score differential at the end of the first period, winning the coin toss is not, in itself, a likely predictor of the ultimate match outcome ($p = 0.196$).

An arguably more-interesting line of questioning is to examine the impact of the defer/do not defer choice. Considering the choice made by the coin toss winner, we find that in the event that the red wrestler is, at the end of the first period, either losing by three or more points or winning by six or more points, the decision to defer or not does not have a statistically significant impact on either the binary outcome of winning or losing the match, or on the set of outcomes that considers the margin of victory differences that impact his team's score. However, should the red wrestler find himself either losing by two or fewer points, tied, or winning by two or fewer points at the end of the first period (a set of conditions defining the state Close), the decision to defer or not has a statistically significant impact on both the binary outcome of winning or losing the match (significant at the 1% level) and on the set of outcomes that considers margin of victory (significant at the 1% level). If the red wrestler finds himself winning by a margin of between three and five points at the end of the first period (defining the state Wmed), the decision to defer or not has a statistically significant, but weaker, impact on both the binary outcome (significant at the 5% level) and on the set of outcomes that considers margin of victory (significant at the 10% level).

In both state Close and state Wmed, the superior decision is to not defer, and instead decide on the restart position at the start of the second period. For a match that is in the Close state at the end of the first period, the red wrestler can increase his expected team points scored from -0.207 if he defers to 0.083

if he does not defer. For a match that is in the Wmed state at the end of the first period, the red wrestler can increase his expected team points from 1.357 if he defers to 1.808 if he does not defer.

In each of the states Close and Wmed, it appears that, having gained the match lead in the first period, the red wrestler should immediately attempt to extend his advantage, and increase his control of the match. In these states, a strategy of retaining the red wrestler's advantage for the latter part of the match by deferring his choice appears to, counterproductively, allow the green wrestler to take control of the match and perhaps wrest momentum away from the red wrestler, with the red wrestler's advantage at the start of the third period not compensating for his loss of match control. These effects appear to be stronger for situations in which a binary (win/lose) outcome are considered, and weaker when considering the effect on margin of victory.

This finding of a benefit to not deferring in close matches is particularly surprising, given that wrestlers do not appear to demonstrate a uniform preference for deferring, with the coin toss winner electing to defer in 46.3% of close matches. This is dissimilar from the experience of soccer and the NFL, with a strong preference for not deferring in soccer, and a preference for deferring in NFL overtime football games.

A potential limitation to this analysis, as discussed earlier, is the fact that the coin toss winner's decision is not randomized and is potentially confounded with other unmeasured factors that are not captured in the score differential. For example, the choice to defer in a close match could be related to a wrestler's ability or experience. Given the lack of data on individual wrestlers in our data set, we were unable to test for such effects. However, as discussed in Section 2.2, the possibility of confounding factors in this analysis only strengthens the finding that many wrestlers are making sub-optimal decisions. Subsequent research in this area is needed to attempt to identify potential confounding factors and to give a more complete examination of the impact of the coin toss winner's decision, including the psychological factors that influence decision making in wrestling.

Further research could also address the implications of specific position choices for the match restart at the beginning of the second and third periods. Recall that the wrestler making this choice can elect to begin in either the neutral, top, or bottom position. As discussed above, such research would be

complicated by the fact that, in a given match, each wrestler might view himself as having a different relative advantage versus his opponent. For example, the red wrestler might view himself as being greatly superior to his opponent from the neutral position in one match, but might view himself as having a particular advantage from the top position versus a different opponent. Nonetheless, there is considerable similarity across matches with respect to the actual restart position choices made by each wrestler in our dataset. The wrestler who selected the restart position at the start of the second period (irrespective of whether he won or lost the coin toss) chose to begin the second period in the neutral position on 43 occasions, in the top position on 22 occasions, and in the bottom position on 533 occasions. Out of the 562 matches that continued through the end of the second period, neutral was chosen 126 times, top was chosen 24 times, bottom was chosen 412 times. In the overwhelming majority of cases, the wrestler selecting the restart position chose the bottom position. This can be explained by the general impression among wrestlers that, without accounting for the specific relative strengths and weaknesses of individual wrestlers, it is typically easier to score match points from the bottom position (in particular, the one point that can be scored by *escaping* from the bottom to a neutral position).

This research could potentially be used as a starting point for an examination of other, business-related, situations in which a competitor has the choice to either call on an advantageous restart position now, or defer the call until a future time, when his opponent's actions have announced their intentions and reduced their flexibility. This analysis suggests that different effects might be expected for participants in a competition with binary outcomes (such as deciding when to use negative advertising in an election) versus those observed for a situation in which margin of difference effects are important (such as deciding when to release an upgraded product in an oligopoly where one is concerned with market capture). Given relatively equal competitors, this analysis suggests a greater benefit for "pressing one's advantage" and selecting an advantageous position sooner, rather than "taking the opponent's best shot" and retaining the advantage for a later time in the future, in particular for binary outcomes. Further research providing a comprehensive examination of coin toss strategies across multiple sports could provide insights into the relationship between the structure of a competition and the best decision.

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