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Erratum

Correction to "Pythagorean fuzzy interaction aggregation operators and their application to multiple attribute decision making"

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Abstract. In this erratum, we point out that two theorems and six equations in a previous paper by G. Wei (2017) are incorrect by the Pythagorean fuzzy operations in detail, and present the modified theorems and equations. The original paper appeared as Wei, G. (2017). Pythagorean fuzzy interaction aggregation operators and their application to multiple attribute decision making. *Journal of Intelligent & Fuzzy Systems*, 33(4), 2119–2132. https://dx.doi.org/10.3233/jifs-162030.

1. Introduction

More recently, Pythagorean fuzzy set (PFS) [1, 2] has emerged as an effective tool for depicting uncertainty of the MADM problems. The PFS is also characterized by the membership degree and the non-membership degree, whose sum of squares is less than or equal to 1, the PFS is more general than the IFS. In some cases, the PFS can solve the problems that the IFS cannot, for example, if a DM gives the membership degree and the non-membership degree as 0.8 and 0.6, respectively, then it is only valid for the PFS. In other words, all the intuitionistic fuzzy degrees are a part of the Pythagorean fuzzy degrees, which indicates that the PFS is more powerful to handle the uncertain problems. Zhang and Xu [3] provided the detailed mathematical expression for PFS and introduced the concept of Pythagorean fuzzy number(PFN). Meanwhile, they also developed a Pythagorean fuzzy TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) for handling the MCDM problem within PFNs. Wei [4] utilized arithmetic and geometric operations to develop some Pythagorean fuzzy interaction aggregation operators: Pythagorean fuzzy interaction weighted average (PFIWA) operator, Pythagorean fuzzy interaction weighted geometric (PFIWG) operator, Pythagorean fuzzy interaction ordered weighted average (PFIOWA) operator, Pythagorean fuzzy interaction ordered weighted geometric (PFIOWG) operator, Pythagorean fuzzy interaction hybrid average (PFIHA) operator and Pythagorean fuzzy interaction hybrid geometric (PFIHG) operator and studied the prominent characteristics of these proposed operators.

The aim of this paper is to point out some errors to the Theorems 1, Theorem 10 and some equations in Wei [4] and we propose the revised theorems, equations and their proof.

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2. Preliminaries

The basic concepts of PFNs [1, 2] are briefly reviewed in this section.

Definition 1. [1, 2] Let X be a fix set. A PFS is an object having the form

$$P = \{\langle x, (\mu_P(x), \nu_P(x))\rangle | x \in X\} \quad (1)$$

where the function $\mu_P : X \to [0, 1]$ defines the degree of membership and the function $\nu_P : X \to [0, 1]$ defines the degree of non-membership of the element $x \in X$ to P, respectively, and, for every $x \in X$, it holds that

$$\left(\mu_p(x)\right)^2 + \left(\nu_p(x)\right)^2 \leqslant 1. \quad (2)$$

For convenience, Zhang and Xu [3] called $\tilde{a} = (\mu, \nu)$ a Pythagorean fuzzy number (PFN).

Definition 2. [4] Let $\tilde{a} = (\mu, \nu)$ be a Pythagorean fuzzy number, a score function S of a Pythagorean fuzzy number can be represented as follows:

$$S(\tilde{a}) = \frac{1}{2}\left(1 + \mu^2 - \nu^2\right), \; S(\tilde{a}) \in [0, 1]. \quad (3)$$

Definition 3. [5] Let $\tilde{a} = (\mu, \nu)$ be a Pythagorean fuzzy number, an accuracy function H of a Pythagorean fuzzy number can be represented as follows:

$$H(\tilde{a}) = \mu^2 + \nu^2, \; H(\tilde{a}) \in [0, 1]. \quad (4)$$

to evaluate the degree of accuracy of the Pythagorean fuzzy number $\tilde{a} = (\mu, \nu)$, where $H(\tilde{a}) \in [0, 1]$. The larger the value of $H(\tilde{a})$, the more the degree of accuracy of the Pythagorean fuzzy number $\tilde{a}$.

Based on the score function S and the accuracy function H, Wei [4] gave an order relation between two Pythagorean fuzzy numbers, which is defined as follows:

Definition 4. [4] Let $\tilde{a}_1 = (\mu_1, \nu_1)$ and $\tilde{a}_2 = (\mu_2, \nu_2)$ be two Pythagorean fuzzy number, $s(\tilde{a}_1)$ and $s(\tilde{a}_2)$ be the scores of $\tilde{a}$ and $\tilde{b}$, respectively, and let $H(\tilde{a}_1)$ and $H(\tilde{a}_2)$ be the accuracy degrees of $\tilde{a}$ and $\tilde{b}$, respectively, then if $S(\tilde{a}) < S\left(\tilde{b}\right)$, then $\tilde{a}$ is smaller than $\tilde{b}$, denoted by $\tilde{a} < \tilde{b}$; if $S(\tilde{a}) = S\left(\tilde{b}\right)$, then

(1) if $H(\tilde{a}) = H\left(\tilde{b}\right)$, then $\tilde{a}$ and $\tilde{b}$ represent the same information, denoted by $\tilde{a} = \tilde{b}$; (2) if $H(\tilde{a}) < H\left(\tilde{b}\right)$, $\tilde{a}$ is smaller than $\tilde{b}$, denoted by $\tilde{a} < \tilde{b}$.

Definition 5. [3] Let $\tilde{a}_1 = (\mu_1, \nu_1)$, $\tilde{a}_2 = (\mu_2, \nu_2)$, and $\tilde{a} = (\mu, \nu)$ be three Pythagorean fuzzy numbers, and the basic operations on them are defined as follows:

(1) $\tilde{a}_1 \oplus \tilde{a}_2 = \left(\sqrt{(\mu_1)^2 + (\mu_2)^2 - (\mu_1)^2(\mu_2)^2}, \nu_1\nu_2\right);$

(2) $\tilde{a}_1 \otimes \tilde{a}_2 = \left(\mu_1\mu_2, \sqrt{(\nu_1)^2 + (\nu_2)^2 - (\nu_1)^2(\nu_2)^2}\right);$

(3) $\lambda\tilde{a} = \left(\sqrt{1 - \left(1 - \mu^2\right)^\lambda}, \nu^\lambda\right), \lambda > 0;$

(4) $(\tilde{a})^\lambda = \left(\mu^\lambda, \sqrt{1 - \left(1 - \nu^2\right)^\lambda}\right), \lambda > 0;$

(5) $\tilde{a}^c = (\nu, \mu).$

3. Pythagorean fuzzy interaction aggregation operators

In this section, we shall give the corrected Theorem 1 and its proof and the corrected Equations (6 and 7).

Theorem 1. *Let $\tilde{a}_j = \left(\mu_j, \nu_j\right)(j = 1, 2, \cdots, n)$ be a collection of Pythagorean fuzzy numbers, then their aggregated value by using the PFIWA operator is also a PFN, and*

$$\mathrm{PFIWA}_\omega(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n) = \overset{n}{\underset{j=1}{\oplus}}\left(\omega_j\tilde{a}_j\right)$$

$$= \left(\begin{array}{l}\sqrt{1 - \prod_{j=1}^{n}\left(1 - \left(\mu_j\right)^2\right)^{\omega_j}}, \\ \sqrt{\prod_{j=1}^{n}\left(1 - \left(\mu_j\right)^2\right)^{\omega_j} - \prod_{j=1}^{n}\left(1 - \left(\left(\mu_j\right)^2 + \left(\nu_j\right)^2\right)\right)^{\omega_j}}\end{array}\right) \quad (5)$$

where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ be the weight vector of $\tilde{a}_j\,(j = 1, 2, \cdots, n)$, and $\omega_j > 0$, $\sum_{j=1}^{n}\omega_j = 1$.

Proof. We prove Equation (5) by mathematical induction on n.

① When $n = 2$, we have

$$\mathrm{PFIWA}_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2) = \omega_1\tilde{\alpha}_1 \oplus \omega_2\tilde{\alpha}_2$$

By Theorem 1, we can see that both $\omega_1\alpha_1$ and $\omega_2\alpha_2$ are PFNs, and the value of $\omega_1\alpha_1 \oplus \omega_2\alpha_2$ is also a PFN. From the operational laws of Pythagorean fuzzy number, we have

$$\omega_1\tilde{\alpha}_1 = \left(\begin{array}{l}\sqrt{1-\left(1-(\mu_1)^2\right)^{\omega_1}}, \\ \sqrt{\left(1-(\mu_1)^2\right)^{\omega_1}-\left(1-\left((\mu_1)^2+(\nu_1)^2\right)\right)^{\omega_1}}\end{array}\right);$$

$$\omega_2\tilde{\alpha}_2 = \left(\begin{array}{l}\sqrt{1-\left(1-(\mu_2)^2\right)^{\omega_2}}, \\ \sqrt{\left(1-(\mu_2)^2\right)^{\omega_2}-\left(1-\left((\mu_2)^2+(\nu_2)^2\right)\right)^{\omega_2}}\end{array}\right)$$

Then

$$\begin{aligned}
\mathrm{PFIWA}_\omega(\tilde{\alpha}_1,\tilde{\alpha}_2) &= \omega_1\tilde{\alpha}_1 \oplus \omega_2\tilde{\alpha}_2 \\
&= \left(\begin{array}{l}\sqrt{1-\left(1-(\mu_1)^2\right)^{\omega_1}}, \\ \sqrt{\left(1-(\mu_1)^2\right)^{\omega_1}-\left(1-\left((\mu_1)^2+(\nu_1)^2\right)\right)^{\omega_1}}\end{array}\right) \\
&\quad \oplus \left(\begin{array}{l}\sqrt{1-\left(1-(\mu_2)^2\right)^{\omega_2}}, \\ \sqrt{\left(1-(\mu_2)^2\right)^{\omega_2}-\left(1-\left((\mu_1)^2+(\nu_1)^2\right)\right)^{\omega_2}}\end{array}\right) \\
&= \left(\sqrt{1-\left(1-(\mu_1)^2\right)^{\omega_1}\left(1-(\mu_2)^2\right)^{\omega_2}},\right. \\
&\quad \left.\sqrt{\begin{array}{l}\left(1-(\mu_1)^2\right)^{\omega_1}\left(1-(\mu_2)^2\right)^{\omega_2}- \\ \left[\left(1-\left((\mu_1)^2+(\nu_1)^2\right)\right)\right]^{\omega_1}\left[\left(1-\left((\mu_2)^2+(\nu_2)^2\right)\right)\right]^{\omega_2}\end{array}}\right) \\
&= \left(\begin{array}{l}\sqrt{1-\prod_{j=1}^{2}\left(1-(\mu_j)^2\right)^{\omega_j}}, \\ \sqrt{\prod_{j=1}^{2}\left(1-(\mu_j)^2\right)^{\omega_j}-\prod_{j=1}^{2}\left(1-\left((\mu_j)^2+(\nu_j)^2\right)\right)^{\omega_j}}\end{array}\right)
\end{aligned}$$

② Suppose that $n = k$, Equation (5) holds, i.e.,

$$\begin{aligned}
PFIWA_\omega(\tilde{\alpha}_1,\ \tilde{\alpha}_2,\cdots,\ \tilde{\alpha}_k) &= \omega_1\tilde{\alpha}_1 \oplus \omega_2\tilde{\alpha}_2 \oplus \cdots \omega_k\tilde{\alpha}_k \\
&= \left(\begin{array}{l}\sqrt{1-\prod_{j=1}^{k}\left(1-(\mu_j)^2\right)^{\omega_j}}, \\ \sqrt{\prod_{j=1}^{k}\left(1-(\mu_j)^2\right)^{\omega_j}-\prod_{j=1}^{k}\left(1-\left((\mu_j)^2+(\nu_j)^2\right)\right)^{\omega_j}}\end{array}\right)
\end{aligned}$$

And the aggregated value is a PFN, Then when $n = k + 1$, by the operational laws of Pythagorean fuzzy number, we have

$$
\begin{aligned}
&PFIWA_{\omega}\left(\tilde{\alpha}_1,\ \tilde{\alpha}_2, \cdots,\ \tilde{\alpha}_{k+1}\right)\\
&= \omega_1\tilde{\alpha}_1 \oplus \omega_2\tilde{\alpha}_2 \oplus \cdots \omega_k\tilde{\alpha}_k \oplus \omega_{k+1}\tilde{\alpha}_{k+1}\\
&= \left(\begin{array}{l}\sqrt{1-\prod_{j=1}^{k}\left(1-\left(\mu_j\right)^2\right)^{\omega_j}},\\ \sqrt{\prod_{j=1}^{k}\left(1-\left(\mu_j\right)^2\right)^{\omega_j}-\prod_{j=1}^{k}\left(1-\left(\left(\mu_j\right)^2+\left(\nu_j\right)^2\right)\right)^{\omega_j}}\end{array}\right)\\
&\quad\oplus\left(\begin{array}{l}\sqrt{1-\left(1-(\mu_{k+1})^2\right)^{\omega_{k+1}}},\\ \sqrt{\left(1-(\mu_{k+1})^2\right)^{\omega_{k+1}}-\left(1-\left((\mu_{k+1})^2+(\nu_{k+1})^2\right)\right)^{\omega_{k+1}}}\end{array}\right)\\
&= \left(\begin{array}{l}\sqrt{1-\prod_{j=1}^{k+1}\left(1-\left(\mu_j\right)^2\right)^{\omega_j}},\\ \sqrt{\prod_{j=1}^{k+1}\left(1-\left(\mu_j\right)^2\right)^{\omega_j}-\prod_{j=1}^{k+1}\left(1-\left(\left(\mu_j\right)^2+\left(\nu_j\right)^2\right)\right)^{\omega_j}}\end{array}\right)
\end{aligned}
$$

by which aggregated value is also a PFN, Therefore, when $n = k + 1$, Equation (5) holds.

Thus, by ① and ②, we know that Equation (5) holds for all n. The proof is completed.

Furthermore, Equations (6 and 7) can be corrected as follows:

$$
\begin{aligned}
&\mathrm{PFIOWA}_w\left(\tilde{a}_1,\ \tilde{a}_2, \cdots,\ \tilde{a}_n\right) = \overset{n}{\underset{j=1}{\oplus}}\left(w_j\tilde{a}_{\sigma(j)}\right)\\
&= \left(\begin{array}{l}\sqrt{1-\prod_{j=1}^{n}\left(1-\left(\mu_{\sigma(j)}\right)^2\right)^{w_j}},\\ \sqrt{\prod_{j=1}^{n}\left(1-\left(\mu_{\sigma(j)}\right)^2\right)^{w_j}-\prod_{j=1}^{n}\left(1-\left(\mu_{\sigma(j)}+\nu_{\sigma(j)}\right)^2\right)^{w_j}}\end{array}\right)
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
&\mathrm{PFIHA}_{w,\omega}\left(\tilde{\alpha}_1,\ \tilde{\alpha}_2, \cdots,\ \tilde{\alpha}_n\right) = \overset{n}{\underset{j=1}{\oplus}}\left(w_j\dot{\tilde{a}}_{\sigma(j)}\right)\\
&= \left(\begin{array}{l}\sqrt{1-\prod_{j=1}^{n}\left(1-\left(\dot{\mu}_{\sigma(j)}\right)^2\right)^{w_j}},\\ \sqrt{\begin{array}{l}\prod_{j=1}^{n}\left(1-\left(\dot{\mu}_{\sigma(j)}\right)^2\right)^{w_j}-\\ \prod_{j=1}^{n}\left(1-\left(\left(\dot{\mu}_{\sigma(j)}\right)^2+\left(\dot{\nu}_{\sigma(j)}\right)^2\right)\right)^{w_j}\end{array}}\end{array}\right)
\end{aligned} \tag{7}
$$

4. Pythagorean fuzzy interaction geometric aggregation operators

In this section, we shall give the corrected Theorem 10 and its proof and the corrected Equations (9 and 10).

Theorem 10. *The aggregated value by using PFIWG operator is also a PFN, where*

$$PFIWG_{\omega}(\tilde{\alpha}_1,\ \tilde{\alpha}_2,\cdots,\ \tilde{\alpha}_n) = \begin{pmatrix} \sqrt{\prod_{j=1}^{k}\left(1-(\nu_j)^2\right)^{\omega_j} - \prod_{j=1}^{k}\left(1-(\mu_j+\nu_j)^2\right)^{\omega_j}}, \\ \sqrt{1-\prod_{j=1}^{k}\left(1-(\nu_j)^2\right)^{\omega_j}} \end{pmatrix} \tag{8}$$

where $\omega = (\omega_1,\ \omega_2,\cdots,\ \omega_n)^T$ be the weight vector of $\alpha_j\,(j = 1,\ 2,\cdots,\ n)$, and $\omega_j > 0$, $\sum_{j=1}^{n}\omega_j = 1$.

Proof. We prove Equation (8) by mathematical induction on n.

① When $n = 2$, we have

$$\mathrm{PFIWG}_{\omega}(\tilde{\alpha}_1,\tilde{\alpha}_2) = (\tilde{\alpha}_1)^{\omega_1}\otimes(\tilde{\alpha}_2)^{\omega_2}$$

By Theorem1, we can see that both $(\tilde{\alpha}_1)^{\omega_1}$ and $(\tilde{\alpha}_2)^{\omega_2}$ are PFNs, and the value of $(\tilde{\alpha}_1)^{\omega_1}\otimes(\tilde{\alpha}_2)^{\omega_2}$ is also a PFN. From the operational laws of Pythagorean fuzzy number, we have

$$(\tilde{\alpha}_1)^{\omega_1} = \begin{pmatrix} \sqrt{\left(1-(\nu_1)^2\right)^{\omega_1} - \left(1-(\mu_1+\nu_1)^2\right)^{\omega_1}}, \\ \sqrt{1-\left(1-(\nu_1)^2\right)^{\omega_1}} \end{pmatrix};$$

$$(\tilde{\alpha}_2)^{\omega_2} = \begin{pmatrix} \sqrt{\left(1-(\nu_2)^2\right)^{\omega_2} - \left(1-(\mu_2+\nu_2)^2\right)^{\omega_2}}, \\ \sqrt{1-\left(1-(\nu_2)^2\right)^{\omega_2}} \end{pmatrix}$$

Then

$$\begin{aligned}
\mathrm{PFIWG}_{\omega}(\tilde{\alpha}_1,\tilde{\alpha}_2) &= (\tilde{\alpha}_1)^{\omega_1}\otimes(\tilde{\alpha}_2)^{\omega_2} \\
&= \left(\sqrt{\left(1-(\nu_1)^2\right)^{\omega_1} - \left(1-(\mu_1+\nu_1)^2\right)^{\omega_1}},\ \sqrt{1-\left(1-(\nu_1)^2\right)^{\omega_1}}\right) \\
&\quad\otimes\begin{pmatrix} \sqrt{\left(1-(\nu_2)^2\right)^{\omega_2} - \left(1-(\mu_2+\nu_2)^2\right)^{\omega_2}}, \\ \sqrt{1-\left(1-(\nu_2)^2\right)^{\omega_2}} \end{pmatrix} \\
&= \left(\sqrt{\begin{array}{l}\left(1-(\nu_1)^2\right)^{\omega_1}\left(1-(\nu_2)^2\right)^{\omega_2} \\ -\left[\left(1-(\mu_1+\nu_1)^2\right)\right]^{\omega_1}\left[\left(1-(\mu_2+\nu_2)^2\right)\right]^{\omega_2}\end{array}},\right. \\
&\quad\left.\sqrt{1-\left(1-(\nu_1)^2\right)^{\omega_1}\left(1-(\nu_2)^2\right)^{\omega_2}}\right) \\
&= \begin{pmatrix} \sqrt{\prod_{j=1}^{2}\left(1-(\nu_j)^2\right)^{\omega_j} - \prod_{j=1}^{2}\left(1-(\mu_j+\nu_j)^2\right)^{\omega_j}}, \\ \sqrt{1-\prod_{j=1}^{2}\left(1-(\nu_j)^2\right)^{\omega_j}} \end{pmatrix}
\end{aligned}$$

② Suppose that $n = k$, Equation) holds, i.e.,

$$PFIWG_{\omega}(\tilde{\alpha}_1, \tilde{\alpha}_2, \cdots, \tilde{\alpha}_k) = (\tilde{\alpha}_1)^{\omega_1} \otimes (\tilde{\alpha}_2)^{\omega_2} \otimes \cdots (\tilde{\alpha}_k)^{\omega_k}$$

$$= \left(\begin{array}{l} \sqrt{\prod_{j=1}^{k} \left(1-(\nu_j)^2\right)^{\omega_j} - \prod_{j=1}^{k} \left(1-(\mu_j+\nu_j)^2\right)^{\omega_j}}, \\ \sqrt{1-\prod_{j=1}^{k} \left(1-(\nu_j)^2\right)^{\omega_j}} \end{array} \right)$$

And the aggregated value is a PFN, Then when $n = k + 1$, by the operational laws of Pythagorean fuzzy number, we have

$$PFIWG_{\omega}(\tilde{\alpha}_1, \tilde{\alpha}_2, \cdots, \tilde{\alpha}_{k+1}) = (\tilde{\alpha}_1)^{\omega_1} \otimes (\tilde{\alpha}_2)^{\omega_2} \otimes \cdots (\tilde{\alpha}_k)^{\omega_k} \otimes (\tilde{\alpha}_{k+1})^{\omega_{k+1}}$$

$$= \left(\begin{array}{l} \sqrt{\prod_{j=1}^{k} \left(1-(\nu_j)^2\right)^{\omega_j} - \prod_{j=1}^{k} \left(1-(\mu_j+\nu_j)^2\right)^{\omega_j}}, \\ \sqrt{1-\prod_{j=1}^{k} \left(1-(\nu_j)^2\right)^{\omega_j}} \end{array} \right)$$

$$\otimes \left(\begin{array}{l} \sqrt{\left(1-(\nu_{k+1})^2\right)^{\omega_{k+1}} - \left(1-(\mu_{k+1}+\nu_{k+1})^2\right)^{\omega_{k+1}}}, \\ \sqrt{1-\left(1-(\nu_{k+1})^2\right)^{\omega_{k+1}}} \end{array} \right)$$

$$= \left(\begin{array}{l} \sqrt{\prod_{j=1}^{k+1} \left(1-(\nu_j)^2\right)^{\omega_j} - \prod_{j=1}^{k+1} \left(1-(\mu_j+\nu_j)^2\right)^{\omega_j}}, \\ \sqrt{1-\prod_{j=1}^{k+1} \left(1-(\nu_j)^2\right)^{\omega_j}} \end{array} \right)$$

by which aggregated value is also a PFN, Therefore, when $n = k + 1$, Equation (8) holds.

Thus, by ① and ②, we know that Equation (8) holds for all n. The proof is completed.

Furthermore, Equations (9 and 10) can be corrected as follows:

$$\mathrm{PFIOWG}_{\omega}(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n) = \underset{j=1}{\overset{n}{\otimes}} \left(\tilde{\alpha}_{\sigma(j)}\right)^{w_j}$$

$$= \left(\begin{array}{l} \sqrt{\begin{array}{l} \prod_{j=1}^{n} \left(1-\left(\nu_{\sigma(j)}\right)^2\right)^{w_j} \\ -\prod_{j=1}^{n} \left(1-\left(\left(\mu_{\sigma(j)}\right)^2+\left(\nu_{\sigma(j)}\right)^2\right)\right)^{w_j} \end{array}}, \\ \sqrt{1-\prod_{j=1}^{n} \left(1-\left(\nu_{\sigma(j)}\right)^2\right)^{w_j}} \end{array} \right) \quad (9)$$

$$\mathrm{PFIHG}_{w,\omega}(\tilde{\alpha}_1, \tilde{\alpha}_2, \cdots, \tilde{\alpha}_n) = \underset{j=1}{\overset{n}{\otimes}} \left(\dot{\tilde{a}}_{\sigma(j)}\right)^{w_j}$$

$$= \left(\begin{array}{l} \sqrt{\begin{array}{l} \prod_{j=1}^{n} \left(1-\left(\dot{\nu}_{\sigma(j)}\right)^2\right)^{w_j} \\ -\prod_{j=1}^{n} \left(1-\left(\left(\dot{\mu}_{\sigma(j)}\right)^2+\left(\dot{\nu}_{\sigma(j)}\right)^2\right)\right)^{w_j} \end{array}}, \\ \sqrt{1-\prod_{j=1}^{n} \left(1-\left(\dot{\nu}_{\sigma(j)}\right)^2\right)^{w_j}} \end{array} \right) \quad (10)$$

5. An approach to multiple attribute decision making with Pythagorean fuzzy information

In this section, we shall give the corrected Equations (11 and 12).

$$\begin{aligned}\tilde{r}_i &= (\mu_i, \nu_i)\\ &= \mathrm{PFIWA}_\omega\left(\tilde{r}_{i1},\ \tilde{r}_{i2}, \cdots,\ \tilde{r}_{in}\right)\\ &= \mathop{\oplus}_{j=1}^{n}\left(\omega_j \tilde{r}_{ij}\right)\\ &= \left(\begin{array}{l}\sqrt{1-\prod_{j=1}^{n}\left(1-\left(\mu_{ij}\right)^2\right)^{\omega_j}},\\ \sqrt{\prod_{j=1}^{n}\left(1-\left(\mu_{ij}\right)^2\right)^{\omega_j}-\prod_{j=1}^{n}\left(1-\left(\left(\mu_{ij}\right)^2+\left(\nu_{ij}\right)^2\right)\right)^{\omega_j}}\end{array}\right),\\ & i = 1,\ 2, \cdots,\ m. \end{aligned} \tag{11}$$

$$\begin{aligned}\tilde{r}_i &= (\mu_i, \nu_i)\\ &= \mathrm{PFIWG}_\omega\left(\tilde{r}_{i1},\ \tilde{r}_{i2}, \cdots,\ \tilde{r}_{in}\right)\\ &= \mathop{\otimes}_{j=1}^{n}\left(\tilde{r}_{ij}\right)^{\omega_j}\\ &= \left(\begin{array}{l}\sqrt{\prod_{j=1}^{n}\left(1-\left(\nu_{ij}\right)^2\right)^{\omega_j}-\prod_{j=1}^{n}\left(1-\left(\left(\mu_{ij}\right)^2+\left(\nu_{ij}\right)^2\right)\right)^{\omega_j}},\\ \sqrt{1-\prod_{j=1}^{n}\left(1-\left(\nu_{ij}\right)^2\right)^{\omega_j}}\end{array}\right),\\ & i = 1,\ 2, \cdots,\ m. \end{aligned} \tag{12}$$

6. Illustrative example and comparative analysis

In this section, with corrected Equations (11 and 12), the Tables 1–3 are corrected as follows:

Step 1. According to Table 1, aggregate all Pythagorean fuzzy numbers $\tilde{r}_{ij}\,(j = 1,\ 2, \cdots,\ n)$ by using the PFIWA (PFIWG) operator to derive the overall Pythagorean fuzzy numbers $\alpha_i\,(i = 1, 2, 3, 4, 5)$ of the alternative A_i. The aggregating results are shown in Table 1.

Step 2. According to the aggregating results shown in Table 1 and the score functions of the ERP systems are shown in Table 2.

Table 1
The aggregating results of the ERP systems by the PFIWA (PFIWG) operators

	PFIWA	PFIWG
A_1	(0.727,0.324)	(0.660,0.446)
A_2	(0.458,0.583)	(0.355,0.651)
A_3	(0.407,0.538)	(0.341,0.579)
A_4	(0.663,0.511)	(0.495,0.674)
A_5	(0.667,0.474)	(0.520,0.632)

Step 3. According to the score functions shown in Table 2 and the comparison formula of score

Table 2
The score functions of the ERP systems

	PFIWA	PFIWG
A_1	0.712	0.619
A_2	0.435	0.351
A_3	0.438	0.391
A_4	0.589	0.396
A_5	0.610	0.435

Table 3
Ordering of the ERP systems

	Ordering
PFIWA	$A_1 > A_5 > A_4 > A_3 > A_2$
PFIWG	$A_1 > A_5 > A_4 > A_3 > A_2$

functions, the ordering of the ERP systems are shown in Table 3. Note that ">" means "preferred to". As we can see, depending on the aggregation operators used, the ordering of the ERP systems is some and the best ERP system is A_1.

7. Conclusion

In this study we utilize arithmetic and geometric operations to investigate some Pythagorean fuzzy interaction aggregation operators in detail, and point out that Theorems 1 and 10 in Wei [4] are incorrect by the Pythagorean fuzzy operational laws. Finally we propose the modifications of these theorems and equations. In the future, we shall continue working in the extension and application of the developed operators to other domains and fuzzy setting, such as picture fuzzy sets, dual hesitant fuzzy sets, and so on [6–22].

References

[1] R.R. Yager, Pythagorean fuzzy subsets, in: *Proceeding of The Joint IFSA Wprld Congress and NAFIPS Annual Meeting*, Edmonton, Canada, 2013, pp. 57–61.
[2] R.R. Yager, Pythagorean membership grades in multicriteria decision making, *IEEE Transactions on Fuzzy Systems* **22** (2014), 958–965.
[3] X.L. Zhang and Z.S. Xu, Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets, *International Journal of Intelligent Systems* **29** (2014), 1061–1078.
[4] G.W. Wei, Pythagorean fuzzy interaction aggregation operators and their application to multiple attribute decision making, *Journal of Intelligent and Fuzzy Systems* **33**(4) (2017), 2119–2132.
[5] R.R. Yager and A.M. Abbasov, Pythagorean membeship grades, complex numbers and decision making, *International Journal of Intelligent Systems* **28** (2013), 436–452.
[6] G.W. Wei and M. Lu, Pythagorean fuzzy maclaurin symmetric mean operators in multiple attribute decision making, *International Journal of Intelligent Systems* (2017). DOI: 10.1002/int.21911
[7] G.W. Wei and M. Lu, Pythagorean fuzzy power aggregation operators in multiple attribute decision making, *International Journal of Intelligent Systems* **33**(1) (2018), 169–186.
[8] S.J. Wu and G.W. Wei, Pythagorean fuzzy Hamacher aggregation operators and their application to multiple attribute decision making, *International Journal of Knowledge-based and Intelligent Engineering Systems* **21**(3) (2017), 189–201.
[9] M. Lu and G.W. Wei, Pythagorean uncertain linguistic aggregation operators for multiple attribute decision making, *International Journal of Knowledge-based and Intelligent Engineering Systems* **21**(3) (2017), 165–179.
[10] G.W. Wei and M. Lu, Dual hesitant Pythagorean fuzzy Hamacher aggregation operators in multiple attribute decision making, *Archives of Control Sciences* **27**(3) (2017), 365–395.
[11] G.W. Wei, Picture fuzzy Hamacher aggregation operators and their application to multiple attribute decision making, *Fundamenta Informaticae* **157**(3) (2018), 271–320. DOI: 10.3233/FI-2018-1628
[12] G.W. Wei, Picture uncertain linguistic Bonferroni mean operators and their application to multiple attribute decision making, *Kybernetes* **46**(10) (2017), 1777–1800.
[13] G.W. Wei, Interval-valued dual hesitant fuzzy uncertain linguistic aggregation operators in multiple attribute decision making, *Journal of Intelligent and Fuzzy Systems* **33**(3) (2017), 1881–1893.
[14] G.W. Wei, Picture 2-tuple linguistic Bonferroni mean operators and their application to multiple attribute decision making, *International Journal of Fuzzy System* **19**(4) (2017), 997–1010.
[15] G.W. Wei, M. Lu, F.E. Alsaadi, T. Hayat and A. Alsaedi, Pythagorean 2-tuple linguistic aggregation operators in multiple attribute decision making, *Journal of Intelligent and Fuzzy Systems* **33**(2) (2017), 1129–1142.
[16] G.W. Wei, F.E. Alsaadi, T. Hayat and A. Alsaedi, Hesitant bipolar fuzzy aggregation operators in multiple attribute decision making, *Journal of Intelligent and Fuzzy Systems* **33**(2) (2017), 1119–1128.
[17] G.W. Wei, Interval valued hesitant fuzzy uncertain linguistic aggregation operators in multiple attribute decision making, *International Journal of Machine Learning and Cybernetics* **7**(6) (2016), 1093–1114.
[18] G.W. Wei, Some cosine similarity measures for picture fuzzy sets and their applications to strategic decision making, *Informatica* **28**(3) (2017), 547–564.
[19] H. Gao, G.W. Wei and Y.H. Huang, Dual hesitant bipolar fuzzy Hamacher prioritized aggregation operators in multiple attribute decision making, *IEEE Access* (2017). DOI: 10.1109/ACCESS.2017.2784963
[20] G.W. Wei and M. Lu, Pythagorean hesitant fuzzy Hamacher aggregation operators in multiple attribute decision making, *Journal of Intelligent Systems* (2017). DOI: https://doi.org/10.1515/jisys-2017-0106
[21] G.W. Wei, F.E. Alsaadi, T. Hayat and A. Alsaedi, Picture 2-tuple linguistic aggregation operators in multiple attribute decision making, *Soft Computing* (2016). DOI: 10.1007/s00500-016-2403-8
[22] G.W. Wei, F.E. Alsaadi, T. Hayat and A. Alsaedi, Projection models for multiple attribute decision making with picture fuzzy information, *International Journal of Machine Learning and Cybernetics* (2016). DOI: 10.1007/s13042-016-0604-1