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# Fair division rules for funds distribution: The case of the Italian Research Assessment Program (VQR 2004–2010)

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**Abstract**. In a great number of applications, it is necessary to distribute resources or tasks to agents collaborating with each other in order to maximize the social welfare of the structure they belong to. The question in these cases is how to divide in a fair way the outcome that the structure eventually earns, say money, to the participating agents.

The paper faces this issue by focusing on a real-world application: the distribution of funds to Italian research structures and substructures, after a research assessment program (known as VQR) that is currently evaluating the Italian research production over years 2004–2010. A number of desirable properties for any reasonable fair division rule are identified and exemplified for the case-study application. For instance, it is argued that the money distribution should be independent of possible alternative allocations of research products, and it should consider the actual contribution of every researcher to the outcome of the structure (s)he belongs to. Moreover, the whole process starting from the preliminary phase of products selection is dealt with, in order to prevent possible strategic behaviors from researchers that may lead their structure to miss its best possible score in the evaluation process.

A fair solution based on the notion of Shapley value is described and analyzed. It turns out that the proposed solution enjoys all the desirable properties of fair division rules, and it could effectively be implemented in the VQR, as well as in related division-rules applications.

Keywords: Social choice, coalitional game theory, mechanism design, Shapley value

## 1. Introduction

In 2012, the National Agency for the Evaluation of Universities and Research Institutes (ANVUR) promoted the ‘VQR’ assessment program to evaluate the quality of the whole Italian research production in the period 2004–2010. Every research structure $R$ has to select some research products, and submit them to ANVUR. While doing so, the structure $R$ is in competition with all other Italian research structures, as the outcome of the evaluation will be used to proportionally transfer the funds allocated by the Ministry to support research activities in the next years (until the subsequent evaluation process). Every structure $R$ is then interested in selecting and submitting its *best* research products. The program is articulated in three phases:

1. Authors affiliated to $R$ are asked to self-evaluate their products, according to some evaluation criteria defined by groups of experts chosen by ANVUR. Here, it is assumed that, having such criteria, every author is able to equip each product with a quality score.[1]

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[1]The set of the possible scores is defined in the VQR guidelines. To our ends, this detail is immaterial and scores are just viewed as (arbitrary) real numbers.

2. Based on the self-evaluations being collected, $R$ selects and submits to ANVUR (at most) three products for each one of its authors[2] in such a way that any product is formally associated with one author at most.
3. ANVUR formulates its independent quality judgment about the submitted publications, and the sum of their "true" scores (i.e., those resulting by ANVUR actual evaluation) is then the VQR score of $R$. Eventually, $R$ will receive funds in the next years proportionally to this score. Moreover, by using such products scores, ANVUR should evaluate all substructures, too (e.g., all departments, if $R$ is a University).

As far as the latter point is concerned, it is then natural for the structure $R$ to exploit the VQR scores of its substructures to divide funds among them. However, up to now, it is not clear at all how the evaluation of the research products, designed for the evaluation of the whole structure, may be extended for the evaluation of substructures and, possibly, of single researchers.

In this paper, we first show that finding a *good rule* for scores/funds redistribution is not an easy task, and a careful design is in order. In particular, we point out that the simplest rules that one may think of, which are also the ones that are currently believed to be applied, are unsatisfactory for a number of reasons. They are indeed *unfair* for substructures (and for researchers, as well) that may complain (with good arguments) about the score assigned to them. Moreover, we pinpoint that, without an agreement on a fair division rule, the self-evaluation performed at point 1 above is not affordable, and the research structure may miss the optimal possible submission to ANVUR, hence loosing precious funds.

Inspired by Marco Cadoli's example [4], who combined in a nice way techniques from game theory and artificial intelligence, we define a *fair division rule* that enjoys a number of desirable properties. We remark that all technical ingredients that are needed to prove such properties come from our recent contribution on mechanisms with verification [8] (see, e.g., [1, 2, 5, 6, 9, 11], for related work on this subject). We focus here on the specific application of the fund distribution with respect to the research evaluation program VQR 2004-2010, and we refer the reader interested in the more general framework of allocation problems to [8].

[2]The number of publications is not always three, in some specific exceptions. Again, this is not a relevant issue.

**Organization.** The paper is organized as follows. In Section 2 we discuss a formal mathematical model of the VQR program. Then, we present our division rule and we show that it enjoys several properties that are desirable in this context, to formally capture the concept of fair division. To this end, we use a number of notions from coalitional game theory, which are discussed in Section 3. While in the first part of the work we focus on the research products submitted to ANVUR by every structure $R$, we then show in Section 4 that the proposed rule enjoys the desired fairness properties also with respect to possible alternative optimal allocations that could have been submitted by $R$. Finally, some strategic issues arising in the VQR program are analyzed in Section 5, and the robustness of the proposed rule w.r.t. them is evidenced. That is, it is shown that with the proposed rule, there is no incentive for any researcher or group of researchers to cheat in the self-evaluation step 1. As a consequence, the structure $R$ is always able to submit to ANVUR its actual best products.[3] A few concluding remarks are discussed in Section 6.

## 2. The Italian Research Assessment Program (VQR) 2004–2010

The current Italian research evaluation process is performed by ANVUR, the National Agency for the Evaluation of Universities and Research Institutes (www.anvur.org). Both structures and substructures (e.g., universities and departments).

### 2.1. Allocations and division rules

Let $R$ be a research structure, and let $\mathcal{R}$ be the set of researchers affiliated with $R$. For each researcher $r \in \mathcal{R}$, let *products*$(r, R)$ (or just *products*$(r)$, if $R$ is understood from the context) be the set of the research products of $r$ in the given period 2004–2010. An *allocation* for a set of researchers $\mathcal{S} \subseteq \mathcal{R}$ is a function $\psi$ mapping each researcher $r \in \mathcal{S}$ to a set of publications $\psi(r) \subseteq$ *products*$(r)$ with $|\psi(r)| \leq 3$ and with $\psi(r) \cap \psi(r') = \emptyset$, for each $r' \in \mathcal{S} \setminus \{r\}$. An allocation for $R$ is an allocation for all researchers $\mathcal{R}$, while an allocation for a substructure $S$ of $R$ is an allocation for the researchers affiliated to $S$.

[3]Of course, we mean here the best products according to a truthful honest self-evaluation of researchers. Recall that in the ANVUR process some products will undergo a peer-review process, from which possible (unpredictable) discrepancies may arise.

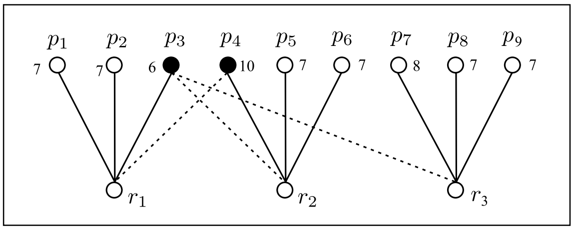


Fig. 1. Running example in Section 2.

In the VQR program, every research structure $R$ has to submit to ANVUR for its evaluation a set of products $\mathcal{P}_\psi$ such that $\mathcal{P}_\psi = \bigcup_{r \in \mathcal{R}} \psi(r)$, for some allocation $\psi$ for all researchers $\mathcal{R}$ affiliated to $R$. Then, for each $p \in \mathcal{P}_\psi$, ANVUR calculates a quality score $score_{\text{VQR}}(p)$, so that $R$ will receive funds proportionally to its overall score $score_{\text{VQR}}(\psi) = \sum_{p \in \mathcal{P}_\psi} score_{\text{VQR}}(p)$.

**Example 2.1.** Let us consider the simple scenario that is illustrated in Fig. 1, by exploiting an intuitive graphical notation based on a weighted bipartite graph, whose vertices are the researchers and the products, and whose weights are the VQR scores of the products. Assume that there are just three researchers, $r_1$, $r_2$, and $r_3$, affiliated to $R$. Moreover, consider the allocation $\psi$ such that $\psi(r_1) = \{p_1, p_2, p_3\}$, $\psi(r_2) = \{p_4, p_5, p_6\}$, and $\psi(r_3) = \{p_7, p_8, p_9\}$. Thus, we have that $\mathcal{P}_\psi = \{p_1, ..., p_9\}$ is the set of publications submitted for the evaluation. In particular, we also assume that $products(r_1) \cap \mathcal{P}_\psi = \{p_1, p_2, p_3, p_4\}$, $products(r_2) \cap \mathcal{P}_\psi = \{p_3, p_4, p_5, p_6\}$, and $products(r_3) \cap \mathcal{P}_\psi = \{p_3, p_6, p_7, p_8\}$. Thus, $p_3$ is co-authored by $r_1$, $r_2$, and $r_3$, while $p_4$ is co-authored by $r_1$ and $r_2$. Then, according to the VQR score assigned to each product (i.e., the weight of the corresponding vertex in Fig. 1), we get that $score_{\text{VQR}}(\psi) = 66$. $\lhd$

While the first aim of the VQR program is to evaluate the various Italian research structures, it is known that the obtained information will be used to evaluate substructures, too (e.g., University departments). Thus, following the same principle of binding funds to VQR scores used for the main structure, it is natural to exploit such scores for money distribution inside every research structure. It is therefore of utmost importance the way VQR assigns scores to substructures. Nevertheless, as already mentioned, up to date there is no official information about such an algorithm. We argue that the score of any structure $R$ should *fairly* be distributed over its substructures (and possibly over individuals), in such a way to reflect their actual contribution to the result achieved by the structure $R$. Formally, we need a suitable division rule.

**Definition 2.2.** A *division rule* $\gamma$ for $R$ is a real-valued function that, given a researcher $r \in \mathcal{R}$ and an allocation $\psi$ for $R$, returns its score $\gamma_\psi(r, R) \geq 0$ with respect to the allocation $\psi$.

By slightly abusing notation, for any substructure $\mathcal{S} \subseteq \mathcal{R}$ (here just viewed as the set of its members), we denote by $\gamma_\psi(\mathcal{S}, R)$ the value $\sum_{r \in \mathcal{S}} \gamma_\psi(r, R)$.

Whenever $R$ is understood from the context, we just write $\gamma_\psi(r)$ and $\gamma_\psi(\mathcal{S})$, in place of $\gamma_\psi(r, R)$ and $\gamma_\psi(\mathcal{S}, R)$, respectively. $\square$

Hereafter, we assume that $\mathcal{S}_1, ..., \mathcal{S}_n$ are the substructures of $R$. They exhaustively cover the researchers of $R$, i.e, $\bigcup_{i=1}^n \mathcal{S}_i = \mathcal{R}$, and are pair-wise disjoint, i.e., $\mathcal{S}_i \cap \mathcal{S}_j = \emptyset$, for each $i \neq j$.

Note that such a division rule may naturally be used for evaluating individuals, besides substructures. In fact, we shall discuss all fairness properties with reference to individual researchers, because all issues about individuals immediately extend to the substructures they belong to.

### 2.2. *Basic desirable properties of fair division rules*

In the absence of an official division rule, most researchers believe that the score of any substructure $\mathcal{S}$ will be based on the naive `proj` rule where, for any researcher $r$, $\texttt{proj}_\psi(r)$ is the sum of the VQR scores of the products allocated to $r$ in $\psi$, i.e.,

$$\texttt{proj}_\psi(r) = \sum_{p \in \psi(r)} score_{\text{VQR}}(p).$$

**Example 2.3.** In the setting of Example 2.1., it is immediate to check that $\texttt{proj}_\psi(r_1) = 20$, $\texttt{proj}_\psi(r_2) = 24$, and $\texttt{proj}_\psi(r_3) = 22$. $\lhd$

The rule `proj` satisfies a very basic requirement for every division rule, which we state below.

**(P1) "budget-balance":** *A division rule $\gamma$ must precisely distribute the VQR score of R over all its members, i.e., $\sum_{r \in \mathcal{R}} \gamma_\psi(r) = score_{\text{VQR}}(R)$. Clearly, because the substructures of R define a partition of its researchers, this implies that $\gamma$ completely distributes* *the VQR score of R over all its substructures and does not distribute more than that, i.e.,* $\sum_{i=1}^{n} \gamma_{\psi}(\mathcal{S}_i) = score_{\text{VQR}}(R)$.

However, we claim that this rule is hardly perceivable as a "fair" one. Indeed, proj might lead to scenarios where some researcher $r$ (and in turn her/his substructure) has reasonable arguments against her/his structure because of possible alternative allocations where $r$ would get higher scores. This is next exemplified.

**Example 2.4.** Consider again the allocation $\psi$ depicted in Fig. 1. Consider now the different allocation $\bar{\psi}$ where $r_1$ and $r_2$ just swap the allocation of the two publications they have co-authored. That is, $\bar{\psi}$ is such that $\bar{\psi}(r_1) = \{p_1, p_2, p_4\}$, $\bar{\psi}(r_2) = \{p_3, p_5, p_6\}$, and $\bar{\psi}(r_3) = \{p_7, p_8, p_9\}$.

Of course, we have $\mathcal{P}_{\psi} = \mathcal{P}_{\bar{\psi}}$ and $score_{\text{VQR}}(\psi) = score_{\text{VQR}}(\bar{\psi})$. Therefore, from the perspective of $R$ the choice of $\bar{\psi}$ in place of $\psi$ is completely immaterial. However, we now would have $\texttt{proj}_{\bar{\psi}}(r_1) = 24$ and $\texttt{proj}_{\bar{\psi}}(r_2) = 20$. Thus, $r_2$ may well complain to her/his structure, if $\bar{\psi}$ is selected in place of $\psi$. Symmetrically, $r_1$ is not happy with $\psi$, knowing the existence of the alternative allocation $\bar{\psi}$. $\triangleleft$

As the above example has pointed out, proj fails to satisfy a basic and intuitive requirement of *fairness*, which we formalize as follows.

**(P2) "Independence of the product allocation":** *Assume that $\psi$ and $\bar{\psi}$ are two allocations of the same research products, i.e., $\mathcal{P}_{\psi} = \mathcal{P}_{\bar{\psi}}$. A division rule $\gamma$ must be independent of the allocation being selected, i.e., for each $r \in \mathcal{R}$, $\gamma_{\psi}(r) = \gamma_{\bar{\psi}}(r)$. In fact, this implies that, for each substructure $\mathcal{S}_i$, $\gamma_{\psi}(\mathcal{S}_i) = \gamma_{\bar{\psi}}(\mathcal{S}_i)$ also holds.*

The above criterion is definitely desirable. However, a closer look reveals that it is not enough, as there is a trivial way to circumvent this kind of fairness: just consider the rule trivial assigning the overall VQR score to one fixed researcher, say $r_1$ (independently of its actual contribution), i.e.,

$$\texttt{trivial}_{\psi}(r) = \begin{cases} \sum_{p \in \mathcal{P}_{\psi}} score_{\text{VQR}}(p), & \text{if } r = r_1 \\ 0 & \text{if } r \in \mathcal{R} \setminus \{r_1\} \end{cases}$$

A problem with this rule is that it is not "symmetric," in that a researcher $r_2$ that has co-authored precisely the same set of products as $r_1$ would be treated differently, just because of her name. Avoiding these cases and guaranteeing an *equal treatment of the equals* is another very basic requirement, formalized as follows.

Let $\pi : \mathcal{R} \mapsto \mathcal{R}$ be a permutation of the researchers in $\mathcal{R}$. Let $R_{\pi}$ be the research structure over the researchers in $\mathcal{R}$ where, for each $r \in \mathcal{R}$, $products(r, R_{\pi}) = products(\pi(r), R)$. Moreover, if $\psi$ is an allocation for $R$, then let $\psi_{\pi}$ be the allocation such that, for each $r \in \mathcal{R}$, $\psi_{\pi}(r, R_{\pi}) = \psi(\pi(r), R)$. Thus, $R_{\pi}$ and $\psi_{\pi}$ are derived by applying the permutation $\pi$, whose role is just to rename the researchers in $\mathcal{R}$. With these notions in place, we can now state the following property, which is in fact not enjoyed by trivial.

**(P3) "impartiality":** *Let $\pi$ be an arbitrary permutation over $\mathcal{R}$. A division rule $\gamma$ must be such that, for each $r \in \mathcal{R}$ and each allocation $\psi$, $\gamma_{\psi_{\pi}}(r, R_{\pi}) = \gamma_{\psi}(\pi(r), R)$ holds.*

Yet again this is not enough. To see that, consider the very impartial rule uniform, where the overall ANVUR score is distributed uniformly over the various researchers, i.e.,

$$\texttt{uniform}_{\psi}(r) = \frac{score_{\text{VQR}}(R)}{|\mathcal{R}|}.$$

**Example 2.5.** In the setting of Example 2.1., it is immediate to check that each researcher would get score $\frac{20+24+22}{3}$ according to the uniform rule. $\triangleleft$

Clearly, uniform is unsatisfying because it does not capture our intuition that a division rule should reflect the actual contribution of each researcher to the overall evaluation of the structure. In the rest of this section, we will elaborate on this issue by using the notion of marginal contribution, which fits well our intuition of actual contributions of individual or groups to the performance of a given structure.

### *2.3. Marginal contribution*

Let $R$ be a research structure and assume that $\psi$ is the allocation selected by the structure $R$, so that the set of products $\mathcal{P}_{\psi}$ have been submitted and evaluated by ANVUR. Let $\mathcal{S} \subseteq \mathcal{R}$ be any set of researchers. An allocation $\psi_{\mathcal{S}}$ for $\mathcal{S}$ is *$\psi$-legal* if $\psi_{\mathcal{S}}(r) \subseteq \mathcal{P}_{\psi}$, for each $r \in \mathcal{S}$. That is, any legal allocation only considers for the assignment the products already evaluated by ANVUR. The allocation $\psi_{\mathcal{S}}$ is *$\psi$-optimal* if there is no $\psi$-legal allocation $\psi'_{\mathcal{S}}$ such that $\sum_{r\in\mathcal{S}}\sum_{p\in\psi'_{\mathcal{S}}(r)} score_{\text{VQR}}(p) > \sum_{r\in\mathcal{S}}\sum_{p\in\psi_{\mathcal{S}}(r)} score_{\text{VQR}}(p)$.

Given the above notions, we can equip $\mathcal{S}$ with the following score, which is meant to assess the overall VQR score that researchers in $\mathcal{S}$ would have achieved if the research structure had been constituted by them only (i.e., without caring about their co-authors outside $\mathcal{S}$):

$$\texttt{best}_{\psi}(\mathcal{S}) = \sum_{r\in\mathcal{S}}\sum_{p\in\psi_{\mathcal{S}}(r)} score_{\text{VQR}}(p),$$

where $\psi_{\mathcal{S}}$ is any $\psi$-*optimal* allocation.

Note that in the extreme case where $\mathcal{S} = \mathcal{R}$ we just obtain $\texttt{best}_{\psi}(\mathcal{R}) = score_{\text{VQR}}(R)$. That is, when all researchers of $R$ are considered, $\texttt{best}_{\psi}$ precisely returns the overall VQR score.

**Example 2.6.** Consider again our running example, and check that the following expressions hold:

- $\texttt{best}_{\psi}(\{r_1\}) = 24$, $\texttt{best}_{\psi}(\{r_2\}) = 24$, and $\texttt{best}_{\psi}(\{r_3\}) = 22$;
- $\texttt{best}_{\psi}(\{r_1, r_2\}) = 44$, $\texttt{best}_{\psi}(\{r_1, r_3\}) = 46$, and $\texttt{best}_{\psi}(\{r_2, r_3\}) = 46$;
- $\texttt{best}_{\psi}(\{r_1, r_2, r_3\}) = 66$.

For instance, $\texttt{best}_{\psi}(\{r_1\})$ is in fact the VQR score if the structure had been constituted by $r_1$ only, so that all the best products co-authored by $r_1$ can freely be assigned to her/him. In particular, this is the case for the excellent publication $p_4$ that is a joint work with $r_2$. However, when they are considered together in the computation of $\texttt{best}_{\psi}(\{r_1, r_2\})$, only one of them may get $p_4$ and the other one must take the bad publication $p_3$, so that their overall value is $20 + 24 = 44$. ◁

We can now formalize the notion of marginal contribution.

**Definition 2.7.** Let $\psi$ be an allocation for $R$. Given two sets of researchers $\mathcal{S}_1, \mathcal{S}_2 \subseteq \mathcal{R}$ with $\mathcal{S}_1 \subseteq \mathcal{S}_2$, the *marginal contribution* of $\mathcal{S}_1$ to $\mathcal{S}_2$ (in $R$ and $\psi$) is the value:

$\texttt{marg}_{\psi}(\mathcal{S}_1, \mathcal{S}_2) = \texttt{best}_{\psi}(\mathcal{S}_2) - \texttt{best}_{\psi}(\mathcal{S}_2 \setminus \mathcal{S}_1)$. □

Intuitively, the marginal contribution $\texttt{marg}_{\psi}(\mathcal{S}_1, \mathcal{S}_2)$ quantifies the loss of VQR score for the group of researchers $\mathcal{S}_2$ (e.g., a substructure) if the groups of researchers $\mathcal{S}_1$ were not part of it. In particular, $\texttt{marg}_{\psi}(\{r\}, \mathcal{R})$ measures the loss for the whole structure $R$, if the single researcher $r$ were not part of it.

**Example 2.8.** In our running example, by first focusing on the whole set $\mathcal{R}$ of researchers in $R$, we obtain the following values:

- $\texttt{marg}_{\psi}(\{r_1\}, \mathcal{R}) = 66 - 46 = 20$;
- $\texttt{marg}_{\psi}(\{r_2\}, \mathcal{R}) = 66 - 46 = 20$;
- $\texttt{marg}_{\psi}(\{r_3\}, \mathcal{R}) = 66 - 44 = 22$;
- $\texttt{marg}_{\psi}(\{r_1, r_2\}, \mathcal{R}) = 66 - 22 = 44$;
- $\texttt{marg}_{\psi}(\{r_1, r_3\}, \mathcal{R}) = \texttt{marg}_{\psi}(\{r_2, r_3\}, \mathcal{R}) = 66 - 24 = 42$.

Moreover, we have

- $\texttt{marg}_{\psi}(\{r_1\}, \{r_1, r_2\}) = \texttt{marg}_{\psi}(\{r_2\}, \{r_1, r_2\}) = 44 - 24 = 20$;
- $\texttt{marg}_{\psi}(\{r_1\}, \{r_1, r_3\}) = \texttt{marg}_{\psi}(\{r_2\}, \{r_2, r_3\}) = 46 - 22 = 24$;
- $\texttt{marg}_{\psi}(\{r_3\}, \{r_1, r_3\}) = \texttt{marg}_{\psi}(\{r_3\}, \{r_2, r_3\}) = 46 - 24 = 22$.

For instance, note that $\texttt{marg}_{\psi}(r_3, \mathcal{R}) = 22$, i.e., without researcher $r_3$, $R$ will entirely lose the value of the best products of this researcher. Instead, $\texttt{marg}_{\psi}(r_2, \mathcal{R}) = \texttt{marg}_{\psi}(r_1, \mathcal{R}) = \texttt{marg}_{\psi}(\{r_1\}, \{r_1, r_2\}) = \texttt{marg}_{\psi}(\{r_2\}, \{r_1, r_2\}) = 20$, i.e., the marginal contribution of each of these two players is less than the value of the products assigned to them. This is because, if $r_2$ were not in the structure, $r_1$ could still use the best co-authored products for the evaluation of the remaining group—and viceversa. Finally, observe that $r_1$ and $r_2$ are completely interchangeable (nothing changes if one switches their names). ◁

We are now ready to define the last fairness property, which takes care of the actual contribution of individuals and groups.

**(P4) "marginality":** *A division rule $\gamma$ must be such that, for each group of researchers $\mathcal{S} \subseteq \mathcal{R}$ and each allocation $\psi$, $\gamma_{\psi}(\mathcal{S}) \geq \texttt{marg}_{\psi}(\mathcal{S}, \mathcal{R})$. Therefore, every group is granted at least its marginal contribution to the performance of the structure R.*

In particular, the above property entails that groups without interactions with other researchers, e.g., departments without collaborations with other departments of the same university, get precisely the total scores of the products assigned to them according to $\psi$.

**Example 2.9.** Interestingly, in our running example, every possible division rule $\gamma$ satisfying the fairness properties **P1–P4** must give the same outcome, given the products evaluated according to $\psi$: $\gamma_\psi(r_1) = \gamma_\psi(r_2) = 21$ and $\gamma_\psi(r_3) = 22$.

Indeed, because the rule must be budget balanced (**P1**), $\gamma_\psi(\mathcal{R}) = 66$. From property **P4**, $\gamma_\psi(r_3) \geq 22$ and $\gamma_\psi(\{r_1, r_2\}) \geq 44$, which entails that $\gamma_\psi(r_3) = 22$, while $r_1$ and $r_2$ must share 44. Moreover, from properties **P2** and **P3** the scores of these latter researchers depend neither on the specific allocation $\psi$, nor on the "name" of the researchers. It follows that $r_1$ and $r_2$ are completely undistinguishable for such a division rule $\gamma$, and thus must get the same score $44/2$. $\lhd$

## 3. A fair solution from coalitional game theory

In this section we show that a division rule enjoying all the above fairness properties actually exists. The proposed solution is based on well-known notions defined for coalitional games, as we can view every group of researchers as a coalition of agents with a suitable associated worth.

### *3.1. Coalitional games*

Coalitional games were introduced by [14] in order to reason about scenarios where players can collaborate by forming coalitions with the aim of obtaining higher worths than by acting in isolation. In the Transferable Utility (TU) setting, coalition worths can be freely distributed amongst agents, while in the non-Transferable Utility setting (NTU) coalitions are allowed to distribute worths only in some specified configurations, called consequences [10].

Here, we consider the classical TU setting, and thus by saying *game* we always mean hereafter *coalitional game with transferable utility*. Such a game can abstractly be modeled as a pair $\mathcal{G} = \langle N, \varphi \rangle$, where $N = \{1, ..., n\}$ is a finite set of players, and $\varphi$ is a function associating with each coalition $C \subseteq N$ a real-value $\varphi(C) \in \mathbb{R}$, with $\varphi(\{\}) = 0$, which is meant to encode the worth that agents in $C$ obtain by collaborating with each other. The function $\varphi$ is *supermodular* (resp., *submodular*) if $\varphi(R \cup T) + \varphi(R \cap T) \geq \varphi(R) + \varphi(T)$ (resp., $\varphi(R \cup T) + \varphi(R \cap T) \leq \varphi(R) + \varphi(T)$) holds, for each pair of coalitions $R, T \subseteq N$.

The outcome of $\mathcal{G}$ is a vector of payoffs $\mathbf{x} = (x_1, ..., x_n) \in \mathbb{R}^n$ meant to specify the distribution of the total worth $\varphi(N)$ granted to each player in $N$. In particular, outcomes are required to be *efficient*, i.e., $\sum_{i \in N} x_i = \varphi(N)$. However, infinitely many outcomes can be associated to a coalitional game. Therefore, a fundamental problem is to single out the most desirable ones in terms of appropriate notions of worth distributions, which are usually called *solution concepts*. Traditionally, this question was studied in economics and game theory with the aim of providing arguments and counterarguments about why such proposals are reasonable mathematical renderings of the intuitive concepts of fairness and stability. Well-known and widely-accepted solution concepts are the *Shapley value*, the *core*, the *kernel*, the *bargaining set*, and the *nucleolus* (see, e.g., [10] for definitions and discussions about such notions, and [7] for an analysis of their computational complexity). Each solution concept defines a set of outcomes that are referred to with the name of the underlying concept. For instance, the *core* of a game is the set of those efficient outcomes satisfying the following condition:

$$\forall C \subseteq N, \sum_{i \in C} x_i \geq \varphi(C).$$

Therefore, according to the core solution concept, the total worth assigned to the agents of any coalition must be at least the worth that the coalition may claim. In particular, every single agent must get at least what (s)he may claim as a singleton coalition. This condition is usually referred to as *individual rationality*. Intuitively, outcomes belonging to the core are *stable*, in that no coalition has interest in leaving the other agents to play on its own (in order to get the worth that it deserves). It is thus highly desirable that a game has a non-empty core.

Note that, whenever the core is not empty, it may contain an infinite number of possible outcomes, so that many proposals define instead a single desirable outcome. In our analysis, we consider the *Shapley value* of $\mathcal{G} = \langle N, \varphi \rangle$, which is the following (unique) outcome:

$$\phi_i(\mathcal{G}) = \sum_{C \subseteq N} \frac{(|N| - |C|)!(|C| - 1)!}{|N|!} (\varphi(C) - \varphi(C \setminus \{i\})),$$

for each $i \in N$.

Roughly, the Shapley value of each player $i$ is determined by the average marginal contribution of $i$ over all possible coalitions (s)he may participate in.

Well-known properties (see, e.g., [10, 15]) of the Shapley value of any game $\mathcal{G} = \langle N, \varphi \rangle$ are the following:

**(I)** $\sum_{i \in N} \phi_i(\mathcal{G}) = \varphi(N)$ (efficiency);
**(II)** If $\varphi$ is *supermodular*, the Shapley value belongs to the core of the game (stability);
**(III)** If $\mathcal{G}' = \langle N, \varphi' \rangle$ is a game such that $\varphi'(C) \geq \varphi(C)$, for each $C \subseteq N$, then $\phi_i(\mathcal{G}') \geq \phi_i(\mathcal{G})$, for each agent $i \in \mathcal{A}$ (monotonicity).

### *3.2. Application to the VQR Program*

It is now natural to formalize the considered application as a coalitional game where the agents are the researchers, and the worth of any coalition is the best result that the researchers in that coalition may achieve if acting in isolation. Formally, for a research structure $R$ and an allocation $\psi$, define the coalitional game $\mathcal{G}_\psi = \langle \mathcal{R}, \mathtt{best}_\psi \rangle$. Then, we propose the following division rule, which is easily seen to be precisely the Shapley value of $\mathcal{G}_\psi$:

$$\gamma^*_\psi(r) = \sum_{\mathcal{S} \subseteq \mathcal{R}} \frac{(|\mathcal{R}| - |\mathcal{S}|)!(|\mathcal{S}| - 1)!}{|N|!} \mathtt{marg}_\psi(r, \mathcal{S}),$$

$$\text{for each } r \in \mathcal{R}.$$

We show that this division rules satisfies all fairness properties defined in the previous section.

**Theorem 3.1.** *Let $\psi$ be any allocation for a given structure R. Then, $\gamma^*_\psi$ is a division rule satisfying properties* **P1***,* **P2***,* **P3***, and* **P4***.*

*Proof Idea.* Because each researcher gets the Shapley value of $\mathcal{G}_\psi$, property **P1** follows immediately from the efficiency property of this solution concept. Property **P2** holds because the allocation $\psi$ is only used to determine the set of products $\mathcal{P}_\psi$ evaluated by ANVUR and hence available for the allocations to the various coalitions. Therefore, every alternative initial allocation based on the same set of products will lead to the same Shapley value for each researcher. Anonymity (**P3**) is also a known property of the Shapley value, because it depends only on the marginal contribution of each agent (and not on her/his "name").

Property **P4** is a little more involved, and follows from the analysis provided in [8] for the more general setting of fair allocation problems. There, it is considered a related game $\bar{\mathcal{G}}_\psi$ over the same set of agents $\mathcal{R}$ as $\mathcal{G}_\psi$, but where the worth of every coalition $C$ is $\mathtt{marg}_\psi(C, \mathcal{R})$. It is also shown that, while $\mathcal{G}_\psi$ is a submodular game, $\bar{\mathcal{G}}_\psi$ is supermodular and thus its Shapley value belongs to its core. Moreover, the Shapley value of the two games coincide, and thus we conclude by observing that property **P4** requires in fact that the outcome of our division rule belongs to the core of $\bar{\mathcal{G}}_\psi$.

## 4. Further properties: Fairness in computing allocations

So far, we focused on a setting where the allocation $\psi$ selected by the structure $R$ is already in place. However, if we really care about fairness, then we should be able to guarantee that no researcher (or substructure) gets a score lower than (s)he deserves just because of the preliminary selection of the products $\mathcal{P}_\psi$. Recall that these products have been determined by the allocation $\psi$ selected in the first phase by the structure $R$, by using a sort of self-evaluation of its researchers (having neither resources nor time to perform a detailed evaluation of the products on its own). In particular, for the current edition (2004–2010) the selection phase is now concluded, and we have registered the following two main approaches in the computation of $\psi$:

- In some structures (or substructures), a centralized approach to prepare the submission has been carried out. In practice, authors have been asked to assign a quality score to each of their products, and the central authority has been in charge to compute the optimal allocation based on them.
- In other structures, researchers resolved in a peer-to-peer manner the conflicts related to co-authored publications. Thus, they presented to their own structures three (or just a small number of) publications, so that the allocation problem was immaterial there, from a centralized perspective. However, while reaching such agreements, researchers still implicitly assigned a quality score to the publications. Therefore, this setting basically aims at finding an optimal allocation via distributed computations.

By abstracting from the peculiarities of this two settings, we can ideally think that each researcher $r \in \mathcal{R}$ associates a quality score $score_r(p) \in \mathbb{R}$ with each product $p \in products(r)$. Recall that an allocation for $R$ is a function $\psi$ mapping each researcher $r \in R$ to a set of publications $\psi(r) \subseteq products(r)$ with $|\psi(r)| \leq 3$ and with $\psi(r) \cap \psi(r') = \emptyset$, for each $r' \in \mathcal{R} \setminus \{r\}$. Then, the preliminary selection phase can be seen as a phase where, given this information on the research products, the structure $R$ computes an *optimal* allocation, i.e., an allocation $\psi^*$ maximizing the *social* *welfare*, i.e., such that $\sum_{r\in\mathcal{R}}\sum_{p\in\psi*(r)} score_r(p) \geq \sum_{r\in\mathcal{R}}\sum_{p\in\psi(r)} score_r(p)$, for each possible allocation $\psi$. Note that this optimization phase is based on the scores declared by researchers. Therefore, the goal of $R$ will be achieved if authors correctly/truthfully self-evaluate their products. Throughout this section we assume that this is always the case. The analysis of the scenario where discrepancies might emerge because researchers finds convenient to adopt untruthful strategies is deferred to Section 5.

**Example 4.1.** Let us consider the simple scenario that is illustrated in Fig. 2(I). Assume that there are just two researchers, $r_1$ and $r_2$, affiliated to $R$. Moreover, assume that $products(r_1) = \{p_1, ..., p_5\}$ and $products(r_2) = \{p_4, ..., p_8\}$, and notice that products $p_4$ and $p_5$ have been co-authored by $r_1$ and $r_2$. For each $p_i \in products(r_1)$ (resp., $p_i \in products(r_2)$), let the self-assessed score $score_{r_1}(p_i)$ (resp., $score_{r_2}(p_i)$) be the one associated with the edge connecting $r_1$ (resp., $r_2$) to $p_i$. Given this setting, it is then easily seen that an optimal allocation for $R$ is $\psi^*$ such that $\psi^*(r_1) = \{p_1, p_2, p_4\}$ and $\psi^*(r_2) = \{p_5, p_7, p_8\}$—see Fig. 2(II). Compare now $\psi^*$ with the allocation $\hat{\psi}^*$ of Fig. 2(III). Note that $\hat{\psi}^*$ is another optimal allocation. ◁

As the above example pointed out, several alternative optimal allocations can be selected by the structure $R$. The choice of a specific optimal allocation is immaterial for $R$ but, depending on the division rule being adopted, can be rather sensible to the researchers. We exemplify this issue by considering again the rule `proj`.

**Example 4.2.** In Fig. 2, it is easily seen that $\texttt{proj}_{\psi^*}(r_1) = 25$, whereas $\texttt{proj}_{\hat{\psi}^*}(r_1) = 26$. Thus, $r_1$ would complain with her/his structure, if $\psi^*$ is selected in place of $\hat{\psi}^*$. ◁

The above observation suggests that a division rule should be indifferent w.r.t. the optimal allocation being selected by $R$ and submitted to ANVUR. Indeed, if the score of a researcher may change over different preliminary allocations, a fair division rule should take care of this fact (which would be nor easy, because it would need to mix scores certified by ANVUR with scores about unselected publications).

**(P5) "independence of the products selection":** *Let $\psi_1, \psi_2$ be any pair of optimal allocations for $R$. A division rule $\gamma$ must be such that, for each $r \in \mathcal{R}$, $\gamma_{\psi_1}(r) = \gamma_{\psi_2}(r)$.*

Interestingly, the rule $\gamma^*$ discussed in this paper satisfies this strong requirement, too. The result is a specialization of the results recently discussed by [8] in the context of fair division problems.

**Theorem 4.2.** *The division rule $\gamma^*$ satisfies* **(P5)**.

*Proof Idea.* In [8], it is shown that a set of products $\mathcal{P}_{\psi_1}$ determined by any optimal allocation $\psi_1$ is enough for computing optimal allocations for every subset of researchers $\mathcal{S} \subseteq \mathcal{R}$. That is, the value $\texttt{best}_{\psi_1}(\mathcal{S})$ computed by using the products in $\mathcal{P}_{\psi_1}$ is at least as good as the value that could be obtained by considering the full set of products coauthored by the researchers in $R$. Therefore, it clearly holds that $\texttt{best}_{\psi_1}(\mathcal{S}) \geq \texttt{best}_{\psi_2}(\mathcal{S})$ for any other optimal allocation $\psi_2$ and thus these values must be equal, because of the arbitrary choice of $\psi_1$. It easily follows that $\forall \mathcal{S} \subseteq \mathcal{R}$, $\texttt{marg}_{\psi_1}(\mathcal{S}, \mathcal{R}) = \texttt{marg}_{\psi_2}(\mathcal{S}, \mathcal{R})$, and thus $\gamma^*_{\psi_1}(r) = \gamma^*_{\psi_2}(r)$. □

## 5. Strategic issues and further desiderata

In this section, we complete the picture by removing the assumption that researchers necessarily report in a truthful way their self-evaluation of research products.

Indeed, while it is clear that the main objective function is to maximize the total value of the products submitted to ANVUR, whenever the same product has different co-authors some strategic issues come into play, and co-authors' personal interest may induce them to cheat on the quality of their products. Clearly, such a wrong information may lead the structure $R$ to perform a product selection quite far from the optimum (total) value. As a matter of fact, depending on the specific division rule being selected, co-authors may be competitors and might want to act strategically to improve their own score. Thus, a division rule must prevent manipulation, which is an issue considered by a field of research known as *mechanism design* (see, e.g, [9, 12, 13]).

**(P6) "truthfulness":** *A division rule $\gamma$ must provide no incentive in misreporting the score of the research products.*

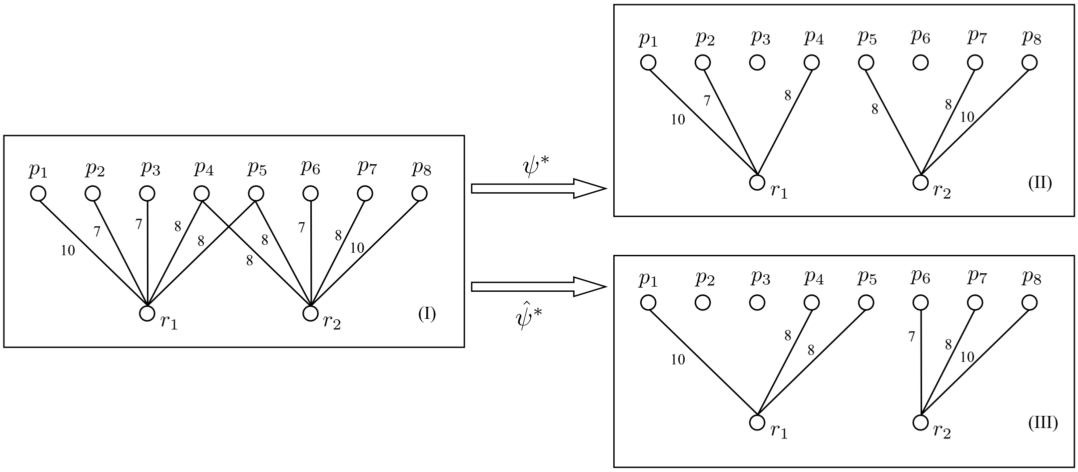


Fig. 2. Running example in Section 2.

Note that, having defined no (fair) division rule, ANVUR declared that for the current evaluation process (2004–2010) only aggregated information about structures and substructures will be made available, rather than the individual scores of the submitted products. However, we next argue that this is not a good choice, and in any case does not prevent ANVUR to receive complaints from (sub)structures.

First, it is clearly a waste of money to conduct such a thorough evaluation of the quality of the Italian research, without then providing the output of the results about research products. Indeed, this kind of information could be useful for a number of purposes, in particular for the evaluation of the researchers that is mandatory according to the current law (binding part of their salary to the productivity).[4] This is so evident that many researchers still believe that such an information will be used for their personal evaluation (soon or later), and thus adopt strategic behaviors to have allocated the best products (usually, under the assumption that the rule `proj` will be used). For instance, a full professor might "force" allocations where (s)he gets best products, leaving bad products to assistant professors, and leading their structure to miss the global optimum, in the same way as the example below.

Second, disclosing only aggregated information does not prevent at all the emergence of strategic behaviors. Indeed, such strategic issues still emerge as soon as two researchers from different substructures co-authored some product, with each of them being interested in providing as much as contribution as possible to her/his own substructure. Again, this might not lead to the global optimum, as we next exemplify for the rule `proj`.

[4]Actually, this applies only to tenured positions at Universities.

**Example 5.1.** Consider again the setting in Fig. 2. Assume that $r_1$ and $r_2$ belong to different substructures. Consider the rule `proj`, and assume that researcher $r_1$ declares that her/his products $p_2$ and $p_3$ are of poor quality (e.g., $score_{r_1}(p_2) = score_{r_1}(p_3) = 2$), as it is illustrated in Fig. 3(I). Then, an optimal allocation $\psi^*_{\mathrm{p}}$ is the one shown in Fig. 3(II), where the set $\{p_1, p_4, p_5, p_6, p_7, p_8\}$ of products is submitted to ANVUR. Assume that, for all these products, there is an agreement between declared scores and ANVUR ones. It follows that $\mathtt{proj}_{\psi^*_{\mathrm{p}}}(r_1) = 26$. On the other hand, recall that in the allocation $\psi^*$ of Fig. 2(II), which has been computed based on the declaration that $score_{r_1}(p_2) = score_{r_1}(p_2) = 7$, it holds that $\mathtt{proj}_{\psi^*}(r_1) = 25$. Thus, $r_1$ finds convenient to misreport the true scores of $p_2$ and $p_3$, and underestimate them. Note however that the overall score of the structure $R$ is still 51 and, in fact, $\psi^*_{\mathrm{p}}$ coincides with the optimal allocation $\hat{\psi}^*$ depicted in Fig. 2(III) and discussed in Example 4.2.

Then, consider a slight variation of the problem instance depicted in Fig. 3 where the actual value of product $p_7$ is 6 (instead of 8). Then, the above egoistic behavior of agent $r_1$ also damages its research structure because it leads to a sub-optimal allocation. Indeed, due to the low declared values for $p_2$ and $p_3$, product $p_7$ is selected and allocated to $r_2$ in the unique (wrong) optimal allocation, whose total score is now 49 (instead of 51). ◁

For another example, consider the following division rule.

`owner`: assign to each author the sum of the "normalized" scores of the submitted products (s)he has co-authored, where by normalization we just mean here dividing the score of any product by the number of its authors. For instance, in the setting of Example 4.1., we have that, in the optimal allocation $\psi^*$ of Fig. 2(II), half of the score associated with $p_4$ (equivalently, $p_5$) is given to $r_1$, and the remaining half to $r_2$. However, even this attempt of having a fair division rule is unsuccessful, as this approach does not satisfy property **(P5)**: just check that $\mathtt{owner}_{r_2}(\psi^*) = 26$ while $\mathtt{owner}_{r_2}(\hat{\psi}^*) = 33$. Indeed, according to this division rule, the score of each researcher depends on the number of publications (s)he has coauthored and $R$ has submitted to ANVUR, which may be very different in the various allocations.

Again, the emergence of strategic issues with the rule `owner` can easily be seen.

**Example 5.2.** Assume that $r_1$ and $r_2$ belong to different substructures. Consider now the rule `owner`, and assume that researcher $r_1$ declares that $score_{r_1}(p_2) = score_{r_1}(p_3) = 9$. Then, consider the optimal allocation $\psi^*_{\circ}$ shown in Fig. 4(II), and note that, in this case, the set $\{p_1, p_2, p_3, p_5, p_7, p_8\}$ of products is submitted to ANVUR.

Now, assume that the VQR scores of these products are those illustrated in Fig. 2(I), i.e., the same ones as those discussed in Example 4.1.. Thus, $r_1$ has cheated with the aim of overestimating the products of which (s)he is the sole author. In fact, this is convenient to her/him, since according to the rule `owner`, $r_1$ now gets $\mathtt{owner}_{\psi^*_{\circ}}(r_1) = 24 + 4$, because of the products in $\{p_1, p_2, p_3\}$ allocated to her/him (with overall score 24) and of the product $p_5$ co-authored with $r_2$ (whose overall score is then shared with $r_2$). On the other hand, just recall that in the allocation $\psi^*$ of Fig. 2(II), which has been computed based on the "truthful" declaration that $score_{r_1}(p_2) = score_{r_1}(p_3) = 7$, it holds that $\mathtt{owner}_{\psi^*}(r_1) = 25$. It follows that $r_1$ finds convenient to cheat under `owner`, in order to increment the number of products submitted to the VQR that (s)he has coauthored. However, the egoistic behavior of agent $r_1$ again damages its research structure, as we now have that the total VQR score is 50 (instead of 51)—see again Fig. 4(II). ◁

As a matter of fact, the use of unfair division rules and the emergence of strategic issues across substructures risk to penalize, in the long term period, collaborations and cross-fertilizations.[5] To prevent all these problems, a fair and truthful mechanism is of course definitively needed.

In fact, as the above examples might have already suggested to the careful reader, truthfulness can be achieved by exploiting the fact that ANVUR evaluation can be seen as a "verification ability" available in the setting. Therefore, one might think of punishing (e.g., by assigning 0 as overall score) all those researchers whose reported values are found different from the verified ones (usually interpreted as "lying agents"), as it is in the spirit of most of the literature on mechanisms with verification (see, e.g., [1, 2, 5, 6, 11] and the references therein). Indeed, under the intimidation of a punishment, any (reasonable) division rule can be turned into a truthful one. However, in the application scenario we are considering, a punishment approach would be hardly "politically" acceptable—just think that a number of researchers have already announced that they will not participate to the VQR program because they disagree with some of the evaluation criteria made available by ANVUR, which are in fact perceived as imposed by "law" rather than as being the outcome of a public discussion on the subject. Moreover, charging researchers because of some discrepancy between their self-evaluation about some paper and the one by ANVUR experts would require some convincing proof of their malicious behavior. Therefore, any punishing approach would be quite hard to implement in practice, for this real-world case study. For this reason, we avoid this brute-force approach, and ask that the following property holds.

**(P7) "no punishment":** *A division rule $\gamma$ must be such that, for each $r \in \mathcal{R}$ and each allocation $\psi^*$, the value $\gamma_{\psi^*}(r)$ is indifferent w.r.t. self-assessed scores, in particular, w.r.t. discrepancies possibly emerging between such scores and VQR ones.*

Note that, in the light of the above requirement, we look for a method to enforce truthfulness where verification is used in a rather limited sense. Moreover, it is relevant

[5] It is not by chance that the authors of this paper belong to different substructures of the same university.

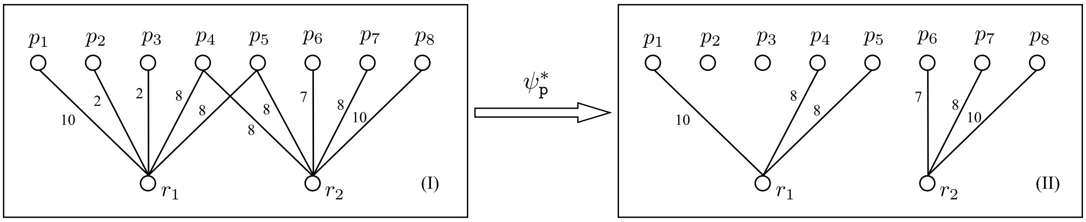


Fig. 3. Strategic manipulations with the rule `proj`.

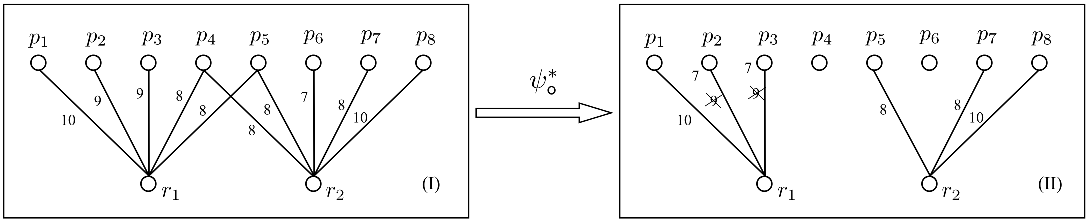


Fig. 4. Strategic manipulations with the rule `owner`.

to observe that if the division rule is not well-designed, then cases might emerge where there is no way at all to exploit verification (even in its strongest form where punishment is allowed).

**Example 5.3.** Consider again the use of the rule `proj` in Example 5.1., and recall that $r_1$ finds convenient to underestimate the true scores of $p_2$ and $p_3$. However, since $p_2$ and $p_3$ does not occur in $\psi^*_{\mathtt{p}}$, as we can see in Fig. 3(II), then there is no way to discover that $r_1$ has actually cheated. Therefore, in this case, verification on the selected products provides no-extra power, and truthfulness is not achieved. $\lhd$

Our final result is that the proposed division rule is robust even to strategic issues, in that there is no advantage for individuals or group of researchers to misreport the scores of their products. To formalize the result, we shall use the well-known concept of *strong Nash equilibrium* [3]. We say that truthtelling is a strong Nash equilibrium for $\mathcal{R}$ w.r.t. a division rule $\gamma$ if there exists no set of researchers $\mathcal{S} \subseteq \mathcal{R}$ such that: every $r \in \mathcal{S}$ provides a wrong information on her/his products and, with respect to an allocation $\psi_w$ computed from this information, (s)he gets a score $\gamma_{\psi_w}(r) > \gamma_{\psi^*}(r)$, for some optimal allocation $\gamma_{\psi^*}(r)$ computed according to the true scores of all research products.

**Theorem 5.3.** *Truthtelling is a strong Nash equilibrium for $\mathcal{R}$ w.r.t. the division rule $\gamma^*$ (property* **P6***). Moreover, the rule satisfies* **P7***, too.*

*Proof.* The fact that the division rule $\gamma^*$ satisfies **P7** is immediate, because the products that were not submitted play no role in the computation, and for the submitted products only the ANVUR actual evaluation is considered (independently of possible different preliminary declarations of researchers).

As far as property **P6** is concerned, we prove a slightly more general result from which the statement clearly follows: Every researchers' declaration that leads to an optimal allocation $\psi^*$ is a Strong Nash equilibrium for the researchers in $\mathcal{R}$ with respect to the division rule $\gamma^*$.

First recall from Theorem 4.3 that $\gamma^*$ satisfies property **P5**, so that the choice of the specific optimal allocation $\psi^*$ is immaterial. Assume, by contradiction, that the declaration of the researchers leading to $\psi^*$ is not a Strong Nash equilibrium. Then, there exists a set of researchers $\mathcal{S} \subseteq \mathcal{R}$ that can improve their scores by changing somehow the declaration for their products. Assume that $\psi_w$ is the allocation for the structure $R$ resulting from this modified declaration of its researchers. Then, it must hold that, for every $r \in \mathcal{S}$, $\gamma_{\psi_w}(r) > \gamma_{\psi^*}(r)$. Again because of property **P5**, note that such a difference may exists only if $\psi_w$ is not an optimal allocation for $R$.

Recall also from the proof of Theorem 4.3 the technical result stated in [8] that the set of products $\mathcal{P}_{\psi^*}$ determined by the optimal allocation $\psi^*$ is enough for computing optimal allocations for every subset of researchers $\mathcal{S} \subseteq \mathcal{R}$. It follows that, for every set $C \subseteq \mathcal{R}$, $\mathtt{best}_{\psi^*}(C) \geq \mathtt{best}_{\psi_w}(C)$. Then, by the monotonicity property of the Shapley value (III) applied to the coalitional games $\langle \mathcal{R}, \texttt{best}_{\psi^*} \rangle$ and $\langle \mathcal{R}, \texttt{best}_{\psi_w} \rangle$, there exists no player $r \in \mathcal{R}$ whose Shapley value in the latter game is strictly higher than her/his Shapley value in the former game (having a better worth function). We thus get a contradiction, because recall that $\gamma^*$ precisely assigns to researchers the Shapley value of such a coalitional game. □

## 6. Conclusion

We described a fair division rule for assigning scores to researchers and substructures, with reference to the Italian research assessment program (in particular, the current edition considering years 2004–2010). We have shown that the proposed rule enjoys a number of desirable properties, such as the independence of the specific allocation of research products, the independence of the preliminary (optimal) products selection, the dependence on the actual (marginal) contribution, and so on. We also explored strategic issues that may occur if such a rule is not used, and we provided a number of examples showing the drawbacks of some alternative rules (that we hope will not be applied by ANVUR). The paper carefully considers practical aspects of the problem, too. For instance, only scores of products actually evaluated by ANVUR are used by the proposed algorithm, and no rules based on punishment are employed to force researchers to truthfully reports the scores of their products in the preliminary selection phase.

The results in this paper may be really useful for the research evaluation process, especially because of the particular period the Italian research system is going through. Indeed, the composition of almost all substructures in all Italian universities is changed since the preliminary selection for the VQR 2004–2010 has been performed. Therefore, the funds assigned according to the result of the evolution process will be given to the new substructures, and it is of utmost importance that such an assignment does depend only on the actual contribution of researchers, and not on the preliminary allocation (where researchers were distributed over substructures in a different way).

We point out that the proposed approach may be useful in more general contexts as well, and we refer the interested reader to [8], for more general examples, further mechanisms where truthtelling is always a dominant strategy, and technical details.

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