A 2-order additive fuzzy measure identification method based on intuitionistic fuzzy sets and its application in credit evaluation

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Abstract. In view of the problem that it is difficult to quantitatively assess the interactivity between attributes in the identification process of 2-order additive fuzzy measure, this work uses the intuitionistic fuzzy sets (IFSs) to describe and deal with the interactivity between attributes. Firstly, the interactivity between attributes is defined by the supermodular game theory. On this basis, the experts employ the intuitionistic fuzzy number (IFN) to assess the interactivity between attributes, Secondly, the opinions of all experts are aggregated by using the intuitionistic fuzzy weighted average operator (IFWA). Finally, based on the aggregated results, the intuitionistic fuzzy interaction degree between attributes is defined and calculated by the score function of IFN. Thus, a 2-order additive fuzzy measure identification method based on IFSs is further proposed. Based on the proposed method, using the Choquet fuzzy integral as nonlinear integration operator, a multi-attribute decision making (MADM) process is presented. Taking the credit evaluation of the big data listed companies in China as an application example, the feasibility and effectiveness of the proposed method is verified by the analysis results of application example.

Keywords: Interactivity between attributes, intuitionistic fuzzy sets, 2-order additive fuzzy measure, choquet fuzzy integral, multi-attribute decision making, credit evaluation

1. Introduction

Due to the interaction among attributes, such as complementarity and repeatability, the classical weighted arithmetic mean method is often invalid in the process of multi-attribute decision making (MADM) [1]. In order to overcome this limitation,

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and effectively describe and deal with any interaction among attributes, Sugeno [2] proposed the concept of fuzzy measure and fuzzy integral. Since then, after nearly 20 years of development, the theory of fuzzy measures had been formed [3, 4]. However, in the practical application process, when there are n attributes, the general fuzzy measure generally needs to determine $2^n - 2$ parameters [4, 5]. This complexity greatly limits its practical application ability.

In order to reduce the complexity, Sugeno [2] proposed the λ fuzzy measure, that only needs to determine n parameters, however, the λ fuzzy

measure can only represent a certain kind of interaction among attributes, which weakens its performance ability. Thereafter, in order to face with the complexity of discrete fuzzy measures, Grabisch [6] proposed the k-order additive fuzzy measures, including usual additive measures and fuzzy measures. Every discrete fuzzy measure is a k-order additive fuzzy measure for a unique k. The k-order additive fuzzy measures cover all fuzzy measures with any complexity from classical additive measure ($k = 1$) to general fuzzy measure ($k = n$). Among them, the 2-order additive fuzzy measure not only merely needs to determine $n(n + 1)/2$ parameters, but also merely involves the relative importance of attributes and the interaction between attributes, which well solves the contradiction between complexity and performance ability, so it has been widely used [7].

However, because the interaction between attributes is difficult to explain and understand [8], combined with the uncertainty of decision-makers' cognition, the decision-makers often cannot give accurate quantitative assessment on the interactivity between attributes, but with a certain fuzziness and hesitation, which brings difficulties to decision-making. The existing 2-order additive fuzzy measure identification methods use the subjective methods [9–15] and the objective methods [7, 16, 17] to describe and deal with the interactivity between attributes. The subjective methods are more explanatory than the objective methods, so they have been applied more widely. However, the above methods cannot reflect the fuzziness and hesitation of decision-makers in the assessment of interactivity between attributes.

In [18], Atanassov proposed the concept of intuitionistic fuzzy set (IFS). IFS is an extension of traditional fuzzy set. It considers the information of membership degree, non-membership degree and hesitation degree, so it is more flexible and practical than traditional fuzzy set in dealing with fuzziness and uncertainty [19]. IFS can better and more completely reflect the fuzziness and hesitation of the decision-maker's judgment of the objective things, so it has been successfully applied in the field of uncertain MADM [20–22].

Based on this observation, in view of the problem that it is difficult to quantitatively assess the interactivity between attributes, the present work uses the IFSs to describe and deal with the interactivity between attributes. As a result, a 2-order additive fuzzy measure identification method based on IFSs is proposed. Based on the proposed method, using

the Choquet fuzzy integral as nonlinear integration operator [23], a MADM process is further presented.

This paper is structured as follows: Section 2 introduces the preparatory knowledge employed in this work. Section 3 proposes the 2-order additive fuzzy measure identification method based on IFSs. Section 4 presents the MADM process based on the proposed method. Section 5 describes the analysis results of application example. Section 6 discusses the results obtained and Section 7 concludes this paper.

2. Preparatory knowledge

This section introduces the related definitions of IFSs, 2-order additive fuzzy measure and Choquet fuzzy integral reported in the literatures. This is the basis of Section 3 and Section 4.

Definition 1 [18]. Let X be a non-empty set, then $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \, | x \in X \}$ is called the IFS, where $\mu_A(x)$ and $\nu_A(x)$ are the membership degree and non-membership degree of element x in X belonging to A respectively, then $\mu_A : X \to [0, 1]$, $\nu_A : X \to [0, 1]$ holds, and satisfies $0 \le \mu_A(x) + \nu_A(x) \le 1, \forall x \in X$. In addition, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the hesitation degree of element x in X belonging to A, and satisfies $0 \le \pi_A(x) \le 1, \forall x \in X$. For convenience, let $\alpha = (\mu_\alpha, \nu_\alpha)$ be an IFN, where $\mu_\alpha \in [0, 1]$, $\nu_\alpha \in [0, 1]$, $0 \le \mu_\alpha + \nu_\alpha \le 1$, and let Θ be the set of all IFNs.

Definition 2 [18]. Let $a_1 = (\mu_1, \nu_1)$ and $a_2 = (\mu_2, \nu_2)$ be two IFNs on a given domain; let λ be a real number, and $\lambda > 0$, then the operational rules of IFNs are as follows:

1) $a_1 \oplus a_2 = (\mu_1 + \mu_2 - \mu_1\mu_2, \nu_1\nu_2)$; 2) $a_1 \otimes a_2 = (\mu_1\mu_2, \nu_1 + \nu_2 - \nu_1\nu_2)$; 3) $\lambda a_1 = (1 - (1 - \mu_1)^\lambda, \nu_1^\lambda)$; 4) $a_1^\lambda = (\mu_1^\lambda, 1 - (1 - \nu_1)^\lambda)$.

Definition 3 [24]. Let $\alpha_j = (\mu_{\alpha_j}, \nu_{\alpha_j})$ $(j = 1, 2, \cdots, n)$ be a set of IFNs, w_j is the weight of α_j, where $w_j \in [0, 1]$, and $\sum_{j=1}^{n} w_j = 1$. Let $IFWA : \Theta^n \to \Theta$, if

$$IFWA_w(\alpha_1, \alpha_2, \cdots, \alpha_n) = (1 - \prod_{j=1}^{n}(1 - \mu_{\alpha_j})^{w_j}, \prod_{j=1}^{n} \nu_{\alpha_j}^{w_j})$$

(1)

then IFWA is called the intuitionistic fuzzy weighted average operator.

Definition 4 [25]. Let $U = \{x_1, x_2, \cdots, x_n\}$ be a given domain, $A = \sum_{i=1}^{n} \langle \mu_A(x_i), \nu_A(x_i) \rangle / x_i$ is an IFS, then the intuitionistic fuzzy entropy of A is

$$E(A) = \frac{1}{n} \sum_{i=1}^{n} \frac{\min(\mu_A(x_i), \nu_A(x_i)) + \pi_A(x_i)}{\max(\mu_A(x_i), \nu_A(x_i)) + \pi_A(x_i)} \quad (2)$$

Definition 5 [26, 27]. Let $\alpha = (\mu_\alpha, \nu_\alpha)$ be an IFN, where $\mu_\alpha \in [0, 1]$, $\nu_\alpha \in [0, 1]$, $0 \leq \mu_\alpha + \nu_\alpha \leq 1$, when the hesitation degree is not considered, the score function of IFN α can be expressed as [26]

$$S(\alpha) = \mu_\alpha - \nu_\alpha, \quad S(\alpha) \in [-1, 1] \quad (3)$$

When considering the hesitation degree, the score function of IFN α can be expressed as [27]

$$S_L(\alpha) = \mu_\alpha + \mu_\alpha(1 - \mu_\alpha - \nu_\alpha), \quad S_L(\alpha) \in [0, 1] \quad (4)$$

Definition 6 [2, 6]. Let $X = \{x_1, x_2, \cdots, x_n\}$ be a set of attributes, let $X^* = \{1, 2, \cdots, n\}$ be a set of subscripts of attributes. $P(X)$ is the power set of X. If the set function $g : P(X) \to [0, 1]$ satisfies the following two conditions:

1) $g(\varnothing) = 0$, $g(X) = 1$; 2) If $K \in P(X)$, $T \in P(X)$, $K \subseteq T$, then $g(K) \leq g(T)$.
then g is called a fuzzy measure on $P(X)$ [2].

Grabisch [6] proposed the *k*-order additive fuzzy measures based on pseudo-Boolean function and Möbius transformation. On this basis, the 2-order additive fuzzy measure is then defined as

$$g(K) = \sum_{i \in K^*} m_i + \sum_{\{i, j\} \subset K^*} m_{ij}, \quad \forall K \subseteq X \quad (5)$$

where m_i is the Möbius transformation coefficient of x_i $(i = 1, 2, \cdots, n)$, which is an overall importance; m_{ij} is the Möbius transformation coefficient of $\{x_i, x_j\}$ $(i, j = 1, 2, \cdots, n; i \neq j)$, which represents the extent of interaction between x_i and x_j.

Definition 7 [13]. Let $X = \{x_1, x_2, \cdots, x_n\}$ be a set of attributes, $W = \{w_1, w_2, \cdots, w_n\}$ is the weight set of X, the Möbius transformation coefficients of x_i and $\{x_i, x_j\}$ are respectively

$$\begin{cases} m_i = {w_i}/{P} \\ m_{ij} = {\xi_{ij} w_i w_j}/{P} \end{cases}, \quad i, j = 1, 2, \cdots, n \, ; i \neq j \quad (6)$$

where $P = \sum_{i \in X^*} w_i + \sum_{\{i, j\} \subset X^*} \xi_{ij} w_i w_j$ is the sum of the importance of all x_i and $\{x_i, x_j\}$, ξ_{ij}

is the interaction degree between x_i and x_j, $\xi_{ij} \in [-1, 1]$.

Definition 8 [23]. Let f be a nonnegative function defined on X, F is a $\sigma-$ algebra composed of subsets of X (when X is finite, F is the power set $P(X)$ of X), g is a fuzzy measure defined on F, then the Choquet fuzzy integral of function f on set X for fuzzy measure g is defined as $(c) \int f dg = \int_0^\infty g(F_\alpha) d\alpha$, where $F_\alpha = \{x \, | \, f(x) \geq \alpha, x \in X\}$, $\alpha \in [0, \infty]$; $\int_0^\infty g(F_\alpha) d\alpha$ is the Riemann integral.

When X is a finite set, the elements in X are rearranged as $\{x_{(1)}, x_{(2)}, \cdots, x_{(n)}\}$, which makes $f(x_{(1)}) \leq f(x_{(2)}) \leq \cdots \leq f(x_{(n)})$. Let $H = (c) \int f dg$, then the Choquet fuzzy integral has the following simplified formula:

$$H = (c) \int f dg = \sum_{i=1}^{n} \left[f(x_{(i)}) - f(x_{(i-1)}) \right] g(X_{(i)}) \quad (7)$$

where $X_{(i)} = \{x_{(i)}, x_{(i+1)}, \cdots, x_{(n)}\}$, $(i) = (1), (2), \cdots, (n)$; $f(x_{(0)}) = 0$.

3. A 2-order additive fuzzy measure identification method based on IFSs

Based on IFSs, this section proposes a 2-order additive fuzzy measure identification method. In addition, the correctness of the proposed method is proved theoretically.

Let $A = \{A_1, A_2, \cdots, A_m\}$ be a finite set of alternatives, and $C = \{C_1, C_2, \cdots, C_n\}$ be a set of attributes to compare the alternatives, let $C^* = \{1, 2, \cdots, n\}$ be a set of subscripts of attributes. The weight vector of attributes is $W_C = (w_1, w_2, \cdots, w_n)$, where $w_i \in [0, 1]$, and $\sum_{i=1}^{n} w_i = 1$. Let $D = \{D_1, D_2, \cdots, D_p\}$ be a set of experts, the weight vector of experts is $W_D = (w_1, w_2, \cdots, w_p)$, where $w_l \in [0, 1]$, and $\sum_{l=1}^{p} w_l = 1$. Based on IFSs, the identification process of 2-order additive fuzzy measure is as follows:

Step 1: Defines the interactivity between attributes.

According to the supermodular game theory, the interactivity between attributes is defined as follows:

Definition 9. Let any two attributes C_i and C_j $(i \neq j)$ in attribute set C have partial order relation, the supremum $C_i \vee C_j$ and the infimum $C_i \wedge C_j$ are in C, then C is called a sub-lattice [28]. Let f be a real-valued function defined on the sub-lattice $C, C \subseteq R^n$.

For $\forall C_i, C_j \in C$, when $f(C_i \vee C_j) + f(C_i \wedge C_j) > f(C_i) + f(C_j)$, f is a supermodular function [28], then it is said that there is complementarity between C_i and C_j $(i \neq j)$; when $f(C_i \vee C_j) + f(C_i \wedge C_j) < f(C_i) + f(C_j)$, f is a submodular function [28], then it is said that there is repeatability between C_i and C_j $(i \neq j)$; particularly, when $f(C_i \vee C_j) + f(C_i \wedge C_j) = f(C_i) + f(C_j)$, then it is said that there is independence between C_i and C_j $(i \neq j)$.

Step 2: Calculate the individual assessment result of each expert on the interactivity between C_i and C_j $(i \neq j)$.

According to Definition 9, the expert D_l $(l = 1, 2, \cdots, p)$ employs the IFN $\alpha = (\mu_\alpha, \nu_\alpha)$ to assess the interactivity between C_i and C_j $(i \neq j)$ $(C_n^2$ pairs in total). When the expert thinks that there is complementarity between C_i and C_j $(i \neq j)$, $\mu_\alpha > 0.5$, and the larger μ_α is, the stronger complementarity is. When the expert thinks that there is repeatability between C_i and C_j $(i \neq j)$, $\nu_\alpha > 0.5$, and the larger ν_α is, the stronger repeatability is. In addition, when the expert thinks that C_i and C_j $(i \neq j)$ are independent of each other, $\mu_\alpha = \nu_\alpha = 0.5$. Thus, the individual assessment result $\tilde{\xi}_{ijl} = (\mu_{\tilde{\xi}_{ijl}}, \nu_{\tilde{\xi}_{ijl}})$ $(l = 1, 2, \cdots, p)$ of the interactivity between C_i and C_j $(i \neq j)$ is then given.

Step 3: Calculate the group assessment result of p experts on the interactivity between C_i and C_j $(i \neq j)$.

Given the weight vector W_D of experts, using Equation (1) to aggregate the opinions of p experts, the group assessment result $\tilde{\xi}_{ij}$ of the interactivity between C_i and C_j $(i \neq j)$ is further calculated as

$$\tilde{\xi}_{ij} = (\mu_{\tilde{\xi}_{ij}}, \nu_{\tilde{\xi}_{ij}}) = (1 - \prod_{l=1}^{p}(1 - \mu_{\tilde{\xi}_{ijl}})^{w_l}, \prod_{l=1}^{p} \nu_{\tilde{\xi}_{ijl}}^{w_l})$$
(8)

Step 4: Determine the intuitionistic fuzzy interaction degree $S(\tilde{\xi}_{ij})$ between C_i and C_j $(i \neq j)$.

With Equation (3), the intuitionistic fuzzy interaction degree $S(\tilde{\xi}_{ij})$ between C_i and C_j $(i \neq j)$ is defined as follows:

Definition 10. Let $\tilde{\xi}_{ij} = (\mu_{\tilde{\xi}_{ij}}, \nu_{\tilde{\xi}_{ij}})$ be the group assessment result of the interactivity between C_i and C_j $(i \neq j)$, according to Equation (3), if $S(\tilde{\xi}_{ij})$ satisfies

$$S(\tilde{\xi}_{ij}) = \mu_{\tilde{\xi}_{ij}} - \nu_{\tilde{\xi}_{ij}}, \quad S(\tilde{\xi}_{ij}) \in [-1, 1] \quad (9)$$

then $S(\tilde{\xi}_{ij})$ is called the intuitionistic fuzzy interaction degree between C_i and C_j $(i \neq j)$. If there is comple-

mentarity between C_i and C_j $(i \neq j)$, then $S(\tilde{\xi}_{ij}) > 0$, and the larger $S(\tilde{\xi}_{ij})$ is, the stronger complementarity is. If there is repeatability between C_i and C_j $(i \neq j)$, then $S(\tilde{\xi}_{ij}) < 0$, and the smaller $S(\tilde{\xi}_{ij})$ is, the stronger repeatability is. If C_i and C_j $(i \neq j)$ are independent of each other, then $S(\tilde{\xi}_{ij}) = 0$.

Thus, according to Definition 10, the intuitionistic fuzzy interaction degree $S(\tilde{\xi}_{ij})$ between C_i and C_j $(i \neq j)$ can be determined.

Step 5: Calculate the Möbius transformation coefficients m_i and m_{ij} of attributes.

According to the weight w_i of the i-th attribute and the intuitionistic fuzzy interaction degree $S(\tilde{\xi}_{ij})$ between C_i and C_j $(i \neq j)$, using Equation (6), the Möbius transformation coefficients m_i and m_{ij} of attributes can be calculated as

$$\begin{cases} m_i = w_i / P \\ m_{ij} = S(\tilde{\xi}_{ij})w_i w_j / P \end{cases}, \quad i, j = 1, 2, \cdots, n; i \neq j$$
(10)

where $P = \sum\limits_{i \in C^*} w_i + \sum\limits_{\{i,j\} \subset C^*} S(\tilde{\xi}_{ij})w_i w_j$ is the sum of the importance of all C_i and $\{C_i, C_j\}$ $(i \neq j)$.

Step 6: Identify the 2-order additive fuzzy measure g_K.

According to the Möbius transformation coefficients m_i and m_{ij} of attributes, using Equation (5), the 2-order additive fuzzy measure g_K can be identified as

$$g_K = g(K) = \sum_{i \in K^*} m_i + \sum_{\{i,j\} \subset K^*} m_{ij}, \quad \forall K \subseteq C$$
(11)

Theorem 1. *The fuzzy measure identified by steps 1 to 6 is a 2-order additive fuzzy measure.*

To prove that the fuzzy measure identified by steps 1 to 6 is a 2-order additive fuzzy measure, it is only necessary to prove that the determined Möbius transformation coefficients satisfy the following constrained conditions [6]:

1) $m(\varnothing) = 0$; 2) $m_i \geq 0$, $\forall i \in C^*$; 3) $\sum\limits_{i \in C^*} m_i + \sum\limits_{\{i,j\} \subset C^*} m_{ij} = 1$; 4) $m_i + \sum\limits_{j \in K^* \setminus \{i\}} m_{ij} \geq 0, \forall K \subset C$.

Proof.

1) $m(\varnothing) = 0$, obviously holds.
2) Because $S(\tilde{\xi}_{ij}) = S(\tilde{\xi}_{ji})$, and $\sum_{i=1}^{n} w_i = 1$, P can be further written as

$$P = 1 + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1, i \neq j}^{n} S(\tilde{\xi}_{ij}) w_i w_j$$

$$= 1 + \frac{1}{2} \sum_{i=1}^{n} \left[w_i \sum_{j=1, i \neq j}^{n} S(\tilde{\xi}_{ij}) w_j \right]$$

Since $S(\tilde{\xi}_{ij}) \in [-1, 1]$, and $w_j \in [0, 1]$, we have

$$-w_j \leq S(\tilde{\xi}_{ij}) w_j \leq w_j, i \neq j$$

Sum the two sides of the above inequality to j, we obtain

$$-(1 - w_i) \leq \sum_{j=1, i \neq j}^{n} S(\tilde{\xi}_{ij}) w_j \leq 1 - w_i$$

Multiply both sides of the above inequality by w_i, we can get

$$-(w_i - w_i^2) \leq w_i \sum_{j=1, i \neq j}^{n} S(\tilde{\xi}_{ij}) w_j \leq w_i - w_i^2$$

Sum the two sides of the above inequality to i, the following inequality can be given

$$-(1 - \sum_{i=1}^{n} w_i^2) \leq \sum_{i=1}^{n} \left[w_i \sum_{j=1, i \neq j}^{n} S(\tilde{\xi}_{ij}) w_j \right]$$

$$\leq 1 - \sum_{i=1}^{n} w_i^2$$

Therefore, we have

$$\frac{1}{2} + \frac{1}{2} \sum_{i=1}^{n} w_i^2 \leq 1 + \frac{1}{2} \sum_{i=1}^{n} \left[w_i \sum_{j=1, i \neq j}^{n} S(\tilde{\xi}_{ij}) w_j \right]$$

$$\leq \frac{3}{2} - \frac{1}{2} \sum_{i=1}^{n} w_i^2$$

That is to say

$$0 < \frac{1}{2} + \frac{1}{2} \sum_{i=1}^{n} w_i^2 \leq P$$

Because $P > 0$, and $w_i \geq 0$, we can get $m_i = \frac{w_i}{P} \geq 0$.

3) $\sum_{i \in C^*} m_i + \sum_{\{i, j\} \subset C^*} m_{ij} =$

$$\sum_{i \in C^*} \frac{w_i}{P} + \sum_{\{i, j\} \subset C^*} \frac{S(\tilde{\xi}_{ij}) w_i w_j}{P} =$$

$$\frac{1}{P} \left[\sum_{i \in C^*} w_i + \sum_{\{i, j\} \subset C^*} S(\tilde{\xi}_{ij}) w_i w_j \right] = 1,$$

obviously holds.

4) $m_i + \sum_{j \in K^* \setminus \{i\}} m_{ij} = \frac{w_i}{P} + \sum_{j \in K^* \setminus \{i\}} \frac{S(\tilde{\xi}_{ij}) w_i w_j}{P} =$

$$\frac{w_i}{P} \left[1 + \sum_{j \in K^* \setminus \{i\}} S(\tilde{\xi}_{ij}) w_j \right]$$

Since $S(\tilde{\xi}_{ij}) \in [-1, 1]$, and $w_j \in [0, 1]$, we have $-(1 - w_i) \leq \sum_{j \in K^* \setminus \{i\}} S(\tilde{\xi}_{ij}) w_j \leq 1 - w_i$. Thus, the following inequality can be given

$$\frac{w_i}{P} \left[1 + \sum_{j \in K^* \setminus \{i\}} S(\tilde{\xi}_{ij}) w_j \right]$$

$$\geq \frac{w_i}{P} [1 - (1 - w_i)] \geq \frac{w_i^2}{P} \geq 0$$

Hence, we get $m_i + \sum_{j \in K^* \setminus \{i\}} m_{ij} \geq 0, \ \forall K \subset C$.

Q.E.D.

4. A MADM process based on the proposed method

Using the Choquet fuzzy integral as nonlinear integration operator, this section presents a MADM process based on the proposed method.

Step 1: Calculate the group intuitionistic fuzzy assessment value $\tilde{x}_{jik}$.

Let $C_i = \{C_{i1}, C_{i2}, \cdots, C_{ik}, \cdots, C_{in_i}\}$ ($k = 1, 2, \cdots, n_i$) be a set of sub-attributes under the i-th attribute. According to the type of sub-attribute (such as positive, negative and neutral), the expert D_l ($l = 1, 2, \cdots, p$) employs the IFN to assess the performance of the alternative A_j ($j = 1, 2, \cdots, m$) under the sub-attribute C_{ik} ($i = 1, 2, \cdots, n$), the individual intuitionistic fuzzy assessment value $\tilde{x}_{jikl} = (\mu_{\tilde{x}_{jikl}}, \nu_{\tilde{x}_{jikl}})$ of the k-th sub-attribute of the alternative A_j under the i-th attribute is then given.

Given the weight vector W_D of experts, using Equation (1) to aggregate the opinions of p experts, the group intuitionistic fuzzy assessment value $\tilde{x}_{jik}$ of the k-th sub-attribute of the alternative A_j under the i-th attribute is further calculated as

$$\tilde{x}_{jik} = (\mu_{\tilde{x}_{jik}}, \nu_{\tilde{x}_{jik}}) = (1 - \prod_{l=1}^{p} (1 - \mu_{\tilde{x}_{jikl}})^{w_l}, \prod_{l=1}^{p} \nu_{\tilde{x}_{jikl}}^{w_l})$$

$$(12)$$

Step 2: Calculate the overall weight w_i of the i-th attribute and the relative weight w'_{ik} of the k-th sub-attribute under the i-th attribute.

By using Equation (2), the intuitionistic fuzzy entropy E_{ik} of the k-th sub-attribute under the i-th attribute is calculated as

$$E_{ik} = \frac{1}{m} \sum_{j=1}^{m} \frac{\min(\mu_{\tilde{x}_{jik}}, \nu_{\tilde{x}_{jik}}) + \pi_{\tilde{x}_{jik}}}{\max(\mu_{\tilde{x}_{jik}}, \nu_{\tilde{x}_{jik}}) + \pi_{\tilde{x}_{jik}}} \quad (13)$$

According to the definition of information entropy, if the information entropy of an attribute is smaller, the information quantity provided by the attribute is larger, and the weight of the attribute is also greater [29]. Therefore, the entropy weight w_{ik} of the k-th sub-attribute under the i-th attribute can be expressed as [30]

$$w_{ik} = \frac{1 - E_{ik}}{\sum_{i=1}^{n} n_i - \sum_{i=1}^{n} \sum_{k=1}^{n_i} E_{ik}} \quad (14)$$

That is to say, the overall weight vector of sub-attributes is $W_i = (w_{i1}, w_{i2}, \cdots, w_{ik}, \cdots, w_{in_i}), i = 1, 2, \cdots, n$. The overall weight w_i of the i-th attribute is then obtained as

$$w_i = \sum_{k=1}^{n_i} w_{ik} \quad (15)$$

It can be proved that $\sum_{i=1}^{n} w_i = 1$ holds. Therefore, the overall weight vector of attributes is $W_C = (w_1, w_2, \cdots, w_i, \cdots, w_n)$. The relative weight w'_{ik} of the k-th sub-attribute under the i-th attribute is then obtained as

$$w'_{ik} = \frac{w_{ik}}{w_i} \quad (16)$$

Step 3: Calculate the intuitionistic fuzzy assessment score $S_L(\tilde{x}_{ji})$ of the alternative A_j under the i-th attribute.

According to $\tilde{x}_{jik} = (\mu_{\tilde{x}_{jik}}, \nu_{\tilde{x}_{jik}})$ and w'_{ik}, with Equation (1), the intuitionistic fuzzy assessment value $\tilde{x}_{ji}$ of the alternative A_j under the i-th attribute is calculated as

$$\tilde{x}_{ji} = (\mu_{\tilde{x}_{ji}}, \nu_{\tilde{x}_{ji}}) = (1 - \prod_{k=1}^{n_i}(1 - \mu_{\tilde{x}_{jik}})^{w'_{ik}}, \prod_{k=1}^{n_i} \nu_{\tilde{x}_{jik}}^{w'_{ik}})$$
$$(17)$$

Hence, with Equation (4), the intuitionistic fuzzy assessment score $S_L(\tilde{x}_{ji})$ of the alternative A_j under the i-th attribute is further calculated as

$$S_L(\tilde{x}_{ji}) = \mu_{\tilde{x}_{ji}} + \mu_{\tilde{x}_{ji}}(1 - \mu_{\tilde{x}_{ji}} - \nu_{\tilde{x}_{ji}}), \ S_L(\tilde{x}_{ji}) \in [0, 1]$$
$$(18)$$

Step 4: Identify the 2-order additive fuzzy measure g_K.

Given the weight vector W_D of experts, the intuitionistic fuzzy interaction degree $S(\tilde{\xi}_{ij})$ between C_i and C_j $(i \neq j)$ can be determined by steps 1 to 4 in Section 3. Thus, according to w_i and $S(\tilde{\xi}_{ij})$, the 2-order additive fuzzy measure g_K can be further identified by step 5 and step 6 in Section 3.

Step 5: Calculate the Choquet fuzzy integral values and the ranking of alternatives.

Reordering the intuitionistic fuzzy assessment score $S_L(\tilde{x}_{ji})$ $(i = 1, 2, \cdots, n)$ of the alternative A_j $(j = 1, 2, \cdots, m)$ under the i-th attribute from small to large, the $S_L(\tilde{x}_{j(i)})$ can be then obtained. Substituting the $S_L(\tilde{x}_{j(i)})$ and the 2-order additive fuzzy measure g_K into Equation (7), the Choquet fuzzy integral value H_j of the alternative A_j can be calculated. Simultaneously, the ranking of alternatives can be given, where the larger H_j is, the better the alternative A_j is.

5. Application example

This section takes the credit evaluation of the big data listed companies in China as an application example to illustrate the feasibility and effectiveness of the proposed method (see Section 3).

5.1. Credit evaluation Index system and sample data

Considering the characteristics of big data enterprises [31], and following the principles of selecting indicators, such as scientificalness, objectivity, systematization, functionality, dynamics, relative independence, feasibility (or operability), comparability and so on, a credit evaluation index system for big data enterprises was constructed, as shown in Table 1.

We selected the big data listed companies in the Growth Enterprise Market (GEM) in China – Wangsu Science & Technology Co., Ltd. (300017), Beijing Lanxum Technology Co., Ltd. (300010) and Wuhan Tianyu Information Industry Co., Ltd. (300205) to form a set of alternatives, denoted by $A = A_1, A_2, A_3$. Where the alternative A_1 and A_2 belong to the software service industry, and the alternative A_3 belongs to the electronic components industry. The sample data were the section data of 2016, and the original data were shown in Table 2. Where the original

Table 1
Credit evaluation index system for big data enterprises

Primary Indicators	Secondary Indicators
Debt Paying Ability C_1	Current Ratio C_{11}; Quick Ratio C_{12}; Asset-liability Ratio C_{13}
Operational Capability C_2	Turnover Rate of Accounts Receivable C_{21}; Turnover Rate of Total Assets C_{22}; Inventory Turnover C_{23}
Profitability C_3	Profit Margin of Main Business C_{31}; Return on Equity C_{32}; Return on Total Assets C_{33}
Growth Capability C_4	Net Profit Growth Rate C_{41}; Growth Rate of Main Business C_{42}
Technological Innovation Capability C_5	Development Expenditure C_{51}; Growth Rate of Intangible Assets C_{52}; Number of Invention Patent Applications Announced C_{53}
Industry Growth C_6	Network Attention of Industry C_{61}; Industry Average Net Profit Growth Rate C_{62}

Table 2
Original data

Secondary Indicators (Unit, Types)	A_1	A_2	A_3
C_{11} (—, *)	6.73	1.67	2.48
C_{12} (—, *)	6.49	1.38	1.76
C_{13} (%, −)	15.08	28.52	33.54
C_{21} (time, +)	5.48	2.81	3.40
C_{22} (time, +)	0.73	0.37	0.97
C_{23} (time, +)	17.32	2.78	2.88
C_{31} (%, +)	28.07	15.94	0.28
C_{32} (%, +)	17.08	5.21	1.79
C_{33} (%, +)	20.52	5.83	0.27
C_{41} (%, +)	50.52	130.25	12.38
C_{42} (%, +)	51.67	84.10	9.71
C_{51} (10 thousand yuan, +)	5605	9868	0
C_{52} (%, +)	60.57	51.24	-6.73
C_{53} (piece, +)	83	0	24
C_{61} (time, +)	159	159	457
C_{62} (%, +)	30.77	30.77	33.42

Note: ①"+" indicates positive index, "−" indicates negative index, "*" indicates neutral index; ②The Network Attention of Industry is the overall daily average of Baidu Search Index with the industry name as the key word.

data of the Number of Invention Patent Applications Announced were from Tian Yan Cha website, the original data of the Network Attention of Industry were from Baidu Index website, and the rest of the original data were from East Money website.

5.2. Process and results of credit evaluation

Step 1: Calculate the group intuitionistic fuzzy assessment value $\tilde{x}_{jik}$ of the k-th sub-attribute of the alternative A_j under the i-th attribute.

Table 3
Group intuitionistic fuzzy assessment values of sub-attributes

Sub-attributes	A_1	A_2	A_3
C_{11}	(0.399, 0.429)	(0.868, 0.108)	(0.800, 0.150)
C_{12}	(0.248, 0.664)	(0.868, 0.108)	(0.704, 0.183)
C_{13}	(0.551, 0.266)	(0.720, 0.178)	(0.763, 0.164)
C_{21}	(0.868, 0.108)	(0.610, 0.226)	(0.704, 0.183)
C_{22}	(0.704, 0.183)	(0.308, 0.565)	(0.763, 0.164)
C_{23}	(0.950, 0.050)	(0.566, 0.255)	(0.566, 0.255)
C_{31}	(0.704, 0.183)	(0.500, 0.300)	(0.050, 0.950)
C_{32}	(0.924, 0.070)	(0.459, 0.350)	(0.248, 0.664)
C_{33}	(0.950, 0.050)	(0.566, 0.255)	(0.050, 0.950)
C_{41}	(0.763, 0.164)	(0.950, 0.050)	(0.551, 0.266)
C_{42}	(0.763, 0.164)	(0.950, 0.050)	(0.459, 0.350)
C_{51}	(0.596, 0.235)	(0.868, 0.108)	(0.050, 0.950)
C_{52}	(0.924, 0.070)	(0.868, 0.108)	(0.050, 0.950)
C_{53}	(0.924, 0.070)	(0.050, 0.950)	(0.566, 0.255)
C_{61}	(0.610, 0.226)	(0.610, 0.226)	(0.924, 0.070)
C_{62}	(0.763, 0.164)	(0.763, 0.164)	(0.885, 0.097)

According to the type of sub-attribute (such as positive, negative and neutral), three experts employed the IFN to assess the performance of the alternative A_j under the sub-attribute C_{ik} respectively, the individual intuitionistic fuzzy assessment value $\tilde{x}_{jikl}$ of the k-th sub-attribute of the alternative A_j under the i-th attribute was then given.

Adopting the cycle mutual evaluation method [32], the weight vector of experts was calculated as $W_D = (0.3976, 0.3012, 0.3012)$ (see Appendix A for full calculation principle and process).

Using Equation (12) to aggregate the opinions of three experts, the group intuitionistic fuzzy assessment value $\tilde{x}_{jik}$ of the k-th sub-attribute of the alternative A_j under the i-th attribute was further obtained, as shown in Table 3.

Step 2: Determine the weights of attributes.

Based on Table 3, with Equations (13) and (14), the intuitionistic fuzzy entropy E_{ik} and entropy weight w_{ik} of the k-th sub-attribute under the i-th attribute were calculated. In addition, with Equation (15), the overall weight vector W_C of attributes was also obtained. With Equation (16), the relative weight w'_{ik} of the k-th sub-attribute under the i-th attribute was then obtained. The above calculation results were shown in Table 4.

Step 3: Calculate the intuitionistic fuzzy assessment value $\tilde{x}_{ji}$ of the alternative A_j under the i-th attribute.

Based on Tables 3 and 4, using Equation (17), the intuitionistic fuzzy assessment value $\tilde{x}_{ji}$ of the alternative A_j under the i-th attribute was calculated as $\tilde{x}_{11} = (0.405, 0.433)$, $\tilde{x}_{12} = (0.876, 0.099)$, $\tilde{x}_{13}$

Table 4
Weights of attributes

Attri-butes	Overall Weights	Sub-attributes	Intuitionistic Fuzzy Entropy	Entropy Weights	Relative Weights
C_1	0.170	C_{11}	0.445	0.052	0.304
		C_{12}	0.319	0.063	0.373
		C_{13}	0.412	0.055	0.323
C_2	0.171	C_{21}	0.338	0.062	0.361
		C_{22}	0.425	0.054	0.314
		C_{23}	0.406	0.055	0.325
C_3	0.181	C_{31}	0.376	0.058	0.321
		C_{32}	0.453	0.051	0.282
		C_{33}	0.229	0.072	0.397
C_4	0.121	C_{41}	0.316	0.064	0.528
		C_{42}	0.389	0.057	0.472
C_5	0.226	C_{51}	0.243	0.071	0.312
		C_{52}	0.094	0.084	0.374
		C_{53}	0.239	0.071	0.314
C_6	0.131	C_{61}	0.363	0.059	0.453
		C_{62}	0.231	0.072	0.547

$= (0.900, 0.083)$, $\tilde{x}_{14} = (0.763, 0.164)$, $\tilde{x}_{15} = (0.872, 0.102)$, $\tilde{x}_{16} = (0.703, 0.189)$; $\tilde{x}_{21} = (0.832, 0.127)$, $\tilde{x}_{22} = (0.517, 0.313)$, $\tilde{x}_{23} = (0.517, 0.294)$, $\tilde{x}_{24} = (0.950, 0.050)$, $\tilde{x}_{25} = (0.755, 0.214)$, $\tilde{x}_{26} = (0.703, 0.189)$; $\tilde{x}_{31} = (0.756, 0.166)$, $\tilde{x}_{32} = (0.688, 0.197)$, $\tilde{x}_{33} = (0.111, 0.859)$, $\tilde{x}_{34} = (0.510, 0.302)$, $\tilde{x}_{35} = (0.257, 0.629)$, $\tilde{x}_{36} = (0.905, 0.083)$.

Step 4: Calculate the intuitionistic fuzzy assessment score $S_L(\tilde{x}_{ji})$ of the alternative A_j under the i-th attribute.

According to the intuitionistic fuzzy assessment value $\tilde{x}_{ji}$ of the alternative A_j under the i-th attribute, with Equation (18), the intuitionistic fuzzy assessment score $S_L(\tilde{x}_{ji})$ of the alternative A_j under the i-th attribute was calculated as $S_L(\tilde{x}_{11}) = 0.4709$, $S_L(\tilde{x}_{12}) = 0.8976$, $S_L(\tilde{x}_{13}) = 0.9151$, $S_L(\tilde{x}_{14}) = 0.8192$, $S_L(\tilde{x}_{15}) = 0.8949$, $S_L(\tilde{x}_{16}) = 0.7789$; $S_L(\tilde{x}_{21}) = 0.8662$, $S_L(\tilde{x}_{22}) = 0.6048$, $S_L(\tilde{x}_{23}) = 0.6148$, $S_L(\tilde{x}_{24}) = 0.9500$, $S_L(\tilde{x}_{25}) = 0.7786$, $S_L(\tilde{x}_{26}) = 0.7789$; $S_L(\tilde{x}_{31}) = 0.8147$, $S_L(\tilde{x}_{32}) = 0.7671$, $S_L(\tilde{x}_{33}) = 0.1140$, $S_L(\tilde{x}_{34}) = 0.6054$, $S_L(\tilde{x}_{35}) = 0.2868$, $S_L(\tilde{x}_{36}) = 0.9156$.

Step 5: Determine the intuitionistic fuzzy interaction degree $S(\tilde{\xi}_{ij})$ between C_i and C_j ($i \neq j$).

According to step 2 in Section 3, three experts employed the IFN $\alpha = (\mu_\alpha, \nu_\alpha)$ to assess the interactivity between C_i and C_j ($i \neq j$) (C_6^2 pairs in total) respectively. Thus, the individual assessment result $\tilde{\xi}_{ijl}$ of the interactivity between C_i and C_j ($i \neq j$) was then given, as shown in Table 5.

According to step 3 in Section 3, given the weight vector $W_D = (0.3976, 0.3012, 0.3012)$ of the experts, using Equation (8) to aggregate the opinions

of three experts, the group assessment result $\tilde{\xi}_{ij}$ of the interactivity between C_i and C_j ($i \neq j$) was obtained (see Table 5). Taking $\tilde{\xi}_{12}$ as an example, we had

$$\tilde{\xi}_{12} = (1 - (1 - 0.50)^{0.3976} \times (1 - 0.65)^{0.3012}$$
$$\times (1 - 0.50)^{0.3012},\ 0.30^{0.3976} \times 0.20^{0.3012}$$
$$\times 0.30^{0.3012}) = (0.551, 0.266).$$

With Equation (9), the intuitionistic fuzzy interaction degree $S(\tilde{\xi}_{ij})$ between C_i and C_j ($i \neq j$) was calculated, as shown in Table 5. Taking $S(\tilde{\xi}_{12})$ as an example, we had $S(\tilde{\xi}_{12}) = 0.551 - 0.266 = 0.285$.

Step 6: Calculate the Möbius transformation coefficients m_i and m_{ij} of attributes.

Based on Tables 4 and 5, using Equation (10), the Möbius transformation coefficients m_i and m_{ij} of attributes were calculated, as shown in Table 6, where $P = 1.125$. Taking m_{12} as an example, we had

$$m_{12} = (0.285 \times 0.170 \times 0.171)/1.125 = 0.007.$$

Step 7: Identify the 2-order additive fuzzy measure g_K.

Based on Table 6, using Equation (11), the 2-order additive fuzzy measure g_K was calculated, as shown in Table 7. Taking $g_{\{1,2\}}$ as an example, we had

$$g_{\{1,2\}} = 0.151 + 0.152 + 0.007 = 0.310.$$

Step 8: Calculate the Choquet fuzzy integral values and the ranking of alternatives.

Take the alternative A_1 as an example. Reordering the intuitionistic fuzzy assessment score $S_L(\tilde{x}_{1i})$ ($i = 1, 2, \cdots, 6$) of alternative A_1 under the i-th attribute from small to large, we can get

$$S_L(\tilde{x}_{11}) < S_L(\tilde{x}_{16}) < S_L(\tilde{x}_{14}) < S_L(\tilde{x}_{15})$$
$$< S_L(\tilde{x}_{12}) < S_L(\tilde{x}_{13}).$$

It can be denoted by

$$S_L(\tilde{x}_{1(1)}) < S_L(\tilde{x}_{1(2)}) < S_L(\tilde{x}_{1(3)}) < S_L(\tilde{x}_{1(4)})$$
$$< S_L(\tilde{x}_{1(5)}) < S_L(\tilde{x}_{1(6)}).$$

Substituting the $S_L(\tilde{x}_{1(i)})$ and the 2-order additive fuzzy measure g_K into Equation (7), the Choquet fuzzy integral value of the alternative A_1 was

Table 5
Intuitionistic fuzzy interaction degrees between attributes

Attributes C_i and C_j ($i \neq j$)	Individual Assessment Results of Interactivity			Group Assessment Results of Interactivity	Intuitionistic Fuzzy Interaction Degrees
	Expert 1	Expert 2	Expert 3		
$\{C_1, C_2\}$	(0.50, 0.30)	(0.65, 0.20)	(0.50, 0.30)	(0.551, 0.266)	0.285
$\{C_1, C_3\}$	(0.65, 0.20)	(0.80, 0.15)	(0.80, 0.15)	(0.750, 0.168)	0.582
$\{C_1, C_4\}$	(0.50, 0.30)	(0.65, 0.20)	(0.50, 0.30)	(0.551, 0.266)	0.285
$\{C_1, C_5\}$	(0.50, 0.30)	(0.65, 0.20)	(0.50, 0.30)	(0.551, 0.266)	0.285
$\{C_1, C_6\}$	(0.50, 0.30)	(0.50, 0.50)	(0.50, 0.30)	(0.500, 0.350)	0.150
$\{C_2, C_3\}$	(0.65, 0.20)	(0.80, 0.15)	(0.80, 0.15)	(0.750, 0.168)	0.582
$\{C_2, C_4\}$	(0.50, 0.30)	(0.50, 0.30)	(0.65, 0.20)	(0.551, 0.266)	0.285
$\{C_2, C_5\}$	(0.50, 0.30)	(0.50, 0.30)	(0.65, 0.20)	(0.551, 0.266)	0.285
$\{C_2, C_6\}$	(0.50, 0.30)	(0.50, 0.50)	(0.50, 0.30)	(0.500, 0.350)	0.150
$\{C_3, C_4\}$	(0.65, 0.20)	(0.80, 0.15)	(0.65, 0.20)	(0.704, 0.183)	0.521
$\{C_3, C_5\}$	(0.50, 0.30)	(0.65, 0.20)	(0.65, 0.20)	(0.596, 0.235)	0.361
$\{C_3, C_6\}$	(0.50, 0.30)	(0.65, 0.20)	(0.65, 0.20)	(0.596, 0.235)	0.361
$\{C_4, C_5\}$	(0.50, 0.30)	(0.65, 0.20)	(0.65, 0.20)	(0.596, 0.235)	0.361
$\{C_4, C_6\}$	(0.20, 0.65)	(0.15, 0.80)	(0.15, 0.80)	(0.170, 0.736)	−0.566
$\{C_5, C_6\}$	(0.50, 0.30)	(0.50, 0.50)	(0.50, 0.30)	(0.500, 0.350)	0.150

Table 6
Calculation results of Möbius transformation coefficients

Möbius Transformation Coefficients	Coefficient Values	Möbius Transformation Coefficients	Coefficient Values	Möbius Transformation Coefficients	Coefficient Values
m_1	0.151	m_{13}	0.016	m_{26}	0.003
m_2	0.152	m_{14}	0.005	m_{34}	0.010
m_3	0.161	m_{15}	0.010	m_{35}	0.013
m_4	0.108	m_{16}	0.003	m_{36}	0.008
m_5	0.201	m_{23}	0.016	m_{45}	0.009
m_6	0.116	m_{24}	0.005	m_{46}	−0.008
m_{12}	0.007	m_{25}	0.010	m_{56}	0.004

Table 7
Calculation results of 2-order additive fuzzy measure

K	g_K	K	g_K	K	g_K	K	g_K
$\{\emptyset\}$	0.000	$\{3, 4\}$	0.279	$\{2, 3, 4\}$	0.452	$\{1, 3, 4, 5\}$	0.683
$\{1\}$	0.151	$\{3, 5\}$	0.375	$\{2, 3, 5\}$	0.553	$\{1, 3, 4, 6\}$	0.570
$\{2\}$	0.152	$\{3, 6\}$	0.285	$\{2, 3, 6\}$	0.456	$\{1, 3, 5, 6\}$	0.683
$\{3\}$	0.161	$\{4, 5\}$	0.317	$\{2, 4, 5\}$	0.484	$\{1, 4, 5, 6\}$	0.599
$\{4\}$	0.108	$\{4, 6\}$	0.216	$\{2, 4, 6\}$	0.376	$\{2, 3, 4, 5\}$	0.684
$\{5\}$	0.201	$\{5, 6\}$	0.321	$\{2, 5, 6\}$	0.486	$\{2, 3, 4, 6\}$	0.571
$\{6\}$	0.116	$\{1, 2, 3\}$	0.503	$\{3, 4, 5\}$	0.501	$\{2, 3, 5, 6\}$	0.684
$\{1, 2\}$	0.310	$\{1, 2, 4\}$	0.428	$\{3, 4, 6\}$	0.395	$\{2, 4, 5, 6\}$	0.600
$\{1, 3\}$	0.328	$\{1, 2, 5\}$	0.531	$\{3, 5, 6\}$	0.503	$\{3, 4, 5, 6\}$	0.621
$\{1, 4\}$	0.264	$\{1, 2, 6\}$	0.433	$\{4, 5, 6\}$	0.430	$\{1, 2, 3, 4, 5\}$	0.874
$\{1, 5\}$	0.362	$\{1, 3, 4\}$	0.451	$\{1, 2, 3, 4\}$	0.631	$\{1, 2, 3, 4, 6\}$	0.753
$\{1, 6\}$	0.271	$\{1, 3, 5\}$	0.552	$\{1, 2, 3, 5\}$	0.737	$\{1, 2, 3, 5, 6\}$	0.871
$\{2, 3\}$	0.329	$\{1, 3, 6\}$	0.455	$\{1, 2, 3, 6\}$	0.633	$\{1, 2, 4, 5, 6\}$	0.776
$\{2, 4\}$	0.265	$\{1, 4, 5\}$	0.483	$\{1, 2, 4, 5\}$	0.658	$\{1, 3, 4, 5, 6\}$	0.806
$\{2, 5\}$	0.363	$\{1, 4, 6\}$	0.375	$\{1, 2, 4, 6\}$	0.543	$\{2, 3, 4, 5, 6\}$	0.807
$\{2, 6\}$	0.271	$\{1, 5, 6\}$	0.485	$\{1, 2, 5, 6\}$	0.657	$\{1, 2, 3, 4, 5, 6\}$	1.000

calculated as

$$H_1 = (0.4709 - 0.0000) \times 1.000 + (0.7789 - 0.4709)$$

$$\times 0.807 + (0.8192 - 0.7789) \times 0.684 + (0.8949 - 0.8192)$$

$$\times 0.553 + (0.8976 - 0.8949) \times 0.329 + (0.9151 - 0.8976)$$

$$\times 0.161 = 0.7927.$$

Similarly, we can also obtain $H_2 = 0.7652$ and $H_3 = 0.5157$. Since $H_1 > H_2 > H_3$, then the ranking of alternatives was $A_1 \succ A_2 \succ A_3$.

That is to say, the credit status of the alternative A_1 was relatively good, and the credit status of the alternative A_3 was relatively poor.

Table 8
Scoring interaction degrees between attributes

Attributes C_i and C_j ($i \neq j$)	Individual Scoring Interaction Degrees			Group Scoring Interaction Degrees
	Expert 1	Expert 2	Expert 3	
$\{C_1, C_2\}$	0.20	0.30	0.20	0.23
$\{C_1, C_3\}$	0.30	0.50	0.50	0.42
$\{C_1, C_4\}$	0.20	0.30	0.20	0.23
$\{C_1, C_5\}$	0.20	0.30	0.20	0.23
$\{C_1, C_6\}$	0.20	0.00	0.20	0.14
$\{C_2, C_3\}$	0.30	0.50	0.50	0.42
$\{C_2, C_4\}$	0.20	0.20	0.30	0.23
$\{C_2, C_5\}$	0.20	0.20	0.30	0.23
$\{C_2, C_6\}$	0.20	0.00	0.20	0.14
$\{C_3, C_4\}$	0.30	0.50	0.30	0.36
$\{C_3, C_5\}$	0.20	0.30	0.30	0.26
$\{C_3, C_6\}$	0.20	0.30	0.30	0.26
$\{C_4, C_5\}$	0.20	0.30	0.30	0.26
$\{C_4, C_6\}$	–0.30	–0.50	–0.50	–0.42
$\{C_5, C_6\}$	0.20	0.00	0.20	0.14

Note: The weighted arithmetic mean method was employed to aggregate the opinions of three experts.

5.3. Comparative analysis

For comparison, we also replaced the intuitionistic fuzzy interaction degrees between attributes (see Table 5) in step 5 of Section 5.2 with the scoring interaction degrees between attributes (see Table 8) determined by the interaction degrees scoring method [13]. Ceteris paribus, the Choquet fuzzy integral values of alternatives were calculated as $H_1' = 0.7946$, $H_2' = 0.7686$, $H_3' = 0.5244$. Since $H_1' > H_2' > H_3'$, then the ranking of alternatives was $A_1 \succ A_2 \succ A_3$.

We further investigated the discrimination of the MADM method based on the intuitionistic fuzzy interaction degrees (hereinafter referred to as Method 1) for alternatives, as well as the MADM method based on the scoring interaction degrees (hereinafter referred to as Method 2). Adopting the algorithm of discrimination given in [33], the discrimination of Method 1 for alternatives was calculated as

$$\rho = \frac{0.7927 - 0.7652}{0.7927} + \frac{0.7927 - 0.5157}{0.7927}$$
$$+ \frac{0.7652 - 0.5157}{0.7652} = 0.7102.$$

Simultaneously, the discrimination of Method 2 for alternatives was calculated as

$$\rho' = \frac{0.7946 - 0.7686}{0.7946} + \frac{0.7946 - 0.5244}{0.7946}$$
$$+ \frac{0.7686 - 0.5244}{0.7686} = 0.6903.$$

Although the ranking result of Method 2 is consistent with that of Method 1, the discrimination of Method 1 for alternatives is higher than that of Method 2. Which means that the decision-making result of Method 1 is better than that of Method 2.

6. Discussion

From the results and analysis of the previous section, we observed that Method 1 was able to obtain the higher discrimination value than Method 2 (Method 1 was 0.7102, and Method 2 was 0.6903), and the slightly lower Choquet fuzzy integral mean value than Method 2 (Method 1 was 0.6912, and Method 2 was 0.6959).

Compared with Method 2, Method 1 can obtain the higher discrimination value. Since the variance of interaction degrees determined by Method 1 (its value was equal to 0.0684) was higher than that of Method 2 (its value was equal to 0.0355). According to Equation (6), the variances of m_i and m_{ij} increase with the increase of the variance of interaction degrees. From Equations (5) and (7), we can further see that the variance of $g(K)$ and the variance of H also increase correspondingly. Thus, according to the algorithm of discrimination [33], the discrimination value becomes larger.

Compared with Method 2, Method 1 can obtain the slightly lower Choquet fuzzy integral mean value. Since the average value of interaction degrees determined by Method 1 (its value was equal to 0.2718) was higher than that of Method 2 (its value was equal to 0.2087). According to Equation (6), when the average value of interaction degrees increases, the mean value of P increases, meanwhile, the mean value of m_i decreases and the mean value of m_{ij} increases. However, because $|m_i| \gg |m_{ij}|$, from Equation (5), we can further see that the mean value of $g(K)$ also decreases correspondingly. Thus, according to Equation (7), the mean value of H becomes smaller.

The variance or average value of interaction degrees determined by Method 1 was higher than that of Method 2, which was closely related to that the experts used the IFN to assess the interactivity between attributes, so it can fully express their opinions.

7. Conclusion

In this paper, the proposed method defines the interactivity between attributes by using the

supermodular game theory, so that the interaction between attributes is easier to explain and understand, which lays a solid foundation for experts to qualitatively assess the interactivity between attributes. The proposed method allows the experts to assess the interactivity between attributes by using the IFN, which better preserves the assessment information of experts and embodies the fuzziness and hesitation of interactivity assessment. Furthermore, the proposed method uses the IFWA to aggregate the opinions of all experts, which considers the weights of experts, thereby ensuring the rationality of decision-making. In addition, the proposed method uses the score function of IFN to define and calculate the intuitionistic fuzzy interaction degree between attributes, so the transformation from qualitative description to quantitative characterization is finally realized. Therefore, using the IFSs, this work successfully solves the problem that it is difficult to quantitatively assess the interactivity between attributes in the identification process of 2-order additive fuzzy measure.

This work proposed a 2-order additive fuzzy measure identification method based on IFSs. Obviously, on the one hand, compared with the objective method describing and dealing with the interactivity between attributes (as in [7, 16] and [17]), the proposed method has subjectivity. On the other hand, compared with the subjective method describing and dealing with the interactivity between attributes (as in [9–14] and [15]), the proposed method has fuzziness and hesitation.

Future application example analysis will consider increasing the number of samples and experts to improve the persuasiveness of application example analysis results and the stability of interactivity assessment. Furthermore, because the sample time may affect the analysis results of application example, it is necessary to update the sample data from 2016 to 2020 in the future. In addition, Method 1 should be compared with other methods except Method 2, such as the diamond pairwise comparisons method [9, 10, 14], proportional scaling method [11], multicriteria correlation preference information method [12], etc.

According to the supermodular game theory, Definition 9 gives the definition of interactivity between attributes. However, this definition is still relatively general. The detailed theoretical analysis of the connotation of interactivity between attributes needs to be completed in the future. The existing 2-order additive fuzzy measure identification methods use the subjective methods and the objective methods to describe and deal with the interactivity between

attributes. Thus, combining the results of subjective methods and objective methods is one of the main research directions in the future. In addition, constructing a Group Decision Support System will reduce the complexity of Method 1 in practical application.

Acknowledgments

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Appendix A

In step 1 of Section 5.2, the weight vector of experts was calculated by using the cycle mutual evaluation method [32]. Its calculation principle is as follows:

Suppose there are p experts, b_{ij} is the mutual evaluation weight given by expert i to expert j, then the mutual evaluation weight matrix is $B = (b_{ij})_{p \times p}$, where $0 \le b_{ij} \le 1$, and $\sum_{j=1}^{p} b_{ij} = 1$. Then the weight of expert j given by the group is $q_j = p^{-1} \sum_{i=1}^{p} b_{ij}$. Therefore, the final weights of experts are determined through the cycle evaluation. Let t be the number of mutual evaluations, when $t = 1$, the weight vector of experts is $q^1 = (q_1, q_2, \cdots, q_p)$; when $t > 1$, the weight vector of experts is $q^t = q^{t-1}B$, until q^t converges to the stable value.

In this paper, the mutual evaluation weight matrix was given by three experts, it was

$$B = \begin{bmatrix} 0.50 & 0.25 & 0.25 \\ 0.38 & 0.31 & 0.31 \\ 0.28 & 0.36 & 0.36 \end{bmatrix}$$

When $t = 1$, the weight vector of experts was $q^1 = (0.3866, 0.3067, 0.3067)$; when $t = 5$, the weight vector of experts converged to the stable value $q^5 = (0.3976, 0.3012, 0.3012)$. Therefore, the final weight vector of experts was $W_D = (0.3976, 0.3012, 0.3012)$.