

# An optimal control model of the spread of the COVID-19 pandemic in Iraq: Deterministic and chance-constrained model

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**Abstract.** Many studies have attempted to understand the true nature of COVID-19 and the factors influencing the spread of the virus. This paper investigates the possible effect the COVID-19 pandemic spreading in Iraq considering certain factors, that include isolation and weather. A mathematical model of cases representing inpatients, recovery, and mortality was used in formulating the control variable in this study to describe the spread of COVID-19 through changing weather conditions between 17th March and 15th May, 2020. Two models having deterministic and an uncertain number of daily cases were used in which the solution for the model using the Pontryagin maximum principle (PMP) was derived. Additionally, an optimal control model for isolation and each factor of the weather factors was also achieved. The results simulated the reality of such an event in that the cases increased by 118%, with an increase in the number of people staying outside of their house by 25%. Further, the wind speed and temperature had an inverse effect on the spread of COVID-19 by 1.28% and 0.23%, respectively. The possible effect of the weather factors with the uncertain number of cases was higher than the deterministic number of cases. Accordingly, the model developed in this study could be applied in other countries using the same factors or by introducing other factors.

**Keywords:** COVID-19 pandemic, optimal control, pontryagin maximum principle, chance-constrained, isolation, weather factors

## 1. Introduction

The COVID-19 pandemic has resulted in a huge loss of human life and affected economies worldwide. Most nations have entered into a “lockdown” situation, isolating their economies from others in facing the virus and decrease the loss of human life. Several studies have investigated COVID-19 to understand the composition of the virus and determine the most suitable treatment. However, the danger of COVID-19 is represented by its spread without any clinical

symptoms during the early stages of contracting the virus and long period incubation [19]. Chen et al. [2] compared two groups of patients; COVID-19 and SARS-CoV-2-negative, where it was revealed that COVID-19 patients suffered from high fever and cough more than SARS-CoV-2 patients. Procalcitonin (PCT) levels of SARS-CoV-2 patients (approximately 2 out of 5 patients) were shown to be higher compared to COVID-19 patients. Also, COVID-19 patients had lower creatinine levels compared with SARS-CoV-2 patients. In the case of those at a young age, the distinction between the two diseases can be diagnosed depending on the fever, cough, urea and creatinine levels, and parameters associated with routine blood workup. COVID-19,

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SARS and MERS have mostly similar pathological features [23]. However, the harm caused by COVID-19 may be caused by SARS-CoV-2 or through liver damage. Ai et al. [1] compared chest CT scans and reverse-transcription chain reaction to diagnose the virus in a sample of patients (1,014 patients) in China. Chest CT scans were shown to be highly sensitive in the diagnosis of the virus. X. Chen et al. [3] assessed the state of pregnant women, having a positive test for COVID-19, in which fever and cough were the main symptoms that were evident in the pregnant women without vertical transmission of the virus in late pregnancy. The World Health Organisation (WHO) confirmed the effect of COVID-19 on the mental health of human; especially children and the elderly [22].

The drug used for the treatment of malaria (Chloroquine phosphate) has found to be effective in the treatment of COVID-19 in some 100 patients through clinical trials conducted in hospitals across seven cities in China [6]. Lipsitch et al. [10] discussed the approach adopted in the treatment of an influenza pandemic (2009). While this approach may possibly be used in the treatment of Covid-19, it depends on many factors, such as existing surveillance systems. In a separate case, viral RNA samples were collected from survivors and people who had died to explore the factors that caused patients to die in a hospital setting by employing multivariable logistic regression [24].

In fact, several studies have formulated mathematical models to simulate actual cases of epidemics and pandemics around the world. In some studies, the optimal control model was widely used to determine the effect of vaccination and isolation. Lee et al. [9] clarified the effect of treatment and isolation on the control of fast transmitting diseases, such as influenza. In their study, they discussed isolation strategies having insufficient antiviral resources. Three models of optimal control, namely, vaccination, isolation and the mixed model of the SIR epidemic were investigated by Hansen and Day [8] to minimise the magnitude of the outbreak. Tuite et al. [18] explored control strategies that could aid in understanding the spreading processes associated with the cholera epidemic in Haiti to reduce the potential effects. Rodrigues et al. [15] discussed a dengue vaccine as a control variable with distinct levels and two methods of treatment. The first method was for paediatric patients, and the second method was for random mass vaccination. A comparison between a different scenario regarding tuberculosis epidemiological features was conducted by P. Rodrigues et al.

[16], aiming to reduce total implementation costs and the number of infected cases. Also, regarding tuberculosis, Moualeu et al. [11] formulated a mathematical model that considers infection (diagnosed and undiagnosed), lost-sight, and latently infected aspects. Pang et al. [12] formulated a mathematical model that simulated the actual cases of measles transmission in the United States (US) for the period between 1951 and 1962 to determine the optimal strategy for vaccination. The spread of the Ebola virus in West Africa was investigated by Rachah and Torres [13, 14]. The first paper investigated the effect of different cases of vaccination on the virus spreading over time, while the second paper addressed in addition to vaccination, several strategies to reduce the number of infected and exposed individuals. In another study by Gao and Huang, they incorporated three controls as part of a strategy from among several initially developed strategies to minimise intervention costs and reduce the burden of tuberculosis [5].

The structure of this paper is organised into five sections. The first section, already discussed, provided a brief introduction and background information on COVID-19. Section 2 provides further information on the spread of COVID-19, followed by Section 3 that presents two models; the optimal control model and a model presenting an explicit solution using the Pontryagin maximum principle (PMP). The results and explanation of several models are presented and discussed in Section 4. Lastly, Section 5 presents the overall conclusions and recommendations for future research.

## 2. The spread of COVID-19

In December 2019, the first cases of COVID-19 surfaced in Wuhan, Hubei, China [2]. Since then, four other Asian countries confirmed a further 282 cases of the virus on 20th January 2020, two cases in Thailand, one case Japan and South Korea, and the remaining number of cases were reported in China. All cases originated from Wuhan City [20].

After several months, the virus spread to other cities in China, and another 33 countries worldwide. On 24th February, the number of deaths reported in China amounted to 2,663, and 33 in other countries [1]. Towards the end of February 2020, the population in Europe and North America showed signs of the virus, albeit a different strain or foci of the virus, including countries in Asia, and the Middle East.

At the same time, the first case of COVID-19 was recorded in Africa and Latin America [21]. At the beginning of March 2020, more than 10,000 patients had died from the virus in 10 countries, including Iran, Italy, and South Korea [19]. A significant increase in the harm caused by the virus in the Middle East was confirmed during the middle of March. As a result of the rapid spread of the virus, the WHO classified the COVID-19 epidemic as a global pandemic [21].

The first case of COVID-19 in Iraq appeared in the Najaf province, south of the capital Baghdad on 24th February 2020; an Iranian student studying Islamic science in Najaf. Many Iraqi people travel to Iran to visit many of the holy shrines and enjoy tourist attractions. It was believed that the virus originated from Iran. On 26th February, four new cases were reported with the number increasing to 13 on 1st March, before reporting 93 further cases in the middle of March with nine deaths.

### 3. Optimal control model

This section presents two models having a known and uncertain number of new daily cases; the deterministic model and the chance-constrained model, respectively.

#### 3.1. Deterministic model

Optimal control for optimization is defined by:

$$Min.J = \sum_{t=0}^T \{1 + y(t)\}^2 \tag{1}$$

Subject to the state equations of cases, inpatients, recovered cases and death:

$$\Delta C(t) = \{1 + y(t)\} \beta(t) C(t) \tag{2}$$

$$\Delta I(t) = \{1 + y(t)\} \beta(t) C(t) - \{\mu(t) + \mathcal{N}(t)\} I(t) \tag{3}$$

$$\Delta R(t) = \mu(t) I(t) \tag{4}$$

$$\Delta D(t) = \mathcal{N}(t) I(t) \tag{5}$$

where,

$y(t)$ : The control variable.

$C(t)$ : Percent of confirmed cases.

$I(t)$ : Percent of inpatients.

$R(t)$ : Percent of recovered cases.

$D(t)$ : Percent of death cases.

$\beta(t)$ : Percent of the new cases to the inpatient cases.

$\mu(t)$ : Percent of the recover cases to the inpatient cases.

$\mathcal{N}(t)$ : Percent of the death cases to the inpatient cases.

$$\Delta C(t) = C(t) - C(t-1)$$

Equation (2) signifies the total number of cases that increased based on new cases daily. The total number of inpatients increases by the addition of new cases and decreases by the number of recovery and deaths each day (Equation 3). Equations (4 & 5) represent the total number of recovered patients and deaths, respectively. The control variable represents the percentage of isolation or each weather factor.

By using the PMP, determining the solution of the optimal control model can be achieved. The Lagrangian function is expressed as follows [17]:

$$L = \sum_{t=0}^T -\{1 + y(t)\}^2 + \sum_{t=0}^T \lambda_c(t) [\{1 + y(t)\} \beta(t) C(t) - C(t) + C(t-1)] + \sum_{t=0}^T \lambda_i(t) [\{1 + y(t)\} \beta(t) C(t) - \{\mu(t) + \mathcal{N}(t)\} I(t) - I(t) + I(t-1)] + \sum_{t=0}^T \lambda_r(t) [\mu(t) I(t) - R(t) + R(t-1)] + \sum_{t=0}^T \lambda_d(t) [\mathcal{N}(t) I(t) - D(t) + D(t-1)] \tag{6}$$

A Hamiltonian function is expressed as:

$$H(t) = -\{1 + y(t)\}^2 + \lambda_c(t) [\{1 + y(t)\} \beta(t) C(t)] + \lambda_i(t) [\{1 + y(t)\} \beta(t) C(t) - \{\mu(t) + \mathcal{N}(t)\} I(t)] + \lambda_r(t) [\mu(t) I(t)] + \lambda_d(t) [\mathcal{N}(t) I(t)] \tag{7}$$

Substituting Equation (7) into Equation (6), gives:

$$L = \sum_{t=0}^T [H(t) - \lambda_c(t) \{C(t) - C(t-1)\} - \lambda_i(t) \{I(t) - I(t-1)\} - \lambda_r(t) \{R(t) - R(t-1)\} - \lambda_d(t) \{D(t) - D(t-1)\}] \tag{8}$$

Deriving Equation (8) concerning  $C(t)$ ,  $I(t)$ ,  $R(t)$ ,  $D(t)$ , separately, gives:

$$\{1 + y(t)\} \beta(t) \{\lambda_c(t) + \lambda_i(t)\} + \lambda_c(t) - \lambda_c(t - 1) = 0, \tag{9}$$

$$\{\mu(t) + \mathcal{N}(t)\} \lambda_i(t) - \mu(t) \lambda_r(t) - \mathcal{N}(t) \lambda_i(t) - \lambda_i(t) + \lambda_i(t - 1) = 0, \tag{10}$$

$$\lambda_r(t) - \lambda_r(t - 1) = 0, \tag{11}$$

$$\lambda_d(t) - \lambda_d(t - 1) = 0, \tag{12}$$

By rearranging Equations (9–12), we can determine the adjoint equations as:

$$\Delta \lambda_c(t) = - \{1 + y(t)\} \beta(t) \{\lambda_c(t) + \lambda_i(t)\} \tag{13}$$

$$\Delta \lambda_i(t) = \{\mu(t) + \mathcal{N}(t)\} \lambda_i(t) - \mu(t) \lambda_r(t) - \mathcal{N}(t) \lambda_d(t) \tag{14}$$

$$\Delta \lambda_r(t) = 0, \tag{15}$$

$$\Delta \lambda_d(t) = 0. \tag{16}$$

From Equations (13 - 14), we get:

$$\lambda_c(t) = \frac{1}{1 + \{1 + y(t)\} \beta(t)} [\lambda_c(t - 1) + \{1 + y(t)\} \beta(t) \lambda_i(t)]. \tag{17}$$

$$\lambda_i(t) = \frac{1}{1 - \mu(t) - \mathcal{N}(t)} \{\lambda_i(t - 1) - \mu(t) \lambda_r(t) - \mathcal{N}(t) \lambda_d(t)\}. \tag{18}$$

Rearranging Equation (13), yields:

$$\{1 + y(t)\} \beta(t) = \frac{-\Delta \lambda_c(t)}{\{\lambda_c(t) + \lambda_i(t)\}} \tag{19}$$

By substituting Equation (19) into Equation (2), it yields:

$$\Delta C(t) = \frac{-\Delta \lambda_c(t)}{\{\lambda_c(t) + \lambda_i(t)\}} C(t) \tag{20}$$

From Equation (20), we can determine the total number of cases over time as follows:

$$C(t) = \frac{1}{1 - \left[ \frac{\Delta \lambda_c(t)}{\{\lambda_c(t) + \lambda_i(t)\}} \right]} C(t - 1) \tag{21}$$

By substituting Equation (19) into Equation (3), it yields:

$$\Delta I(t) = \frac{-\Delta \lambda_c(t)}{\{\lambda_c(t) + \lambda_i(t)\}} C(t) - \{\mu(t) + \mathcal{N}(t)\} I(t) \tag{22}$$

From Equation (22), we then get:

$$I(t) = \frac{1}{1 + \mu(t) + \mathcal{N}(t)} \left[ I(t - 1) - \frac{\Delta \lambda_c(t)}{\{\lambda_c(t) + \lambda_i(t)\}} C(t) \right] \tag{23}$$

Then, substituting Equation (21) into Equation (23), it yields:

$$I(t) = \frac{1}{1 + \mu(t) + \mathcal{N}(t)} \left[ I(t - 1) - \left\{ \frac{\Delta \lambda_c(t)}{\{\lambda_c(t) + \lambda_i(t)\}} \right\} \left\{ \frac{1}{1 - \left[ \frac{\Delta \lambda_c(t)}{\{\lambda_c(t) + \lambda_i(t)\}} \right]} \right\} C(t - 1) \right] \tag{24}$$

Equation (24) represents the total number of inpatients over time. We can determine the total number of recovered cases over time by substituting Equation (24) into Equation (4) as follows:

$$R(t) = R(t - 1) + \frac{\mu(t)}{1 + \mu(t) + \mathcal{N}(t)} \left[ I(t - 1) - \left\{ \frac{\Delta \lambda_c(t)}{\{\lambda_c(t) + \lambda_i(t)\}} \right\} \left\{ \frac{1}{1 - \left[ \frac{\Delta \lambda_c(t)}{\{\lambda_c(t) + \lambda_i(t)\}} \right]} \right\} C(t - 1) \right] \tag{25}$$

Substituting Equation (24) into Equation (5) yields the total number of death cases over time as follows:

$$D(t) = D(t - 1) + \frac{\mathcal{N}(t)}{1 + \mu(t) + \mathcal{N}(t)} \left[ I(t - 1) - \left\{ \frac{\Delta \lambda_c(t)}{\{\lambda_c(t) + \lambda_i(t)\}} \right\} \left\{ \frac{1}{1 - \left[ \frac{\Delta \lambda_c(t)}{\{\lambda_c(t) + \lambda_i(t)\}} \right]} \right\} C(t - 1) \right] \tag{26}$$

Thus, the value of the control variable is as follows:

$$y(t) = \begin{cases} -1; \text{no new cases (optimal)} \\ (-1, 0); \text{decrease cases} \\ 0; \text{no effect} \\ > 0; \text{increase cases} \end{cases} \tag{27}$$

From Equation (27),  $y(t) = -1$  means:

$$\Delta \lambda_c(t) = \Delta C(t) = 0 \tag{28}$$

$$\Delta I(t) = - \{\mu(t) + \mathcal{N}(t)\} I(t) \tag{29}$$

$$\lambda_i(T) = 0 \tag{30}$$

Initially the value of the control variable is zero, which means the model represents the actual cases registered in Iraq with the real percentage of new cases, recovered, and deaths as follows:

$$\beta_1 = \frac{C_1 - C_0}{C_1}; \mu_1 = \frac{R_1}{I_1}; \mathcal{N}_1 = \frac{D_1}{I_1} \tag{31}$$

Next, by introducing the effect of wind (for example): changing the value of the control variable and cases depending on the wind degree ( $W$ ) and transition matrix ( $\varphi$ ) (see Appendix) we get:

$$\delta_1 = C_1 - C_0 + ABS(W_1 - W_0) * \varphi \tag{32}$$

$$new\ C_1 = old\ C_0 + \delta_1; new\ C_2 = new\ C_1 + \delta_2 \tag{33}$$

$$y_1 = \frac{\delta_1}{\beta_1 C_1} - 1 \tag{34}$$

Where  $ABS$  = absolute value.

The elements of the transition matrix can be calculated as follows (for example, humidity):

$$\varphi_{ij} = \frac{\overline{NC}_{H_i} - \overline{NC}_{H_j}}{H_i - H_j} \tag{35}$$

Where  $\overline{NC}_{H_i}$  the average of new cases, according to the humidity degree  $H_i$  (Table 3 shows the average of new cases).

Finally, the next tasks include incorporating the value of the control variable into an optimal control model to obtain the solution. The solution of the optimal control model relies on Equations (15–18, 21, 24–26). The solution is found by using the goal seek function in Microsoft Excel with  $\lambda(T) = 0$ . Hence, achieving the condition  $\lambda(T) = 0$  by changing the value of  $\lambda(0)$ .

### 3.2. Chance-constrained model

The number of new cases reported in Iraq depends on the number of samples tested. Therefore, the actual cases may be greater than those recorded cases. In this model, the number of new cases is uncertain given by the equation representing chance-constrained:

$$P_r \{ \Delta C(t) \geq NC \} \geq \alpha \tag{36}$$

Where  $\alpha$  takes values between zero and one (1) and  $NC$  is a random variable (new cases).

In determining the solution, chance-constrained must be converted to deterministic constrained. Here,

the value of  $\alpha$  is equal to zero or one (1) representing an extremely risky or extremely conservative attitude, respectively. The minimum acceptable to achieve the constraint is  $\alpha$ , while  $(1 - \alpha)$  is the maximum acceptable risks [7].

If  $NC$  is a random variable that adheres to a normal distribution with mean  $\overline{NC}$  and standard deviation  $\sigma_{NC}$ , then the deterministic constraint that is equivalent to chance-constraint (36) as follows [4]:

$$P_r \left\{ \frac{\Delta C(t) - \overline{NC}}{\sigma_{NC}} \geq \frac{NC - \overline{NC}}{\sigma_{NC}} \right\} \geq \alpha$$

$$\emptyset \left\{ \frac{\Delta C(t) - \overline{NC}}{\sigma_{NC}} \right\} \geq \alpha \tag{37}$$

$$\Delta C(t) \geq \overline{NC} + \sigma_{NC} \emptyset^{-1}(\alpha)$$

Where  $\emptyset$  is the cumulative distribution function (CDF) of the standard normal distribution.

The number of daily cases can be found from Eq. (37) with the initial value of cases  $C(0)$ . To determine the effect of the weather factors, we apply the deterministic model (Equations 1–5) with the daily cases of chance-constrained (Equation 37).

## 4. Numerical results

Table 1 (see Appendix) shows the number of COVID-19 cases and degrees of temperature, humidity, wind and pressure from 17th March to 15th May. First, we determine the values of  $(\beta, \mu, N)$  by using Equation (31) to fit the actual cases reported in Table (1) with a zero (0) value of the control variable (see Table 2 in the Appendix).

Next, we change the control variable value to determine the results that represent the effect of isolation and weather.

The effect of isolation is determined by presenting the value of the control variable given below:

$$y(t) = \begin{cases} -0.25 \text{ Increased isolation} \\ 0 \text{ Actual cases (A.C.)} \\ 0.25 \text{ Decreased isolation} \end{cases}$$

Figure (1) shows the effect of isolation on the results, with the actual cases (A.C.), increased cases (I.C.), and decreased cases (D.C.). From Fig. (1), we can conclude that the number of cases increases, due to increase in the number of people that did not commit to staying at home ( $y(t) = 0.25$ ), and vice versa.

The increase in the per cent of people that did not commit to staying at home by 25% led to an increase in the number of COVID-19 cases by

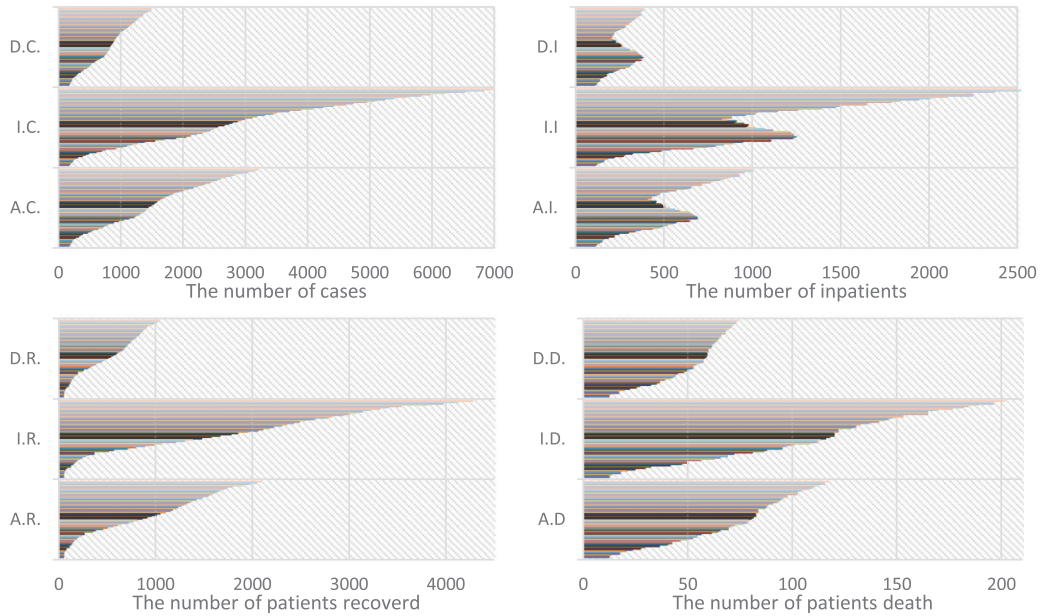


Fig. 1. The effect of isolation.

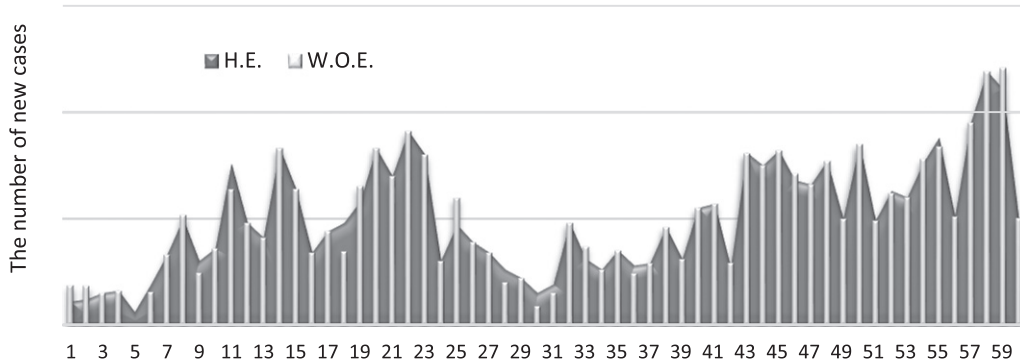


Fig. 2. Numbers of new cases with and without the effect of humidity.

118%, while, the proportion of COVID-19 cases decreased by 53.4% with an increase in the number of citizens staying at home by 25%. The numbers of inpatients (A.I, I.I, D.I), recoverd (A.R, I.R, D.R) and deaths (A.D, I.D, D.D) follow the number of cases.

According to the WHO, symptoms of infection first appear between 2 and 14 days, usually 5 days. In this case, we take the weather for the last 5 days (see Table 3 in the Appendix). We can then find the transition matrix for each factor of the weather (see Tables 4–7 in the Appendix) from Table (3). The values of the main diagonal are zero (0) (without effect) and the other values are negative (cases decrease) and positive (cases increase). The values of the transition matrix represent an increase (or decrease) of COVID-

19 cases with an increase (or decrease) for every one (1) degree of the weather.

From Equations (32–34), and Table (4), we can determine the value of the control variable, according to the effect of humidity. Figure (2) shows the possible effect of humidity on the number of new cases.

The white colour represents the number of new cases without the effect of humidityt, which means the default number. Meanwhile, the actual number of cases, affected by the humidity, is represented by gray colour. The explanation for Fig. (2) is as follows:

Two colours having the same value signifies no change in humidity, thus having no effect.

The white colour higher than the gray colour means humidity increases, while for new cases is decreases.

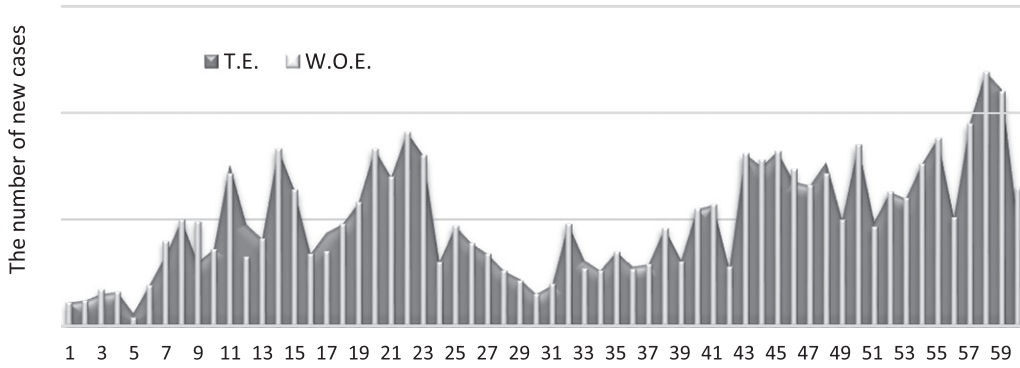


Fig. 3. Numbers of new cases with and without the effect of temperature.

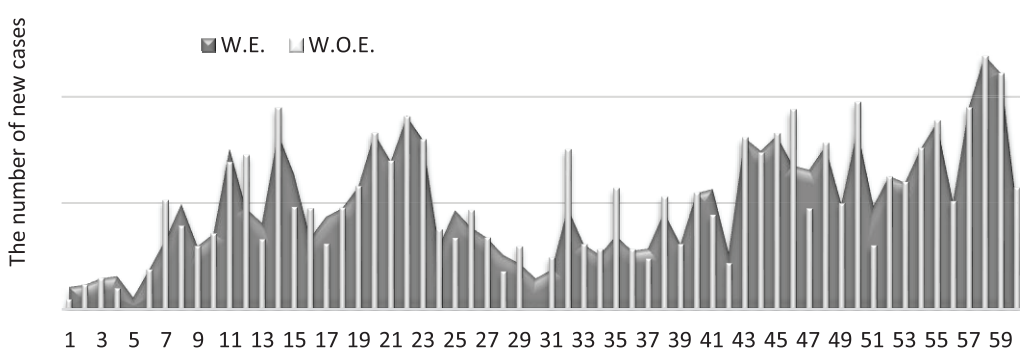


Fig. 4. Numbers of new cases with and without the effect of wind.

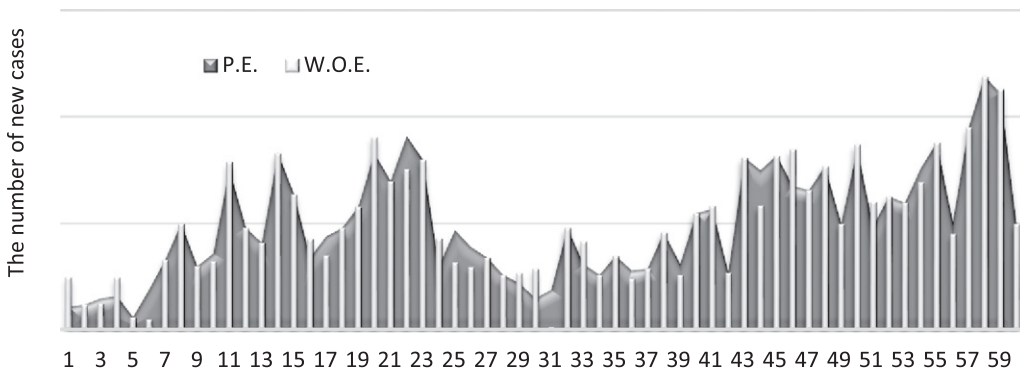


Fig. 5. Numbers of new cases with and without the effect of pressure.

The other case means a decrease in humidity and an increase in new cases.

Similarly, we determine the possible effect of other weather factors, as can be seen in Figs. (3–5):

As evident in Figs. (2–5), the wind shows as having the highest effect on the number of new COVID-19 cases. Figure (6) shows the possible effect of the weather factors at the end of period.

Figure (6) shows the possible effect of weather on the number of new COVID-19 cases. Wind (W.E) and pressure (P.E.) represent the highest and lowest effect, respectively. However, the effect of isolation (see Fig. 1) is more important compared to the weather factors. Figure (7) shows the results at the end of the period without the effect of the weather.

From Fig. (7), it can be seen that the increase in wind speed (W) and temperature (T) has led to

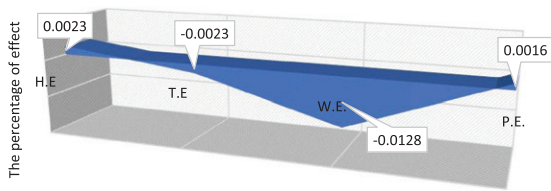


Fig. 6. Weather factors and effect.

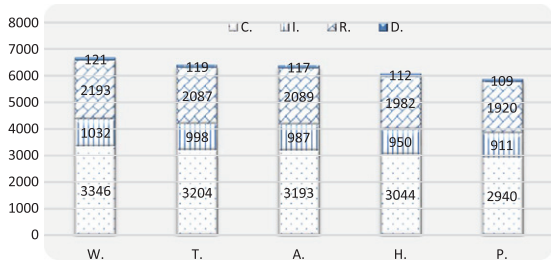


Fig. 7. The results of optimal control model with different values of the control variable.

a decrease in the number of COVID-19 cases. For example, the number of cases is 3,346 and 3,204 without the possible effect of both wind and temperature, respectively. Whereas the number of cases slightly increased with an increase in humidity (H) and pressure (P) compared to the number of actual cases (A).

For the chance-constraint model, the  $\beta$  value is changed according to Eq. (36). At first, we divide the study period into 12 sub-periods with 5 days for each sub-period. Next, a normality test of observations of the sub-periods is conducted, in which all sub-periods adhere to a normal distribution (see Table 8). Eq. (36) is then applied to determine the daily cases with  $\alpha = 0.90$  and  $\phi^{-1}(\alpha) = 1.28$ . Finally, the same steps of the deterministic model are used to find the solution. From Equations (32–34), and Table (9), we can find the value of the control variable and the possible effect of the weather factors, as shown in Figs. (8–11).

By observing Figs. (8–11), the pressure signifies the highest effect on the number of new COVID-19 cases. Whereas, the wind represents the second-highest effect, but with an inverse relation to COVID-19 cases. Figure (12) below illustrates the possible proportional effect of the weather factors at the end of the period.

From observing Fig. (12), we can see that the pressure (P.E) and temperature (T.E.) represent the highest and lowest effect, respectively. The effect of

the weather in the chance-constrained model is higher than the deterministic model because the increase in the number of COVID-19 cases is according to the chance-constrained model.

### 5. Conclusion

In this paper, the possible effect of both weather factors and isolation on the spread of COVID-19 in the context of Iraq was investigated, finding that temperature, humidity, wind, and pressure had a noticeable effect on the spread of the virus. An optimal control model was developed to describe the spread of COVID-19 cases, Deterministic and chance-constrained models were also developed, and the solution of the model using the PMP was also derived. The transition matrix for each of the factor factors was addressed.

Initially, a control variable  $y(t)$  representing the percentage of isolation and weather factors was determined. The zero value of the control variable represented the actual data of COVID-19 cases in Iraq, while the optimal solution was determined with  $y = -1$ . Next, was determining the value of the control variable with respect to five models, before finally, clarifying the possible effect of isolation and the weather factors on the spread of COVID-19.

Accordingly, it was shown that isolation was significant in containing COVID-19 from contagion. On the other hand, the number of cases increased by 118% attributed to an increase in the number of people who ignored the need to stay at home, which rose by 25%. In contrast, the cases reduced by 53.4% for the opposite case where people stayed at home.

These statistics also support the nature of the virus in reality since it is highly contagious, spreading from one person to many. Weather factors also had a noticeable effect on the spread of COVID-19, though lower than self-isolation. Likewise, both wind speed and temperature had the highest effect compared with other weather factors. Further, the wind speed and temperature had an inverse effect on the spread of COVID-19 by 1.28% and 0.23%, respectively, while a positive relationship with humidity and pressure. Humidity and temperature had a similar effect but opposingly. Moreover, increasing the number of daily cases of COVID-19, according to the chance-constrained model, weather factors had a greater effect.

Accordingly, the model developed in this study could be applied in other countries using the same



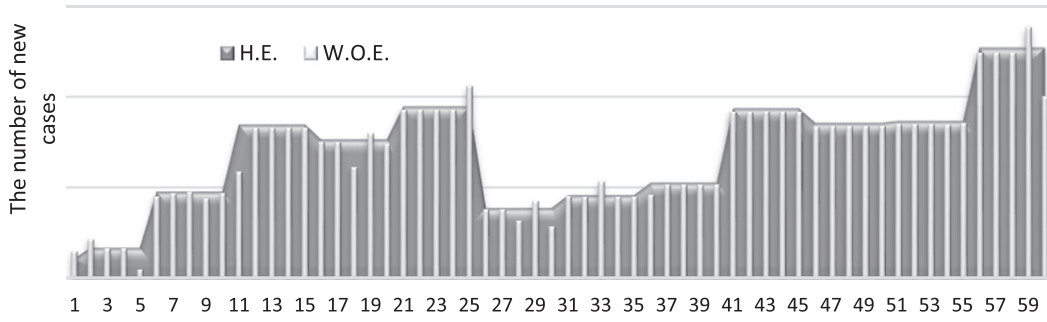


Fig. 8. Number of new cases with and without the effect of humidity (chance-constrained).

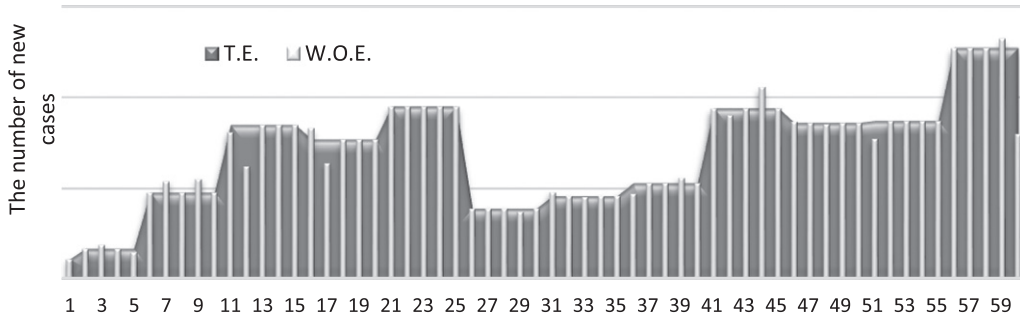


Fig. 9. Number of new cases with and without the wind effect (chance-constrained).

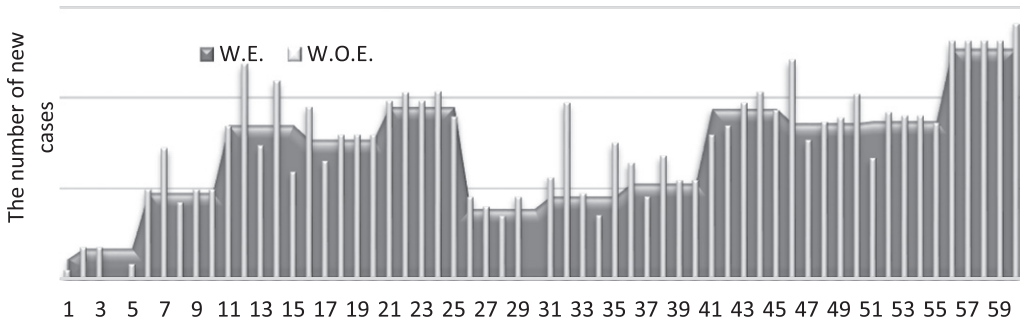


Fig. 10. Number of new cases with and without the pressure effect (chance-constrained).

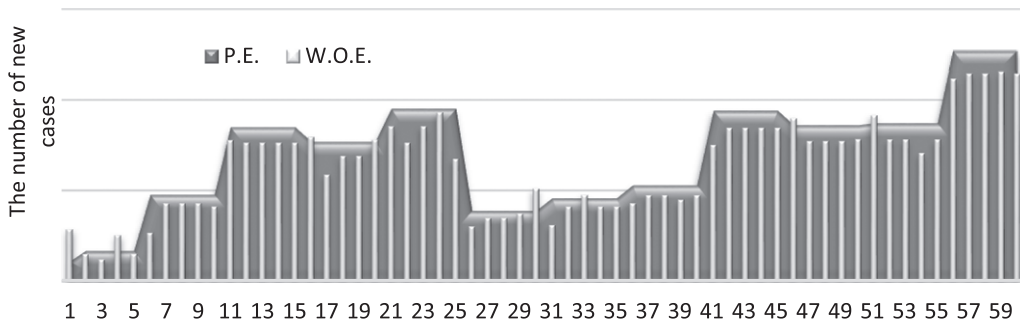


Fig. 11. Number of new cases with and without the pressure effect (chance-constrained).

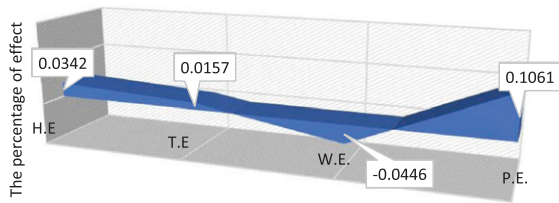


Fig. 12. Proportion, (as a percentage) due to the effect of weather (chance-constrained).

factors or by introducing other factors, such as communication, transportation, and payment.

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## Appendix

Table 1  
The number of COVID-19 cases and weather degrees in Iraq

Date	Cases	Recover	Death	New Cases	Temp.	Wind	Humid.	Pres.
17 March	165	43	12	11	26	7	40	1015
18	177	49	12	12	22	13	39	1010
19	192	49	13	15	18	7	57	1016
20	208	49	17	16	23	9	32	1014
21	214	52	17	6	22	7	45	1006
22	233	57	20	19	17	2	52	1011
23	266	62	23	33	20	9	46	1015
24	316	75	27	50	24	4	34	1014
25	346	103	29	30	27	13	29	1009
26	382	105	36	36	28	20	30	1010
27	458	122	40	76	24	11	40	1014
28	506	131	42	48	24	7	46	1009
29	547	143	42	41	22	7	50	1014
30	630	152	46	83	23	9	33	1012
31	694	170	50	64	26	4	26	1017
1 April	728	182	52	34	28	26	29	1007
2	772	202	54	44	26	9	28	1012
3	820	226	54	48	25	4	33	1016
4	878	259	56	58	25	9	37	1018
5	961	259	61	83	26	4	29	1021
6	1031	344	64	70	29	9	30	1014
7	1122	373	65	91	26	9	37	1012
8	1202	452	69	80	30	7	27	1010
9	1232	496	69	30	26	4	54	1006
10	1279	550	70	47	25	7	35	1008
11	1318	601	72	39	25	9	30	1012
12	1352	640	76	34	26	6	29	1016
13	1378	717	78	26	27	10	33	1017
14	1400	766	78	22	27	13	15	1017
15	1415	812	79	15	29	17	11	1016
16	1434	856	80	19	30	2	17	1013
17	1482	906	81	48	32	7	14	1011
18	1513	953	82	31	31	4	21	1011
19	1539	1009	82	26	28	7	29	1015
20	1574	1043	82	35	32	4	21	1013
21	1602	1096	83	28	33	4	20	1012
22	1631	1146	83	29	33	4	25	1010
23	1677	1171	83	46	37	6	18	1004
24	1708	1204	86	31	29	6	29	1008
25	1763	1224	87	55	32	13	21	1001
26	1820	1263	87	57	26	13	33	1008
27	1847	1286	88	27	28	6	23	1013
28	1928	1319	90	81	31	9	17	1011
29	2003	1346	92	75	24	4	29	1008
30	2085	1375	93	82	24	6	40	1009
1 May	2153	1414	94	68	31	6	26	1014
2	2219	1473	95	66	36	2	16	1011
3	2296	1490	97	77	33	11	20	1013
4	2346	1544	97	50	26	15	45	1011
5	2431	1571	102	85	29	11	21	1018
6	2480	1602	102	49	31	7	20	1016
7	2543	1626	102	63	37	4	13	1009
8	2603	1661	104	60	29	9	33	1008
9	2679	1702	107	76	29	13	20	1008
10	2767	1734	109	88	29	9	21	1010
11	2818	1790	110	51	32	6	17	1015
12	2913	1903	112	95	35	4	13	1013
13	3032	1966	115	119	39	9	8	1011
14	3143	2028	115	111	40	11	10	1009
15	3193	2089	117	50	41	15	10	1008

The ministry of health/ Iraq.

<https://www.timeanddate.com/weather/iraq/baghdad/historic?month=4&year=2020>.

Table 2  
The values of the model parameters

Date	$\beta$	$\mu$	$\mathcal{N}$
17 March	0.066667	0.100000	0.009091
18	0.067797	0.051724	0.000000
19	0.078125	0.000000	0.007692
20	0.076923	0.000000	0.028169
21	0.028037	0.020690	0.000000
22	0.081545	0.032051	0.019231
23	0.124060	0.027624	0.016575
24	0.158228	0.060748	0.018692
25	0.086705	0.130841	0.009346
26	0.094241	0.008299	0.029046
27	0.165939	0.057432	0.013514
28	0.094862	0.027027	0.006006
29	0.074954	0.033149	0.000000
30	0.131746	0.020833	0.009259
31	0.092219	0.037975	0.008439
1 April	0.046703	0.024291	0.004049
2	0.056995	0.038760	0.003876
3	0.058537	0.044444	0.000000
4	0.066059	0.058615	0.003552
5	0.086368	0.000000	0.007800
6	0.067895	0.136437	0.004815
7	0.081105	0.042398	0.001462
8	0.066556	0.116006	0.005874
9	0.024351	0.065967	0.000000
10	0.036747	0.081942	0.001517
11	0.029590	0.079070	0.003101
12	0.025148	0.061321	0.006289
13	0.018868	0.132075	0.003431
14	0.015714	0.088129	0.000000
15	0.010601	0.087786	0.001908
16	0.013250	0.088353	0.002008
17	0.032389	0.101010	0.002020
18	0.020489	0.098326	0.002092
19	0.016894	0.125000	0.000000
20	0.022236	0.075724	0.000000
21	0.0174781	0.125295	0.002364
22	0.0177805	0.124378	0
23	0.0274299	0.059101	0
24	0.0181498	0.078947	0.007177
25	0.0311968	0.044247	0.002212
26	0.0313186	0.082978	0
27	0.0146183	0.048625	0.002114
28	0.0420124	0.063583	0.003853
29	0.0374438	0.047787	0.003539
30	0.0393285	0.047001	0.001620
1 May	0.0315838	0.060465	0.001550
2	0.0297431	0.090629	0.001536
3	0.0335365	0.023977	0.002820
4	0.0213128	0.076595	0
5	0.0349650	0.035620	0.006596
6	0.0197580	0.039948	0
7	0.0247738	0.029447	0
8	0.0230503	0.041766	0.002386
9	0.0283687	0.047126	0.003448
10	0.0318033	0.034632	0.002164
11	0.0180979	0.061002	0.001089
12	0.0326124	0.125835	0.002227
13	0.0392480	0.066246	0.003154
14	0.035316	0.062	0
15	0.015659	0.061803	0.002026

Table 3  
The average of COVID-19 cases, according to humidity, temperature, wind and pressure

Humid.	$\overline{NC}_H$	Temp.	$\overline{NC}_T$	Wind	$\overline{NC}_W$	Pres.	$\overline{NC}_P$
56–75	50	91–100	55	21–25	40	5026–5035	55
76–100	51	101–110	23	26–30	52	5036–5045	79
101–125	60	111–120	31	31–35	54	5046–5055	57
126–150	56	121–130	53	36–40	41	5056–5065	50
151–175	67	131–140	51	41–45	59	5066–5075	38
176–200	48	141–150	58	46–50	61	5076–5085	60
201–225	28	151–160	62	51–55	53		
226–250	22	161–170	64	56–60	34		
251–275	12	171–180	50				

Table 4  
The transition matrix of the number of COVID-19 cases, according to humidity

State	75	100	125	150	175	200	225	250	275
75	0.000	0.030	0.208	0.084	0.170	-0.013	-0.147	-0.160	-0.190
100	-0.030	0.000	0.386	0.111	0.217	-0.024	-0.182	-0.192	-0.221
125	-0.208	-0.386	0.000	-0.164	0.132	-0.161	-0.324	-0.307	-0.323
150	-0.084	-0.111	0.164	0.000	0.428	-0.159	-0.377	-0.343	-0.354
175	-0.170	-0.217	-0.132	-0.428	0.000	-0.745	-0.780	-0.600	-0.550
200	0.013	0.024	0.161	0.159	0.745	0.000	-0.815	-0.527	-0.485
225	0.147	0.182	0.324	0.377	0.780	0.815	0.000	-0.240	-0.320
250	0.160	0.192	0.307	0.343	0.600	0.527	0.240	0.000	-0.400
275	0.190	0.221	0.323	0.354	0.550	0.485	0.320	0.400	0.000

Table 5  
The transition matrix of the number of COVID-19 cases, according to temperature

State	100	110	120	130	140	150	160	170	180
100	0.000	-3.167	-1.200	-0.070	-0.107	0.053	0.652	0.464	-0.036
110	3.167	0.000	0.767	2.958	1.370	1.143	1.143	0.803	0.444
120	1.200	-0.767	0.000	2.191	0.986	0.888	0.787	0.650	0.317
130	0.070	-2.958	-2.191	0.000	-0.218	0.236	0.318	0.265	-0.058
140	0.107	-1.370	-0.986	0.218	0.000	0.690	0.587	0.426	-0.018
150	-0.053	-1.143	-0.888	-0.236	-0.690	0.000	0.484	0.294	-0.254
160	-0.652	-0.978	-0.787	-0.318	-0.587	-0.484	0.000	0.104	-0.623
170	-0.464	-0.803	-0.650	-0.265	-0.426	-0.294	-0.104	0.000	-1.350
180	0.036	-0.444	-0.317	0.058	0.018	0.254	0.623	1.350	0.000

Table 6  
The transition matrix of the number of COVID-19 cases, according to wind

State	25	30	35	40	45	50	55	60
25	0.000	2.400	1.367	0.036	0.958	0.851	0.446	-0.177
30	-2.400	0.000	0.333	-1.145	0.477	0.464	0.055	-0.607
35	-1.367	-0.333	0.000	-2.624	0.549	0.508	-0.015	-0.795
40	-0.036	1.145	2.624	0.000	3.722	2.074	0.855	-0.337
45	-0.958	-0.477	-0.549	-3.722	0.000	0.426	-0.578	-1.690
50	-0.851	-0.464	-0.508	-2.074	-0.426	0.000	-1.582	-2.749
55	-0.446	-0.055	0.015	-0.855	0.578	1.582	0.000	-3.915
60	0.177	0.607	0.795	0.337	1.690	2.749	3.915	0.000

Table 7  
The transition matrix of the number of COVID-19 cases, according to pressure

State	5035	5045	5055	5065	5075	5085
5035	0.000	2.350	0.123	-0.158	-0.436	0.090
5045	-2.350	0.000	-2.104	-1.413	-1.364	-0.475
5055	-0.123	2.104	0.000	-0.721	-0.995	0.068
5065	0.158	1.413	0.721	0.000	-1.268	0.463
5075	0.436	1.364	0.995	1.268	0.000	2.193
5085	-0.090	0.475	-0.068	-0.463	-2.193	0.000

Table 8  
The number of cases, according to chance-constrained model

Date	Cases	$\beta$	Mean	S.D.
17 March	165	0.066667	12.00	3.94
18	182	0.093407		
19	199	0.085427		
20	216	0.078704		
21	233	0.072961		
22	281	0.170819	33.60	11.19
23	329	0.145897		
24	377	0.127321		
25	425	0.112941		
26	473	0.10148		
27	558	0.15233	62.40	17.87
28	643	0.132193		
29	728	0.116758		
30	813	0.104551		
31	898	0.094655		
1 April	975	0.078974	53.40	18.65
2	1052	0.073194		
3	1129	0.068202		
4	1206	0.063847		
5	1283	0.060016		
6	1378	0.06894	63.60	24.83
7	1473	0.064494		
8	1568	0.060587		
9	1663	0.057126		
10	1758	0.054039		
11	1797	0.021703	27.20	9.52
12	1836	0.021242		
13	1875	0.0208		
14	1914	0.020376		
15	1953	0.019969		
16	1999	0.023012	31.80	10.85
17	2045	0.022494		
18	2091	0.021999		
19	2137	0.021526		
20	2183	0.021072		
21	2236	0.023703	37.80	12.07
22	2289	0.023154		
23	2342	0.02263		
24	2395	0.022129		
25	2448	0.02165		
26	2542	0.036979	64.40	23.19
27	2636	0.03566		
28	2730	0.034432		
29	2824	0.033286		
30	2918	0.032214		
1 May	3004	0.028628	69.20	13.14
2	3090	0.027832		
3	3176	0.027078		
4	3262	0.026364		
5	3348	0.025687		
6	3435	0.025328	67.20	15.09
7	3522	0.024702		
8	3609	0.024106		
9	3696	0.023539		
10	3783	0.022998		
11	3910	0.032481	85.20	32.84
12	4037	0.031459		
13	4164	0.0305		
14	4291	0.029597		
15	4418	0.028746		

Table 9  
The average of COVID-19 cases, according to humidity, temperature, wind and pressure (chance-constrained)

Humid.	$\overline{NC}_H$	Temp.	$\overline{NC}_T$	Wind	$\overline{NC}_W$	Pres.	$\overline{NC}_P$
51–75	127	91–100	48	21–25	53	5026–5035	94
76–100	66	101–110	36	26–30	75	5036–5045	94
101–125	81	111–120	43	31–35	72	5046–5055	77
126–150	81	121–130	75	36–40	64	5056–5065	73
151–175	89	131–140	69	41–45	86	5066–5075	56
176–200	69	141–150	70	46–50	76	5076–5085	71
201–225	28	151–160	89	51–55	77		
226–250	25	161–170	84	56–60	43		
251–275	17	171–180	127				

Table 10  
The transition matrix of the number of COVID-19 cases, according to humidity (chance-constrained)

State	75	100	125	150	175	200	225	250	275
75	0.000	-2.430	-0.922	-0.608	-0.380	-0.463	-0.660	-0.583	-0.550
100	2.430	0.000	0.585	0.303	0.303	0.029	-0.306	-0.275	-0.281
125	0.922	-0.585	0.000	0.021	0.162	-0.156	-0.529	-0.447	-0.426
150	0.608	-0.303	-0.021	0.000	0.304	-0.245	-0.712	-0.564	-0.515
175	0.380	-0.303	-0.162	-0.304	0.000	-0.793	-1.220	-0.853	-0.720
200	0.463	-0.029	0.156	0.245	0.793	0.000	-1.647	-0.883	-0.696
225	0.660	0.306	0.529	0.712	1.220	1.647	0.000	-0.120	-0.220
250	0.583	0.275	0.447	0.564	0.853	0.883	0.120	0.000	-0.320
275	0.550	0.281	0.426	0.515	0.720	0.696	0.220	0.320	0.000

Table 11  
The transition matrix of the number of COVID-19 cases, according to temperature (chance-constrained)

State	100	110	120	130	140	150	160	170	180
100	0.000	-1.240	-0.275	0.888	0.530	0.440	0.898	0.596	0.655
110	1.240	0.000	0.690	3.904	1.679	1.147	1.147	0.972	1.523
120	0.275	-0.690	0.000	3.214	1.334	0.917	1.174	0.834	1.408
130	-0.888	-3.904	-3.214	0.000	-0.545	-0.232	0.494	0.239	1.047
140	-0.530	-1.679	-1.334	0.545	0.000	0.082	1.014	0.501	1.445
150	-0.440	-1.147	-0.917	0.232	-0.082	0.000	1.946	0.710	1.900
160	-0.898	-1.347	-1.174	-0.494	-1.014	-1.946	0.000	-0.526	1.877
170	-0.596	-0.972	-0.834	-0.239	-0.501	-0.710	0.526	0.000	4.280
180	-0.655	-1.523	-1.408	-1.047	-1.445	-1.900	-1.877	-4.280	0.000

Table 12  
The transition matrix of the number of COVID-19 cases, according to wind (chance-constrained)

State	25	30	35	40	45	50	55	60
25	0.000	4.400	1.900	0.703	1.635	0.920	0.813	-0.286
30	-4.400	0.000	-0.600	-1.145	0.713	0.050	0.095	-1.067
35	-1.900	0.600	0.000	-1.691	1.369	0.267	0.269	-1.160
40	-0.703	1.145	1.691	0.000	4.429	1.245	0.922	-1.027
45	-1.635	-0.713	-1.369	-4.429	0.000	-1.938	-0.832	-2.846
50	-0.920	-0.050	-0.267	-1.245	1.938	0.000	0.275	-3.300
55	-0.813	-0.095	-0.269	-0.922	0.832	-0.275	0.000	-6.875
60	0.286	1.067	1.160	1.027	2.846	3.300	6.875	0.000

Table 13  
The transition matrix of the number of COVID-19 cases, according to pressure (chance-constrained)

State	5035	5045	5055	5065	5075	5085
5035	0.000	0.000	-0.869	-0.710	-0.943	-0.470
5045	0.000	0.000	-1.738	-1.065	-1.258	-0.588
5055	0.869	1.738	0.000	-0.392	-1.017	-0.204
5065	0.710	1.065	0.392	0.000	-1.643	-0.110
5075	0.943	1.258	1.017	1.643	0.000	1.423
5085	0.470	0.588	0.204	0.110	-1.423	0.000