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A new extension of fuzzy decision by opinion score method based on Fermatean fuzzy: A benchmarking COVID-19 machine learning methods

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Abstract. To date, for the purpose of solving the complex problems in the area of expert system, Multi criteria decision making is the best technique to offer the suitable solution. In the academic literature, the MCDM methods suffered from many challenges. The most important challenges are uncertainty and vagueness. One of the latest MCDM method, called the fuzzy decision by opinion score method (FDOSM). However, there are still some vagueness issues around these methods (mention some of them). According to the advantage of the Fermatean fuzzy set in solving these issues, in this research extends FDOSM into Fermatean-FDOSM so as to effectively benchmark the real-life problem. In this study, we present our methodology in two phases. The first phase presents the mathematical model of Fermatean-FDOSM which is composed of three stages of FDOSM. The second phase applied the new extension to benchmark the COVID-19 machine learning methods. The finding of Fermatean-FDOSM after comparing the result with the basic FDSOM and TOPSIS, is more logical and undergoing a systematic ranking. In the validation process, objective validation is applied to validate the final result of Fermatean-FDOSM. The result of Fermatean-FDOSM is valid, and more logical and in line with decision makers' opinions.

Keywords: Fuzzy decision by opinion score method (FDOSM), machine learning, Fermatean fuzzy, COVID-19, multi-criteria decision-making

1. Introduction

Fundamentally, MCDM is a Decision making is a reasoning approach of ordering available preferences so as to select the highly desirable. The decision making process aims to achieve the most desirable objectives which have the least expected penalties [1]. With the presence of uncertainty, insufficient knowledge, and cases that involve multiple criteria consideration, decision making often becomes more complex. MCDM is the most popular branch of decision making [2]. MCDM refers to decision making in the presence of multiple objectives or attributes[3]. MCDM approach is often used with a view to

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tackle multiple selection problems and/or decision making.

The main aim of MCDM is helping decision makers to nominate the better alternative and rank them based on their effectiveness by sorting the alternatives of the available choices. To do the ranking process, different options needs to be considered to sort the alternatives and select the best of them [4]. Any MCDM problem is generally shown as a matrix, as follows:

$$DM = \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{bmatrix} x_{11} & x_{12} & \ldots & x_{1n} \\ x_{21} & x_{22} & \ldots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \ldots & x_{mn} \end{bmatrix}$$

The above m x n two-dimensional matrix consist of m row and n columns; where the row A1, A2,... Am represents the alternatives and the columns C_1, C_2, ... C_n represents the criteria. The DM matrix ranking the A1 alternative for each Cj criterion [5–7]. Basically, this requires an evaluation and assessments process with respect to the quantitative and/or qualitative analysis by experts (decision makers) to find the best alternative with respect to each criterion. The objective function considers all the criterions with respect to each alternative in the DM matrix by using complex mathematical computations[8]. In addition, single or group decision makers considered in this approach to evaluate the alternatives subjectively for numerous performance with respect to each criteria [9, 10].

Fundamentally, in the MCDM technique there two methods namely, the mathematical approach and the human approach. Where the first one uses the mathematical equations such as Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method) [4, 11].while the second methods considers human preferences in their computations such as Analytic Hierarchy Process (AHP) method [4, 11–14]. Each approach suffered from different issue like: in mathematical approach (i.e. normalization [15, 16], criteria weight [17], and distance measurement [18, 19]). On the other hand, the human approach suffered from the main challenge (i.e. inconsistency ratio [20–23]). The MCDM methods (i.e. mathematical and human approaches) suffered from another challenge is the uncertainty and the vague information. Decision makers (experts) are unable to determine the weight in real number as they utilize the linguistic terms. Consequently, the problems, including this information, become more challenging. Many researchers have addressed this challenge [24–27]. In academic literature, many research recommended to use the fuzzy set numbers so as to handle the problem of the uncertainty and vague information [28–32]. In 2020, fuzzy decision by opinion score method (FDOSM) was proposed as a promising method to solve the abovementioned issues [33]. Similar to other MCDM methods, FDOSM deals with the problem of assisting the decision maker in order to nominate the most promising alternative considering several criteria that depend on the opinion of the decision maker [33]Valls, 2000 #1922. In academic literature, many researchers were extending FDOSM into new fuzzy set number or applied some experiments between the aggregation operators. In [34] the authors applied different scenarios for the basic form of FDOSM and doing a comparatives between different aggregation operators. Also, in [35] the authors extended FDOSM into q-rung orthopair fuzzy and integrate FDOSM with another method (i.e. fuzzy weighted zero inconsistency. In [36], the authors extended FDOSM into Pythagorean fuzzy. Another article [37], the authors extended FDOSM into T-spherical fuzzy numbers. The authors in [30] extended FDOSM into interval type-2 trapezoidal fuzzy. And in [38] the authors extended FDOSM into intuitionistic fuzzy. Another article [39] the authors extend FDOSM using Pythagorean fuzzy numbers. Finally, [40] extended FDOSM using interval-valued Pythagorean fuzzy numbers. In the literature, many researchers mention the benefit of the intuitionistic fuzzy and the extenuations of the intuitionistic (i.e. Pythagorean fuzzy and Fermatean fuzzy)[41, 42]. From the above academic literature, there is no article present the extension of FDOSM using fermatean fuzzy. So, in this paper will extend FDOSM into fermatean-FDOSM, and applied the new extension on covid-19 machine learning case study.

2. Some basic fermatean definitions

Fermatean fuzzy set was proposed by Senapati and Yager in 2020 [43], to handle the uncertainty and vague information easily. Not to mention, the Fermatean fuzzy set is a new type of fuzzy set. Some researchers used Fermatean fuzzy set with MCDM to solve the uncertainty issue and recommended to use it [44]. The Fermatean fuzzy set was derived from the intuitionistic fuzzy sets and Pythagorean fuzzy sets, however the Fermatean fuzzy set is more flexible to handle the uncertainty problem than intuitionistic fuzzy sets and Pythagorean fuzzy sets [44]. Fermatean fuzzy set has three important components (i.e. membership degree, non-membership degree, and indeterminacy). Some basic definitions and operators Fermatean fuzzy set are explained in the following.

Definition 1. let X be a universe of discourse. A fermatean fuzzy set F in X is

$$F = \{\langle x, \alpha_F(x), \beta_F(x)\rangle : x \in X\}, \tag{1}$$

where $\alpha_F(x) : X \to [0, 1]$ and $\beta_F(x) : X \to [0, 1]$, including the condition $0 \leq (\alpha_F(x))^3 + (\beta_F(x))^3 \leq 1$, for all $x \in X$. The number of $\alpha_F(x)$ represented the membership degree, and the number of $\beta_F(x)$ represented the non-membership degree of the element x in the set F. The third component is the degree of indeterminacy is $\pi_F(x) = \sqrt[3]{1 - (\alpha_F(x))^3 - (\beta_F(x))^3)}$. In Fig. 1, can see clearly the differences between the Fermatean Fuzzy set (FFs), Intuitionistic Fuzzy sets (IFs), and Pythagorean Fuzzy sets (PFs) in spaces. This difference provides the FFs more flexibility to address the uncertainty problem.

Definition 2. Let $\tilde{A} = (\alpha_A, \beta_A)$ and $\tilde{B} = (\alpha_B, \beta_B)$ two FFs, and ∂ a positive real number ($\partial > 0$). Than can be define the following operators for FFs [43, 44]:

$$\tilde{A} + \tilde{B} = \left(\sqrt[3]{\alpha_A^3 + \alpha_B^3 - \alpha_A^3\alpha_B^3}, \beta_A\beta_B\right) \tag{2}$$

$$\tilde{A} \times \tilde{B} = \left(\alpha_A\alpha_B, \sqrt[3]{\beta_A^3 + \beta_B^3 - \beta_A^3\beta_B^3}\right) \tag{3}$$

$$\partial.\tilde{A} = \left(\sqrt[3]{1 - \left(1 - \alpha_A^3\right)^{\partial}}, \beta_A^{\partial}\right) \tag{4}$$

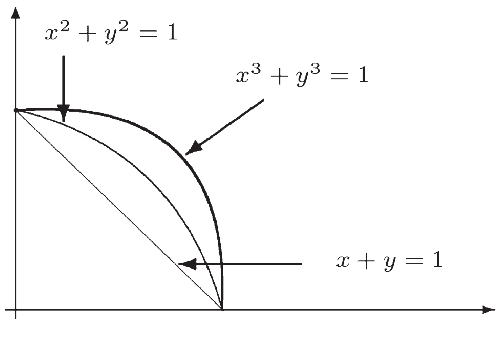

Fig. 1. The differences between Fermatean fuzzy set, intuitionistic fuzzy sets, and Pythagorean fuzzy sets in spaces.

$$\tilde{A}^{\partial} = \left(\alpha_A^3, \sqrt[3]{1 - \left(1 - \beta_A^3\right)^{\partial}}\right) \tag{5}$$

Definition 3. Let $\tilde{A} = (\alpha_A, \beta_A)$ is a FFs, and the score function S and accuracy function T for this FFs are defined as following [44]:

$$S\left(\tilde{A}\right) = \alpha_A^3 - \beta_A^3 \tag{6}$$

$$T\left(\tilde{A}\right) = \alpha_A^3 + \beta_A^3 \tag{7}$$

The above functions can be used to comparing two FFs, $\tilde{A} = (\alpha_A, \beta_A)$ and $\tilde{B} = (\alpha_B, \beta_B)$. There are different conditions to compare these two FFs [44].

1- If $S\left(\tilde{A}\right) < S\left(\tilde{B}\right)$, then $\tilde{A} < \tilde{B}$;
2- If $S\left(\tilde{A}\right) > S\left(\tilde{B}\right)$, then $\tilde{A} > \tilde{B}$;
3- If $S\left(\tilde{A}\right) = S\left(\tilde{B}\right)$, then
   i. $T\left(\tilde{A}\right) < T\left(\tilde{B}\right)$, then $\tilde{A} < \tilde{B}$;
   ii. $T\left(\tilde{A}\right) > T\left(\tilde{B}\right)$, then $\tilde{A} > \tilde{B}$;
   iii. $T\left(\tilde{A}\right) = T\left(\tilde{B}\right)$, then $\tilde{A} = \tilde{B}$.

Definition 4. The complement of FFs $\tilde{A} = (\alpha_A, \beta_A)$ is defined as following [44].

$$Com\left(\tilde{A}\right) = (\beta_A, \alpha_A) \tag{8}$$

Definition 5. The score function of FFs was defined in definition 3. Suppose that $\tilde{A} = (\alpha_A, \beta_A)$ is a FFs. The value of $S\left(\tilde{A}\right)$ can be in the range of –1 to 1. According to [44] define the following function for positive score function:

$$S^p\left(\tilde{A}_{ij}\right) = 1 + S\left(\tilde{A}_{ij}\right) \tag{9}$$

3. Fermatean-FDOSM

This section introduces the steps of Fermatean-FDOSM. In general, FDOSM content three main components (i.e. input unit, data transformation unit, and data processing unit) [33]. Fundamentally, in the FDOSM method, The input part is similar to any MCDM method while solving any decision making problem. Where, the input formatted into m by n matrix. The M represents the alternatives $A_1, \ldots\ldots, A_{m,andthe}$ n set of decision criteria $C_1, \ldots\ldots, C_n$.

These two elements content the decision matrix.

$$DM = \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{bmatrix} x_{11} & x_{12} & \ldots & x_{1n} \\ x_{21} & x_{22} & \ldots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \ldots & x_{mn} \end{bmatrix}$$

The second part is the data transfer unit which can be done within two steps, namely, choosing the optimal solution, and the second one is finding the relevant importance. Where the first process is responsible of choosing the optimal solution using three parameters namely, the critical value, the maximum value, and the minimum value. The critical value defines the optimal solution and it is ranging between the minimum and the maximum value. The minimum value represents criteria cost and the best value is the lower one.

$$A^* = \left\{ \left(\left(\max_i v_{ij} | j \in J \right) . \left(\min_i v_{ij} | j \in J \right) . \right. \right.$$
$$\left. \left. \left(Op_{ij} \in I.J \right) | i = 1.2.3. \ldots .m \right) \right\} \quad (10)$$

Basically, max represents the benefit criteria and the best is the maximum one, min is the perfect solution for cost criteria and finally Op_{ij} = the critical value, where the ideal value is between the minimum and the maximum value.

The second step of data transfer unit is the relevant importance of the differences between the ideal solution and the value of the alternatives which is measured subjectively by using five Likert scale, where decision makers are asked to confirm if the relevant differences have significantly affected the opinion of the decision maker.

$$Op_{Lang} = \left\{ \left(\left(\underset{ij}{\tilde{v}} \otimes v_{ij} | j \in J \right) . | i = 1.2.3. \ldots .m \right) \right\} \quad (11)$$

Where $\otimes$ represents the reference comparison between the potential alternatives and the ideal solution. The final output of this unit is the linguistic term opinion matrix which at this point is ready to be transformed into fuzzy numbers using Fermatean fuzzy set.

$$Op_{Lang} = \begin{array}{c} A_1 \\ \vdots \\ A_m \end{array} \begin{bmatrix} op_{11} & \cdots & op_{1n} \\ \vdots & \ddots & \vdots \\ op_{m1} & \cdots & op_{mn} \end{bmatrix},$$

Whenop is the opinion of the decision maker.

Table 1
Convert the linguistic terms into Fermatean fuzzy set

linguistic terms	FFs
Noidifference	(0.90, 0.10)
Slightidifference	(0.75, 0.20)
Differencei	(0.50, 0.45)
Bigidifference	(0.35, 0.60)
Hugeidifference	(0.10, 0.90)

The final unit in FDOSM is the data processing, this unit contains three steps:

Step 1: in the first step convert the opinion matrix was resulted from the data transformation unit into fuzzy opinion matrix. According to [44], the FFs concerning linguistic terms were defined based on intuitionistic fuzzy set. Therefore, the five Likert scale was used to transfer the linguistic terms into Fermatean fuzzy set showing in Table 1 according to [45].

The result of this step is the fuzzy opinion matrix.

Step 2: Using definition 2 Eq. (2) to aggregate the value for each alternative. The aggregation process used once the fuzzy decision matrix is accomplished.

Step 3: The final step is defuzzification the result to achieve the final score and final rank for the alternatives. Based on definition 3 Eq. (6) and definition 5 Eq. (9) the defuzzification process are achieved. Not to mention, the highest score the best alternative.

3.1. Extend Fermatean-FDOSM into group context

The obtained cumulative decisions from numerous estimators are considered to be essentials for integrating the output benchmarking due to the variation of benchmarking decisions while doing the decision operation by the experts. For this purpose, this study conducted to de the final benchmarking result by integrating ahe dictions by using group decision making. Specifically, FDOSM method based on arithmetic mean was applied to get the final decision-making score. In this context decision making group can be applied to find the best alternative result. [46–48]. After achieving the final score, we combine the decision makers' opinions to result in one final score and one final rank.

$$Group - Fermatean - FDOSM = \oplus R^* \quad (12)$$

R^* = The final result of each decision maker.
$\oplus$ = Arithmetic mean.

4. Applied Fermatean-FDOSM to COVID-19 machine learning methods benchmarking

In this section, we apply the Fermatean-FDOSM to evaluate and benchmark the machine learning methods that were used to classify with the COVID-19 data set. Not to mention, one of the most used approaches in classifying the COVID-19 in artificial intelligent is machine learning methods [49, 50]. Eight machine learning methods (i.e. Neural Network, Support vector machine (SVM), Logistic Regression, k-nearest neighbors (kNN), Random Forest, Naive Bayes, tree, and AdaBoost) and nine evaluation criteria (i.e. Training time (s), Testing time (s), AUC, CA, F1 score, Precision, Recall, Log Loss, Specificity) to create the decision matrix. Table 2 presents the obtained decision matrix.

Each decision maker after determining the ideal solution for each criterion, uses Eq. (10) and Eq. (11) to compare between the ideal solution and the value

Table 2
The decision matrix

Alternatives	Train time [s]	Test time [s]	AUC	CA	F1	Precision	Recall	LogLoss	Specificity
Neural Network	170.281	2.859	0.996348842	0.970532319	0.970536119	0.970634585	0.970532319	0.120653235	0.983664273
SVM	53.793	4.024	0.996283375	0.967680608	0.967633013	0.967913827	0.967680608	0.09635078	0.981727867
Logistic Regression	7.353	1.59	0.994346638	0.958174905	0.958217865	0.958408258	0.958174905	0.233274449	0.976842133
KNN	4.412	5.274	0.98892582	0.937262357	0.937270789	0.938977386	0.937262357	0.339680905	0.964713178
Random Forest	18.635	1.546	0.990371553	0.933460076	0.93361603	0.933882482	0.933460076	0.227589409	0.964689334
Naive Bayes	5.554	1.504	0.966154159	0.900190114	0.900165988	0.900320941	0.900190114	3.150001339	0.947118754
Tree	15.561	0.021	0.916583241	0.891634981	0.891641318	0.891688177	0.891634981	2.123195663	0.943975329
AdaBoost	11.153	1.347	0.901379175	0.868821293	0.869036521	0.869435613	0.868821293	4.530752037	0.933064247

Table 3
The opinion matrix of three decision makers

The opinion matrix of DM1									
Alternatives	Train time	Test time	AUC	CA	F1	Precision	Recall	LogLoss	Specificity
Neural Network	H.D	B.D	NO.D	NO.D	NO.D	NO.D	NO.D	S.D	NO.D
SVM	B.D	H.D	NO.D	S.D	S.D	S.D	S.D	NO.D	S.D
Logistic Regression	S.D	B.D	S.D	S.D	DI	S.D	DI	S.D	DI
kNN	NO.D	H.D	S.D	DI	B.D	DI	B.D	DI	DI
Random Forest	DI	B.D	S.D	DI	B.D	DI	B.D	S.D	DI
Naive Bayes	S.D	B.D	DI	B.D	H.D	B.D	H.D	H.D	B.D
Tree	DI	NO.D	H.D	B.D	H.D	B.D	H.D	B.D	B.D
AdaBoost	DI	DI	H.D	H.D	H.D	H.D	H.D	H.D	H.D
The opinion matrix of DM2									
Alternatives	Train time	Test time	AUC	CA	F1	Precision	Recall	LogLoss	Specificity
Neural Network	H.D	B.D	NO.D	NO.D	NO.D	NO.D	NO.D	S.D	NO.D
SVM	B.D	H.D	NO.D	S.D	S.D	S.D	S.D	NO.D	NO.D
Logistic Regression	S.D	B.D	NO.D	S.D	S.D	S.D	S.D	S.D	S.D
kNN	NO.D	H.D	NO.D	DI	DI	S.D	DI	S.D	DI
Random Forest	DI	B.D	S.D	DI	DI	S.D	DI	S.D	DI
Naive Bayes	NO.D	B.D	S.D	DI	DI	DI	B.D	B.D	B.D
Tree	S.D	NO.D	DI	B.D	B.D	DI	B.D	B.D	B.D
AdaBoost	S.D	B.D	DI	H.D	H.D	DI	H.D	H.D	B.D
The opinion matrix of DM3									
Alternatives	Train time	Test time	AUC	CA	F1	Precision	Recall	LogLoss	Specificity
Neural Network	H.D	B.D	NO.D	NO.D	NO.D	NO.D	NO.D	DI	NO.D
SVM	H.D	H.D	S.D	S.D	S.D	S.D	S.D	NO.D	NO.D
Logistic Regression	DI	DI	S.D	S.D	S.D	S.D	S.D	DI	S.D
kNN	NO.D	H.D	S.D	DI	DI	DI	DI	DI	S.D
Random Forest	B.D	DI	S.D	DI	DI	DI	DI	DI	S.D
Naive Bayes	NO.D	DI	S.D	B.D	DI	DI	B.D	H.D	DI
Tree	B.D	NO.D	DI	H.D	H.D	B.D	B.D	B.D	DI
AdaBoost	B.D	DI	H.D	H.D	H.D	B.D	H.D	H.D	B.D

* NO.D: No Difference / S.D: Slight Difference / DI: Difference / B.D: Big Difference / H.D: Huge Difference.

of the potential alternatives in the same criterion. The decision makers used the five Likert scale (i.e. Noidifference, Slightidifference, Difference, Bigidifference, and hugeidifference). The result of this step is the opinion matrix of each decision maker (DM). In Table 3 reported the opinion matrix of three decision makers.

According to Table 1, the opinion matrix for each decision maker represented in Fermatean fuzzy set value to create the fuzzy opinion matrices. In Table 4 reported the fuzzy opinion matrices for the three decision makers'.

After extracting the fuzzy opinion matrices of the three decision makers', we apply Eq. (2) to aggregate the FFs values for each alternative. And using Eq. (6) to achieve the final crisp score and final rank. In Table 5, the final score and rank for individual decision maker context is reported.

It is clear from the above table the three decision makers concur that "Neural Network" is the best machine learning method in this case with scores (4.839747285, 4.839747285, and 4.542874196) respectively. As well, the three decision makers concur that " AdaBoost" is the worst machine learning method in this case with scores (0.25609141, 0.761624125, and 0.258435924) respectively. The variances can be clearly observed in the final rank of the alternatives (i.e. Random Forest, Naive Bayes, and Tree) of individual decision maker. Those variances are due to the differences in opinion for each decision maker. Group decision making can be used to handle the issue of the variations in final rank. Thus, we apply the group decision making by using Eq. (12). Table 6 reports the final score and the final rank of the group decision making.

According to Table 6, the best machine learning method in this case study is "Neural Network" with score "4.74079", and the worst machine learning method in this case study is "AdaBoost" with score "0.425384".

5. Result of comparative Fermatean-FDOSM, basic FDOSM, and TOPSIS

In this section, we compare the final result of the first decision maker was resulted from Fermatean-FDOSM and the final result for the same decision maker when using basic FDOSM. On the other hand, we extracting the final rank for the machine learning methods using TOPSIS. In Table 7, the results of Fermatean-FDOSM and basic FDOSM, are reported.

Not mention again, the basic FDOSM used fuzzy type-1 with triangular membership [33]. When compare the result of FDOSM with the result of Fermatean-FDOSM can see some differences in the rank in particular with second decision maker and third decision maker. These differences in position of the second alternative and the third alternative. Also, in fifth and sixth alternatives. When compare the score of Fermatean-FDOSM for each decision maker with his opinion matrix we find the result of Fermatean-FDOSM is in line with decision makers opinion and more logical than the result of basic FDOSM. And, On the other hand, the flexibility of Fermetean fuzzy set to deal the uncertainty in the opinion gives us more logical and accurate result. On the other hand, TOPSIS is one of the most methods used in MCDM to extracting rank for the alternatives. In our case, we divide the total of weight (i.e. 1) on the number of the evaluation criteria, to extracting the weight for each criterion equally, then extracting the final score and final rank for the machine learning methods. In Table 8, the final result for the group Fermatean-FDOSM that was presented previously in Table 6 and the final results of TOPSIS are reported.

According to [51], in TOPSIS the highest score the best alternative. When compare the two ranks were presented in Table 8 we can see clearly the differences in the ranks. These differences in the ranks happened because the TOPSIS is a mathematical method doesn't care to the opinion of the decision maker. According to the opinions of the three decision makers were presented in Table 3, the neural network is the best alternative clearly because it has a 6 from 9 ideal solutions for the three decision makers. it is not logical alternative has 6 ideal solutions and present as a worst alternative. For that, the rank of TOPSIS is not logical and totally different from the opinion of the decision makers (experts). We can represent Table 8 in Fig. 2 to clearfiy the differences in final rank between group Fermatean-FDOSM and TOPSIS.

According to the Fig. 2, the differences in the ranks presented clearly. When compare the final rank of group Fermatean-FDOSM with opinions of the three decision makers (experts), we found the rank of group Fermatean-FDOSM is logical and in line with decision makers' opinions. Therefore, the rank of group decision making was reported in Table 6 that can be considered as the final result of Fermatean-FDOSM, which is regarded as the basis of validation process.

Table 4
The fuzzy opinion matrices of the three decision makers'

The fuzzy opinion matrix for first decision maker																		
Alternatives	Train time		Test time		AUC		CA		F1		Precision		Recall		LogLoss		Specificity	
Neural Network	0.10	0.90	0.35	0.60	0.90	0.10	0.90	0.10	0.90	0.10	0.90	0.10	0.90	0.10	0.75	0.20	0.90	0.10
SVM	0.35	0.60	0.10	0.90	0.90	0.10	0.75	0.20	0.75	0.20	0.75	0.20	0.75	0.20	0.90	0.10	0.75	0.20
Logistic Regression	0.75	0.20	0.35	0.60	0.75	0.20	0.75	0.20	0.50	0.45	0.75	0.20	0.50	0.45	0.75	0.20	0.50	0.45
KNN	0.90	0.10	0.10	0.90	0.75	0.20	0.50	0.45	0.35	0.60	0.50	0.45	0.35	0.60	0.50	0.45	0.50	0.45
Random Forest	0.50	0.45	0.35	0.60	0.75	0.20	0.50	0.45	0.35	0.60	0.50	0.45	0.35	0.60	0.75	0.20	0.50	0.45
Naive Bayes	0.75	0.20	0.35	0.60	0.50	0.45	0.35	0.60	0.10	0.90	0.35	0.60	0.10	0.90	0.10	0.90	0.35	0.60
Tree	0.50	0.45	0.90	0.10	0.10	0.90	0.35	0.60	0.10	0.90	0.35	0.60	0.10	0.90	0.35	0.60	0.35	0.60
AdaBoost	0.50	0.45	0.50	0.45	0.10	0.90	0.10	0.90	0.10	0.90	0.10	0.90	0.10	0.90	0.10	0.90	0.10	0.90
The fuzzy opinion matrix for second decision maker																		
Alternatives	Train time		Test time		AUC		CA		F1		Precision		Recall		LogLoss		Specificity	
Neural Network	0.10	0.90	0.35	0.60	0.90	0.10	0.90	0.10	0.90	0.10	0.90	0.10	0.90	0.10	0.75	0.20	0.90	0.10
SVM	0.35	0.20	0.90	0.10	0.90	0.10	0.75	0.20	0.75	0.20	0.75	0.20	0.75	0.20	0.90	0.10	0.90	0.10
Logistic Regression	0.75	0.20	0.35	0.60	0.90	0.10	0.75	0.20	0.75	0.20	0.75	0.20	0.75	0.20	0.75	0.20	0.75	0.20
KNN	0.90	0.10	0.10	0.90	0.90	0.10	0.50	0.45	0.50	0.45	0.75	0.20	0.50	0.45	0.75	0.20	0.50	0.45
Random Forest	0.50	0.45	0.35	0.60	0.75	0.20	0.50	0.45	0.50	0.45	0.75	0.20	0.50	0.45	0.75	0.20	0.50	0.45
Naive Bayes	0.90	0.10	0.35	0.60	0.75	0.20	0.50	0.45	0.50	0.45	0.50	0.45	0.35	0.60	0.35	0.60	0.35	0.60
Tree	0.75	0.20	0.90	0.10	0.50	0.45	0.35	0.60	0.35	0.60	0.50	0.45	0.35	0.60	0.35	0.60	0.35	0.60
AdaBoost	0.75	0.20	0.35	0.60	0.50	0.45	0.10	0.90	0.10	0.90	0.50	0.45	0.10	0.90	0.10	0.90	0.35	0.60
The fuzzy opinion matrix for third decision maker																		
Alternatives	Train time		Test time		AUC		CA		F1		Precision		Recall		LogLoss		Specificity	
Neural Network	0.10	0.90	0.35	0.60	0.90	0.10	0.90	0.10	0.90	0.10	0.90	0.10	0.90	0.10	0.50	0.45	0.90	0.10
SVM	0.10	0.90	0.10	0.90	0.75	0.20	0.75	0.20	0.75	0.20	0.75	0.20	0.75	0.20	0.90	0.10	0.90	0.10
Logistic Regression	0.50	0.45	0.50	0.45	0.75	0.20	0.75	0.20	0.75	0.20	0.75	0.20	0.75	0.20	0.50	0.45	0.75	0.20
KNN	0.90	0.10	0.10	0.90	0.75	0.20	0.50	0.45	0.50	0.45	0.50	0.45	0.50	0.45	0.50	0.45	0.75	0.20
Random Forest	0.35	0.60	0.50	0.45	0.75	0.20	0.50	0.45	0.50	0.45	0.50	0.45	0.50	0.45	0.50	0.45	0.75	0.20
Naive Bayes	0.90	0.10	0.50	0.45	0.75	0.20	0.35	0.60	0.50	0.45	0.50	0.45	0.35	0.60	0.10	0.90	0.50	0.45
Tree	0.35	0.60	0.90	0.10	0.50	0.45	0.10	0.90	0.10	0.90	0.35	0.60	0.35	0.60	0.35	0.60	0.50	0.45
AdaBoost	0.35	0.60	0.50	0.45	0.10	0.90	0.10	0.90	0.10	0.90	0.35	0.60	0.10	0.90	0.10	0.90	0.35	0.60

Table 5
Results of Individual Decision-making Context

Alternatives	Decision Maker 1		Decision Maker 2		Decision Maker 3	
	Score	Rank	Score	Rank	Score	Rank
Neural Network	4.839747285	1	4.839747285	1	4.542874196	1
SVM	3.611249696	2	4.645991426	2	3.569374993	2
Logistic Regression	2.527248881	3	3.724925661	3	2.906238989	3
KNN	1.737625	4	2.802749977	4	2.198749996	4
Random Forest	1.472374997	5	1.933499902	5	1.636624971	6
Naive Bayes	0.721374385	7	1.697374998	6	1.737625	5
Tree	1.028499923	6	1.615249999	7	1.15249999	7
AdaBoost	0.25609141	8	0.761624125	8	0.258435924	8

Table 6
Result of group decision making

Alternatives	Final score	Final rank
Neural Network	4.74079	1
SVM	3.942205	2
Logistic Regression	3.052805	3
kNN	2.246375	4
Random Forest	1.680833	5
Naive Bayes	1.385458	6
Tree	1.265417	7
AdaBoost	0.425384	8

6. Validation results

To prove the final result of group decision-making outcomes obtained by the Fermatean-FDOSM, the objective validation is further applied in this research. The process of objective validation is introduced by splitting the benchmarking machine learning methods into different equal groups. This process is conducted in several MCDM studies [11, 34, 52, 53]. The number of machine learning methods within each group and the number of groups do not affect the objective validation output [54–57]. To validate the group benchmarking machine learning methods results, various steps should be performed as follows: (1) the machine learning methods are sorted according to Group Fermatean-FDOSM decision making results, (2) following sorting, the machine learning methods are separated into two equal groups, and finally (3) the mean ($\bar{x}$) for each group in GDM result is calculated afterwards as defined in Eq. (13).

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{13}$$

Table 7
Fermatean-FDOSM and basic FDOSM

Alternatives	Decision Maker 1		Decision Maker 2		Decision Maker 3	
	Score	Rank	Score	Rank	Score	Rank
Neural Network	0.3	1	0.3	1	0.324074074	1
SVM	0.374074074	2	0.355555556	3	0.392592593	3
Logistic Regression	0.418518519	3	0.327777778	2	0.372222222	2
kNN	0.535185185	4	0.424074074	4	0.466666667	4
Random Forest	0.535185185	4	0.466666667	5	0.490740741	5
Naive Bayes	0.703703704	7	0.538888889	6	0.535185185	6
Tree	0.685185185	6	0.561111111	7	0.644444444	7
AdaBoost	0.801851852	8	0.7	8	0.787037037	8

Table 8
Result of group Fermatean-FDOSM and TOPSIS

Alternatives	Group Fermatean-FDOSM		TOPSIS	
	Final Score	Final Rank	Final Score	Final Rank
Neural Network	4.74079	1	0.450772619	8
SVM	3.942205	2	0.633592209	5
Logistic Regression	3.052805	3	0.860462381	1
kNN	2.246375	4	0.632579533	6
Random Forest	1.680833	5	0.849902136	2
Naive Bayes	1.385458	6	0.657177086	4
Tree	1.265417	7	0.766449366	3
AdaBoost	0.425384	8	0.567469868	7

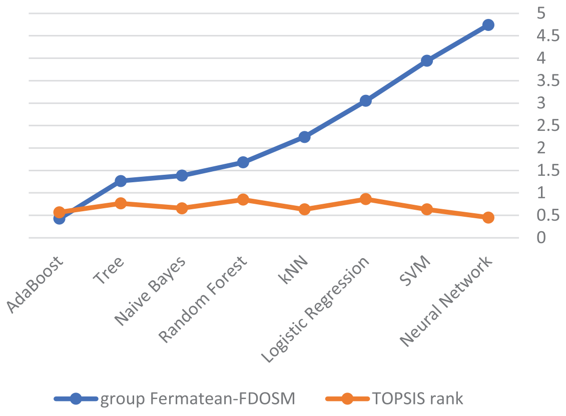

Fig. 2. The differences between ranks of group Fermatean-FDOSM and TOPSIS.

The process of comparison is achieved on the basis of the findings of each group's mean. The method of the comparison is based on the average outcome in each and every group. The minimum values of the mean of each group contribute to relevant outcomes since the lowest linguistic terms are assigned to the optimal solution of each criterion by the decision makers, which is the concept of FDOSM. Therefore, it considered that the first group has the minimum mean of testing the validity of the result, and thus, it is compared with the second group. The mean outcome of the second group ought to be greater than or equal to that of the first group. If the findings of evaluation are consistent with the assumption, the outcomes then, are correct. In Table 9, the results of objective validation for benchmarking machine learning methods based on FDOSM is presented. For the first group, the obtained mean is 2.351851852 which is lower than the mean of the second group with a value of 3.564814815. This shows that the findings of benchmarking machine learning methods based on Fermatean-FDOSM are valid, closest to decision makers' opinions, logical, and having undergone systematic ranking.

As shown in Table 9, based on the effectiveness of the outcome of the group of benchmarking machine learning techniques obtained by the Fermatean-FDOSM, the mean of the first group (i.e. 2.351851852) is lower than that of the second group (i.e. 3.564814815). Therefore, the result of group Fermatean-FDOSM for the benchmarking machine learning methods are valid and underwent systematic ranking.

Table 9
The Objective Validation of Group Fermatean-FDOSM decision making results

Group	machine learning methods	Mean
1st Group	Neural Network	2.351851852
	SVM	
	Logistic Regression	
	kNN	
2nd Group	Random Forest	3.564814815
	Naive Bayes	
	Tree	
	AdaBoost	

7. Conclusion

MCDM methods are the most used in operation research and expert system fields to handle complex real-life problems. In the academic literature, the MCDM methods are suffered from many challenges. One of the most important challenges is uncertainty and vagueness. Several researchers recommended to use the fuzzy environment to solve this challenge. In fuzzy environment, many types and extensions and still the researchers develop new extensions in fuzzy environment. The best MCDM method in the state of art is the FDOSM and it is preferred to have an extension to this valuable method. This method has some challenges with some ambiguity. Fermatean fuzzy set is the best solution to solve this ambiguity problem and can be applied to solve real life problems. The methodology of this study is composed of two phases. The first phase presented the mathematical model of Fermatean-FDOSM considering three stages: input data (i.e. the decision matrix), data transformation (i.e. transfer the decision matrix into opinion matrix according to the philosophy of FDOSM), and finally the data processing stage (i.e. in this stage transferred the opinion matrix into fuzzy opinion matrix and applied the Fermatean fuzzy set equations to create the new extension called Fermatean-FDOSM). The second phase applied the new extension to evaluate and benchmark the COVID-19 machine learning methods. We applied a comparative between the result of Fermatean-FDOSM, the result of the basic FDOSM, and the final result of TOPSIS, we found the result of Fermatean-FDOSM is more logical and in line with the expert opinion. Also, we applied the validation process for the final result of Fermatean-FDOSM, we found the result of Fermatean-FDOSM is more logical and undergoing a systematic ranking and in line with decision makers' opinions. As a future works, we recommend to extend FDOSM into others fuzzy environment. And apply

Fermatean-FDOSM to solve different real life multi criteria decision making problems.

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