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Book Review

Fabio Cuzzolin, *The Geometry of Uncertainty: The Geometry of Imprecise Probabilities* Springer, Cham, Switzerland, 2021

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Need for probabilities. In many real-life situations, we cannot predict what will happen in the future, we can only predict the probabilities of different future events. In most of these cases, the inability to make exact predictions is caused by the fact that we do not have all the information needed for this prediction – this happens, e.g., when we predict tomorrow's weather. In some cases – e.g., in quantum physics – this inability to make exact predictions has a fundamental nature.

In all these cases, we need to describe and predict the probabilities of different events.

In addition to such *objective* probabilities, there are also *subjective* probabilities: numerical values that describe our degree of belief in different events.

Need for imprecise probabilities. Usually, we only have partial information about the initial state of the system. For different values of the missing information, we can have different probability distributions. As a result, in practice, what we can predict is not a unique probability distribution, but the whole class of possible probability distributions. This situation is known as situation of *imprecise probabilities.*

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Similarly, an expert often cannot describe his/her degree of belief in a certain event by a single number, the expert is often more confident by describing it by a range of numbers. Also, different experts may provide us with different degrees of belief. In all these cases, we also encounter imprecise probabilities.

How uncertainties are studied now. Because uncertainty is ubiquitous, there are thousands of research books and papers studying and analyzing uncertainty. If you look at these books and papers, you will see a lot of formulas and very few pictures. This is because methods which are used to study uncertainty are mostly analytical.

Need for visual (geometric) techniques. It is much easier for us humans to understand and process information if it is presented in a visual form. The popular saying that a picture is worth a thousand words is absolutely correct. We do get much more information from pictures than from text. Visualization techniques help us better analyze data. According to psychology, we operate the best if we combine visual representation and visual thinking with logical ones.

At present, in uncertainty studies, there is an overemphasis on analytical methods. Time has come to use more geometric techniques as well.

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Combination of geometric and algebraic methods helped in the past. This is not the first time when in some mathematics-related discipline, there is an overemphasis on one type of representations. The same happened in the old days, when geometry and algebra were two mostly unrelated disciplines – until Descartes came up with the idea of a geometric representation of numerical dependencies – and analytical representation of geometric objects. This revolutionary invention of what we now call Cartesian coordinates – after a Latinized form of Descates' name that he used in his Latin-language research publications – tremendously boosted both algebra and geometry: by graphing functions, we could see their properties like monoticity and convexity very clearly, and by reformulating difficult-to-prove geometric properties in algebraic terms, Descartes found a truly "royal" (much-easier-to-handle) way to geometry, a way that Euclid himself believed not to exist.

This is what the book does. This is exactly what the book does: it describes a new *geometric* approach to uncertainty, an approach that supplements the usual analytical analysis. This is not yet the end of the road: there are many remaining open problems, many challenges – but a lot of interesting results have been already achieved.

What type of uncertainty does the book cover. The main topic covered by the book is the most frequently used imprecise probabilities formalism – the technique of belief functions, also known as evidence theory and as the Dempster-Shafer approach. However, the book also explores the use of geometric techniques in more general imprecise probability approaches.

The book provide an encyclopedic description of different interpretations, analyzes what is the relation between different interpretations, and lists practically all criticisms and counter-criticisms – this book's bibliography has more than 2000 references!

In addition to this, the book provides, in Chapter 5, a guide for the working scientists on how to use the belief theory to solve practical problems – and similar parts are included in the following material as well.

So what *is* the geometric approach? The idea itself is very straightforward – e.g., each belief function is represented by a vector that contains the values $\mathrm{Bel}(S)$ for all possible subsets S of the universal set U. What is new and interesting is that many concepts – such as the Dempster combination rule – can be interpreted in these geometric terms, and this interpretation helps us both to better understand the known results about this rule and to come up with new unexpected results. A similar geometric interpretation can be obtained (and can be useful) for vectors describing other uncertainty characteristics such as mass, plausibility, possibility, etc.

There are many other topics clarified by such geometric interpretations: e.g., transformations between different representations of uncertainty, approximation of one type of representation by another one (when an exact transformation is not possible), "conditioning" (similar to computing conditional probabilities), decision making under different types of uncertainty, etc.; overall there are too many topics to include in a single review.

Summary. This book summarizes the current approaches to uncertainty, and introduces a new geometric approach. It is highly recommended to those who want to (or need to) use the new approach – and upon reading the introductory chapters to this book, many practitioners will understand why, in many cases, they indeed *need* to go beyond traditional probabilistic techniques. The book is highly recommended to students – it is well written, well illustrated, and provides a good introduction to the subject. Finally, it is highly recommended to researchers – in the framework of the new geometric approach, there are many challenging open problems, and the success of the results-so-far is a good indication that solving these problems will not just lead to interesting mathematics – it will, in all probability (pun intended :-), also lead to useful practical applications.