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Prioritized Aggregation Operators of GTHFNs MADM Approach for the Evaluation of Renewable Energy Sources

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Abstract. In this paper, firstly, we propose two new GTHFNs-prioritized aggregation operators called generalized trapezoidal hesitant fuzzy number prioritized weighted average operator and generalized trapezoidal hesitant fuzzy number prioritized weighted geometric operator. Secondly, we investigate the fundamental properties of the operators in detail such as idempotency, boundedness and monotonicity. Thirdly, we propose a method based on the developed GTHF-numbers prioritized aggregation operators for solving an MADM problem with GTHF-numbers. Fourthly, we give a numerical example of the developed method. Finally, a comparative analysis is given with some existing methods in solving an MADM problem with GTHF-numbers.

Key words: hesitant fuzzy set, generalized trapezoidal hesitant fuzzy numbers, prioritized aggregation operator, multiple attribute decision making, renewable energy sources.

1. Introduction

Nowadays, as the society continues developing, practical problems and actual scenarios of both human nature and real-world situations, as well as uncertainty, vagueness, inconsistency and imprecision, seem to be prevalent. Therefore, fuzzy set theory (Zadeh, 1965) has been utilized in various fields with imprecise information. Although the fuzzy set theory is a useful tool for modelling problems, including uncertainty information, it can be too difficult in some cases. To avoid this difficulty, recently, intuitionistic fuzzy sets (Atanassov, 1986), type-2 fuzzy sets (Mizumoto and Tanaka, 1976), type-n fuzzy sets (Rickard *et al.*, 2008) and extensions of fuzzy sets, which express information in different ways, have been defined and researched widely. Also, as a generalization of fuzzy sets, Torra and Narukawa (2009) and Torra (2010) introduced the concept of hesitant fuzzy sets which allows the membership of an element of a set to be represented by several possible values. The relationships among hesitant fuzzy sets were also discussed. Many studies on

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hesitant fuzzy sets have been conducted: Xu and Xia (2011a) and Li *et al.* (2015) developed some distance measures for hesitant fuzzy sets under similarity measures, Xu and Xia (2011b) introduced some distance and correlation measures on hesitant fuzzy sets, including desired properties in detail, Wei (2012) developed a few prioritized aggregation operators for hesitant fuzzy sets, and then applied decision making problems in which the attributes are in a different priority level and so on. Since these studies still cannot provide all original data information for the decision making problems, Deli and Karaaslan (2021) introduced the generalized trapezoidal hesitant fuzzy numbers on R. As a consequence, a large number of studies have been conducted by the following authors: Ali *et al.* (2023), Atanassov (2000), Yager (2008), Xia and Xu (2011), Deli (2021, 2020), Anusha *et al.* (2023), Liao *et al.* (2014), Wei (2012).

As the classical sets, fuzzy sets and the generalization of the collected information for the values of the alternatives based on criteria of aggregation operators are useful to convert the whole data into a single value. Therefore, the aggregation operators have great importance and significance in solving MADM problems in the whole set theory. For example, Wan (2013) developed a new decision method based on power average operators of fuzzy numbers. Aydemir and Yilmaz Gündüz (2020) defined some operational laws of q-rung orthopair fuzzy sets and then proposed Dombi prioritized weighted aggregation operators. Zhao and Wei (2013) proposed the intuitionistic fuzzy Einstein hybrid averaging operator and intuitionistic fuzzy Einstein hybrid geometric operator. Verma and Sharma (2014) introduced the trapezoid fuzzy linguistic prioritized weighted average operators and developed an approach to multiple attribute group decision-making trapezoid fuzzy linguistic information. Liu *et al.* (2016) developed the intuitionistic trapezoidal fuzzy prioritized ordered weighted aggregation operator, then proposed the prioritized multi-criteria decision-making problems under intuitionistic trapezoidal fuzzy information. Jiang (2018) developed some models for interval intuitionistic trapezoidal fuzzy multiple attribute decision making problems in which the attributes are in different priority level and with some prioritized aggregation operators. Fahmi *et al.* (2019, 2021) defined some new operation laws for trapezoidal cubic hesitant fuzzy numbers and then developed some new aggregation operators. Liang *et al.* (2017) proposed some new prioritized aggregation operator, and then some desired properties of the new aggregation operators were studied. Liu *et al.* (2017) proposed two prioritized aggregation operators for hesitant intuitionistic fuzzy linguistic sets, and then, based on these aggregation operators, an approach for multi-attribute decision-making was developed under the hesitant intuitionistic fuzzy linguistic sets. Verma (2017) combined the idea of generalized mean and prioritized weighted average operators. Recently, some authors introduced several prioritized aggregation operators within different mathematical structures (Akram *et al.*, 2020; Garg and Rani, 2023; Jana *et al.*, 2020; Kumar and Chen, 2022; Liu and Gao, 2020; Wang *et al.*, 2022).

1.1. *Novelty*

Some methods have been using generalized trapezoidal hesitant fuzzy (GTHF) numbers such as proposed by Deli and Karaaslan (2021). However, each of these methods can work well in a specific situation when the attributes have the same priority, but can also generate undesirable decision-making results when the attributes have the same priority. Therefore, inspired by the ideal of prioritized aggregation operators (Yager, 2008), we developed an approach to solve a multi-attribute decision-making (MADM) problems with GTHF-numbers in which the attributes are in a different priority level.

1.2. *Motivation and Contribution*

The motivation and contributions of the study are as follows:

1. Prioritized operators of GTHF-numbers are introduced to propose a new method for solving MADM problems with GTHF-numbers in which the attributes are in a different priority level.
2. The main aim is to develop GTHF-numbers-prioritized weighted average operator and GTHF-prioritized weighted geometric operator for MADM problems with GTHF-numbers.
3. A method is constructed with an algorithm for MADM problems with GTHF-numbers.
4. An example for application is presented to demonstrate the effectiveness and advantage of the proposed method.

1.3. *Paper*

Structure

The remainder of the paper is organized as follows:

- ↓ In Section 2, we give a brief introduction on some basic definitions and propositions.
- ↓ In Section 3, we introduce some GTHF-numbers prioritized operators and discuss their desirable properties.
- ↓ In Section 4, we develop an MADM method and then initiate an example in which the attributes are in a different priority level.
- ↓ In Section 5, we give a comparison with some existing methods.
- ↪ In Section 6, we propose a conclusion.

2. Preliminaries

In the following, we briefly describe some basic concepts and basic operational laws related to hesitant fuzzy sets and generalized hesitant trapezoidal fuzzy numbers.

Definition 1 (Zadeh, 1965). Let X be a universe. Then, a fuzzy set is defined as follows:

$$A = \{\mu_A(x)/x : x \in X\}, \tag{1}$$

where $\mu_A : X \to [0, 1]$ such that $0 \leqslant \mu_A(x) \leqslant 1$ for all $x \in X$.

Definition 2 (Wang, 2009). Let $\eta_{\tilde{a}} \in [0, 1]$ and $a, b, c, d \in R$ such that $a \leqslant b \leqslant c \leqslant d$. Then, a trapezoidal fuzzy number (TF-number) $\tilde{a} = \langle (a, b, c, d); \eta_{\tilde{a}} \rangle$ is a special fuzzy set on the real number set R, whose membership function is defined as

$$\mu_{\tilde{a}}(x) = \begin{cases} (x-a)\eta_{\tilde{a}}/(b-a), & a \leqslant x \leqslant b, \\ \eta_{\tilde{a}}, & b \leqslant x \leqslant c, \\ (d-x)\eta_{\tilde{a}}/(d-c), & c \leqslant x \leqslant d, \\ 0, & \text{otherwise.} \end{cases}$$

Definition 3 (Wang *et al.*, 2006). Let $\tilde{a} = \langle (a, b, c, d); \eta_{\tilde{a}} \rangle$ be a TF-number with it's membership function $\mu_{\tilde{a}}(x)$. Centroid point of $\tilde{a}$, denoted by $\tilde{a}^*$, is computed as:

$$\tilde{a}^* = \frac{\int x.\mu_{\tilde{a}}(x)dx}{\int \mu_{\tilde{a}}(x)dx} = \frac{\eta_{\tilde{a}}(d^2 - 2c^2 + 2b^2 - a^2 + dc - ab) + 3(c^2 - b^2)}{3\eta_{\tilde{a}}(d - c + b - a) + 6(c - b)}.$$

Definition 4 (Torra, 2010). Let X be a universe. Then, a hesitant fuzzy set (HFS), denoted by H, is defined as:

$$H = \{\langle x, \xi(x)) \rangle : x \in X\}, \tag{2}$$

where $\xi(x)$ is a set of some values in [0, 1] and $\xi = \xi(x)$ is called a hesitant fuzzy element (HFE).

Definition 5 (Yager, 2008). Let $G = \{G_j : j \in \{1, 2, \dots, n\}\}$ be a set of attributes and let a prioritization between the attribute expressed by the linear order $G_1 \succ G_2 \succ G_3 \succ \cdots \succ G_n$ indicate that attribute G_j has a higher priority than G_k, if $j < k$. The value $G_j(x)$ is the performance of any alternative x under attribute G_j, and satisfies $G_j(x) \in [0, 1]$. Then $PA(G_1(x), G_2(x), \dots, G_n(x))$, called the prioritized average operator, is defined as:

$$PA(G_1(x), G_2(x), \dots, G_n(x)) = \sum_{j=1}^{n} w_j G_j(x),$$

where $w_j = \frac{T_j}{\sum_{i=1}^{n} T_i}$, $T_i = \prod_{i=1}^{i-1} G_i(x)$, $T_j = \prod_{j=1}^{j-1} G_j(x)$ such that $T_1 = 1$.

Definition 6 (Wei, 2012). Let ξ^j $(j \in \{1, 2, \dots, n\})$ be a collection of HFEs. Then,

1. The hesitant fuzzy prioritized weighted average operator of the set ξ^j $(j \in \{1, 2, \dots, n\})$ is defined as:

$$HFPWAO(\xi^1, \xi^2, \ldots, \xi^n) = \bigoplus_{j=1}^{n}\left(\frac{T_j}{\sum_{i=1}^{n} T_i}\xi^j\right) = \bigcup_{h_1^1\in\xi^1, h_1^2\in\xi^2,\ldots,h_1^n\in\xi^n}\left\{1-\prod_{j=1}^{n}\left(1-h_1^j\right)^{\frac{T_j}{\sum_{i=1}^{n} T_i}}\right\}, \quad (3)$$

where $T_i = \prod_{k=1}^{i-1}\sum_{h_1^k\in\xi^k} h_1^k$, $T_j = \prod_{k=1}^{j-1}\sum_{h_1^k\in\xi^k} h_1^k$ $(i, j = 2, \ldots, n)$, $T_1 = 1$.

2. The hesitant fuzzy prioritized weighted geometric operator of the set ξ^j $(j \in \{1, 2, \ldots, n\})$ is defined as:

$$HFPWGO(\xi^1, \xi^2, \ldots, \xi^n) = \bigotimes_{j=1}^{n}(\xi^j)^{\frac{T_j}{\sum_{i=1}^{n} T_i}} = \bigcup_{h_1^1\in\xi^1, h_1^2\in\xi^2,\ldots,h_1^n\in\xi^n}\left\{\prod_{j=1}^{n}(\xi^n)^{\frac{T_j}{\sum_{j=1}^{n} T_j}}\right\}, \quad (4)$$

where $T_i = \prod_{k=1}^{i-1}\sum_{h_1^k\in\xi^k} h_1^k$, $T_j = \prod_{k=1}^{j-1}\sum_{h_1^k\in\xi^k} h_1^k$ $(i, j = 2, \ldots, n)$, $T_1 = 1$.

Theorem 1 (Wei, 2012). *Let ξ^j and $\acute{\xi}^j$ $(j \in \{1, 2, \ldots, n\})$ be two sets of HFEs.*

1. *If all ξ^j $(j = 1, 2, \ldots, n)$ are equal, i.e. $\xi^j = \xi$ for all $j \in \{1, 2, \ldots, n\}$ then,*

$$HFPWGO(\xi^1, \xi^2, \ldots, \xi^n) = HFPWAO(\xi^1, \xi^2, \ldots, \xi^n) = \xi. \quad (5)$$

2. *Let $\xi^- = \min_{j\in\{1,2,\ldots,n\}}\{\xi^j\}$, $\xi^+ = \max_{j\in\{1,2,\ldots,n\}}\{\xi^j\}$ then*

$$\xi^- \leqslant HFPWAO(\xi^1, \xi^2, \ldots, \xi^n), HFPWGO(\xi^1, \xi^2, \ldots, \xi^n) \leqslant \xi^+. \quad (6)$$

3. *If $\xi_i \leqslant \acute{\xi}_j$ for all $j \in \{1, 2, \ldots, n\}$, then*

$$HFPWA(\xi^1, \xi^2, \ldots, \xi^n) \leqslant HFPWA(\acute{\xi}^1, \acute{\xi}^2, \ldots, \acute{\xi}^n), \quad (7)$$
$$HFPWA(\xi^1, \xi^2, \ldots, \xi^n) \leqslant HFPWA(\acute{\xi}^1, \acute{\xi}^2, \ldots, \acute{\xi}^n). \quad (8)$$

Definition 7 (Deli and Karaaslan, 2021). Let $\mathbb{R}$ be a set of real numbers such that $a \leqslant b \leqslant c \leqslant d$. Then, a generalized hesitant trapezoidal fuzzy number (GTHF-number), denoted by $\hbar$, is defined as:

$$\hbar = \langle(a, b, c, d); \xi = \{h_i : h_i \in [0, 1]\}\rangle$$

(h_i is set of some values in [0, 1], $i \in \{1, 2, \dots, n\}$) which is a special hesitant fuzzy set on the real number set $\mathbb{R}$, whose membership functions are defined as

$$\mu^i(x) = \begin{cases} (x-a)h_i/(b-a), & a \leqslant x < b, \\ h_i, & b \leqslant x \leqslant c, \\ (d-x)h_i/(d-c), & c < x \leqslant d, \\ 0, & \text{otherwise.} \end{cases}$$

In the paper, for purposes of focusing on GTHF- numbers, note that the set of all GTHF-numbers on $\mathbb{R}^+$ will be denoted by Θ.

Definition 8 (Deli and Karaaslan, 2021). Let $\hbar = \langle (a, b, c, d); \xi \rangle$, $\hbar^1 = \langle (a_1, b_1, c_1, d_1); \xi^1 \rangle$, $\hbar^2 = \langle (a_2, b_2, c_2, d_2); \xi^2 \rangle \in \Theta$ and $\gamma \neq 0$ be any real number. Then,

1. $\hbar^1 \oplus \hbar^2 = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2); \bigcup_{h_1^1 \in \xi^1, h_1^2 \in \xi^2} \{\xi_1^1 + h_1^2 - h_1^1.h_1^2\} \rangle$;
2. $\hbar^1 \odot \hbar^2 = \langle (a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2); \bigcup_{h_1^1 \in \xi^1, h_1^2 \in \xi^2} \{h_1^1.h_1^2\} \rangle$;
3. $\gamma \hbar = \langle (\gamma a, \gamma b, \gamma c, \gamma d); \bigcup_{h \in \xi} \{1 - (1-h)^\gamma\} \rangle \quad (\gamma \geqslant 0)$;
4. $(\hbar)^\gamma = \langle (a^\gamma, b^\gamma, c^\gamma, d^\gamma); \bigcup_{h \in \xi} \{h^\gamma\} \rangle \quad (\gamma \geqslant 0)$.

Definition 9. Deli and Karaaslan (2021) Let $\hbar_j = \langle (a_j, b_j, c_j, d_j); \xi^j \rangle$, $j \in I_n$ be a set of GTHF-numbers. Then,

1. The hesitant fuzzy weighted geometric operator of the set $\hbar_j$ ($j \in \{1, 2, \dots, n\}$) is defined as:

$$H_w^G(\hbar_1, \hbar_2, \dots, \hbar_n) = \bigotimes_{j=1}^n {\hbar_j}^{w_j} = \Bigg\langle \Bigg(\prod_{j=1}^n a_j^{w_j}, \prod_{j=1}^n b_j^{w_j}, \prod_{j=1}^n c_j^{w_j}, \prod_{j=1}^n d_j^{w_j} \Bigg); \bigcup_{h_1^1 \in \xi^1, h_1^2 \in \xi^2, \dots, h_1^n \in \xi^n} \Bigg\{ \prod_{j=1}^n {h_1^j}^{w_j} \Bigg\} \Bigg\rangle;$$

2. The hesitant fuzzy weighted average operator of collection $\hbar_j$ ($j \in \{1, 2, \dots, n\}$) is defined as:

$$\begin{aligned} & H_w^A(\hbar^1, \hbar^2, \dots, \hbar^n) \\ & = \bigoplus_{j=1}^n w_j.\hbar^j = \Bigg\langle \Bigg(\sum_{j=1}^n w_j.a_j, \sum_{j=1}^n w_j.b_j, \sum_{j=1}^n w_j.c_j, \sum_{j=1}^n w_j.d_j \Bigg); \\ & \bigcup_{h_1^1 \in \xi^1, h_1^2 \in \xi^2, \dots, h_1^n \in \xi^n} \Bigg\{ 1 - \prod_{j=1}^n (1 - h_1^j)^{w_j} \Bigg\} \Bigg\rangle, \end{aligned}$$

where $w = (w_1, w_2, \dots, w_n)^T$ is the weight vector of $\hbar^j$, $j \in I_n$ such that $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$.

3. GTHFN-Prioritized Aggregation Operators

In this section, we developed GTHFN-prioritized average operator and GTHF-prioritized geometric operator and examined some desired properties such as idempotency, boundedness and monotonicity in detail.

Definition 10. Let $\hbar_j = \langle (a_j, b_j, c_j, d_j); \xi^j \rangle$ $(j = 1, 2, \dots, n)$ be a set of GTHF-numbers on $[0, 1] \subseteq \mathbb{R}$, $G = \{G_j : j \in \{1, 2, \dots, n\}\}$ be a set of attributes such that there is a prioritization as in the Definition 5. Then, the GTHFN-prioritized average operator, denoted by $\digamma_A(\hbar_1, \hbar_2, \dots, \hbar_n)$, is defined by

$$\digamma_A(\hbar_1, \hbar_2, \dots, \hbar_n) = \bigoplus_{j=1}^{n} \left(\frac{T_j}{\sum_{i=1}^{n} T_i} \hbar_j \right), \tag{9}$$

where

$$T_1 = 1, T_j = \prod_{k=1}^{j-1} \sum_{h_1^k \in \xi^k} \frac{h_1^k(d_k^2 - 2c_k^2 + 2b_k^2 - a_k^2 + d_k c_k - a_k b_k) + 3(c_k^2 - b_k^2)}{3h_1^k(d_k - c_k + b_k - a_k) + 6(c_k - b_k)} \quad (j = 2, \dots, n)$$

and

$$T_1 = 1, T_i = \prod_{k=1}^{i-1} \sum_{h_1^k \in \xi^k} \frac{h_1^k(d_k^2 - 2c_k^2 + 2b_k^2 - a_k^2 + d_k c_k - a_k b_k) + 3(c_k^2 - b_k^2)}{3h_1^k(d_k - c_k + b_k - a_k) + 6(c_k - b_k)} \quad (i = 2, \dots, n).$$

Theorem 2. *Let $\hbar_j = \langle (a_j, b_j, c_j, d_j); \xi^j \rangle$ $(j = 1, 2, \dots, n)$ be a set of GTHF-numbers on $[0, 1] \subseteq \mathbb{R}$, $G = \{G_j : j \in \{1, 2, \dots, n\}\}$ be a set of attributes such that there is a prioritization as in the Definition 5. Then, their aggregated value by using the $\digamma_A(\hbar_1, \hbar_2, \dots, \hbar_n)$ operator is also a GTHF-number, and*

$$\begin{aligned}\digamma_A(\hbar_1, \hbar_2, \dots, \hbar_n) &= \bigoplus_{j=1}^{n} \left(\frac{T_j \hbar_j}{\sum_{i=1}^{n} T_i} \right) \\ &= \left\langle \left(\sum_{j=1}^{n} \frac{T_j}{\sum_{i=1}^{n} T_i} a_j, \sum_{j=1}^{n} \frac{T_j}{\sum_{i=1}^{n} T_i} b_j, \sum_{j=1}^{n} \frac{T_j}{\sum_{i=1}^{n} T_i} c_j, \sum_{j=1}^{n} \frac{T_j}{\sum_{i=1}^{n} T_i} d_j \right); \right. \\ &\quad \left. \bigcup_{h_1^1 \in \xi^1, h_1^2 \in \xi^2, \dots, h_1^n \in \xi^n} \left\{ 1 - \prod_{j=1}^{n} \left(1 - h_1^j\right)^{\frac{T_j}{\sum_{i=1}^{n} T_i}} \right\} \right\rangle, \end{aligned} \tag{10}$$

where

$$T_1=1,\ T_j=\prod_{k=1}^{j-1}\sum_{h_1^k\in\xi^k}\frac{h_1^k(d_k^2-2c_k^2+2b_k^2-a_k^2+d_kc_k-a_kb_k)+3(c_k^2-b_k^2)}{3h_1^k(d_k-c_k+b_k-a_k)+6(c_k-b_k)}$$
$$(j=2,\dots,n)$$

and

$$T_1=1,\ T_i=\prod_{k=1}^{i-1}\sum_{h_1^k\in\xi^k}\frac{h_1^k(d_k^2-2c_k^2+2b_k^2-a_k^2+d_kc_k-a_kb_k)+3(c_k^2-b_k^2)}{3h_1^k(d_k-c_k+b_k-a_k)+6(c_k-b_k)}$$
$$(i=2,\dots,n).$$

It can be easily proved that the $\digamma_A$ operator has the following properties.

Theorem 3 (Idempotency). *Let* $\hbar_j=\langle(a_j,b_j,c_j,d_j);\xi^j\rangle$ $(j=1,2,\dots,n)$ *be a set of GTHF-numbers on* $[0,1]\subseteq\mathbb{R}$, $G=\{G_j:j\in\{1,2,\dots,n\}\}$ *be a set of attributes such that there is a prioritization as in the Definition 5. If* $\hbar_j=\hbar(\hbar=\langle(a,b,c,d);\xi^j\rangle$ $(j=1,2,\dots,n))$ *for all* $j\in\{1,2,\dots,n\}$*, then*

$$\digamma_A(\hbar_1,\hbar_2,\dots,\hbar_n)=\hbar, \tag{11}$$

where

$$T_1=1,\ T_j=\prod_{k=1}^{j-1}\sum_{h_1^k\in\xi^k}\frac{h_1^k(d_k^2-2c_k^2+2b_k^2-a_k^2+d_kc_k-a_kb_k)+3(c_k^2-b_k^2)}{3h_1^k(d_k-c_k+b_k-a_k)+6(c_k-b_k)}$$
$$(j=2,\dots,n)$$

and

$$T_1=1,\ T_i=\prod_{k=1}^{i-1}\sum_{h_1^k\in\xi^k}\frac{h_1^k(d_k^2-2c_k^2+2b_k^2-a_k^2+d_kc_k-a_kb_k)+3(c_k^2-b_k^2)}{3h_1^k(d_k-c_k+b_k-a_k)+6(c_k-b_k)}$$
$$(i=2,\dots,n).$$

Proof. For $\hbar_j=\hbar$ for all $j\in\{1,2,\dots,n\}$, and by definition of $\digamma_A$ operator, we have

$$\digamma_A(\hbar_1,\hbar_2,\dots,\hbar_n)=\bigoplus_{j=1}^{n}\left(\frac{T_j\hbar_j}{\sum_{i=1}^n T_i}\right)$$
$$=\left\langle\left(\sum_{j=1}^{n}\frac{T_j}{\sum_{i=1}^n T_i}a_j,\sum_{j=1}^{n}\frac{T_j}{\sum_{i=1}^n T_i}b_j,\sum_{j=1}^{n}\frac{T_j}{\sum_{i=1}^n T_i}c_j,\sum_{j=1}^{n}\frac{T_j}{\sum_{i=1}^n T_i}d_j\right);\right.$$

$$\bigcup_{h_1^1\in\xi^1,h_1^2\in\xi^2,\dots,h_1^n\in\xi^n}\left\{1-\prod_{j=1}^{n}\left(1-h_1^j\right)^{\frac{T_j}{\sum_{i=1}^n T_i}}\right\}\Bigg\rangle$$

$$\begin{aligned}
&=\Bigg\langle\Bigg(\sum_{j=1}^{n}\frac{T_j}{\sum_{i=1}^n T_i}a,\sum_{j=1}^{n}\frac{T_j}{\sum_{i=1}^n T_i}b,\sum_{j=1}^{n}\frac{T_j}{\sum_{i=1}^n T_i}c,\sum_{j=1}^{n}\frac{T_j}{\sum_{i=1}^n T_i}d\Bigg);\\
&\qquad\bigcup_{h_1^1\in\xi^1,h_1^2\in\xi^2,\dots,h_1^n\in\xi^n}\left\{1-\prod_{j=1}^{n}\left(1-h_1^j\right)^{\frac{T_j}{\sum_{i=1}^n T_i}}\right\}\Bigg\rangle,\ \Bigg(\sum_{j=1}^{n}\frac{T_j}{\sum_{i=1}^n T_i}=1\Bigg)\\
&=\big\langle(a,b,c,d);\xi^j\big\rangle(j=1,2,\dots,n)\}\rangle\\
&=\hbar,
\end{aligned}\tag{12}$$

where

$$T_1=1,\ T_j=\prod_{k=1}^{j-1}\sum_{h_1^k\in\xi^k}\frac{h_1^k(d_k^2-2c_k^2+2b_k^2-a_k^2+d_kc_k-a_kb_k)+3(c_k^2-b_k^2)}{3h_1^k(d_k-c_k+b_k-a_k)+6(c_k-b_k)}$$
$$(j=2,\dots,n)$$

and

$$T_1=1,\ T_i=\prod_{k=1}^{i-1}\sum_{h_1^k\in\xi^k}\frac{h_1^k(d_k^2-2c_k^2+2b_k^2-a_k^2+d_kc_k-a_kb_k)+3(c_k^2-b_k^2)}{3h_1^k(d_k-c_k+b_k-a_k)+6(c_k-b_k)}$$
$$(i=2,\dots,n).$$

This completes the proof. □

Theorem 4 (Boundedness). *Let $\hbar_j=\langle(a_j,b_j,c_j,d_j);\xi^j\rangle$ $(j=1,2,\dots,n)$ be a set of GTHF-numbers on $[0,1]\subseteq\mathbb{R}$, $G=\{G_j: j\in\{1,2,\dots,n\}\}$ be a set of attributes such that there is a prioritization as in the Definition 5. Let*

$$\begin{aligned}
\hbar^-=\Bigg\langle\Bigg(&\min_{j\in\{1,2,\dots,n\}}\{a_j\},\min_{j\in\{1,2,\dots,n\}}\{b_j\},\min_{j\in\{1,2,\dots,n\}}\{c_j\},\min_{j\in\{1,2,\dots,n\}}\{d_j\}\Bigg);\\
&\min_{j\in\{1,2,\dots,n\}}\left\{\min_{h_1^j\in\xi^j}\{h_1^j\}\right\}\Bigg\rangle
\end{aligned}$$

and

$$\begin{aligned}
\hbar^+=\Bigg\langle\Bigg(&\max_{j\in\{1,2,\dots,n\}}\{a_j\},\max_{j\in\{1,2,\dots,n\}}\{b_j\},\max_{j\in\{1,2,\dots,n\}}\{c_j\},\max_{j\in\{1,2,\dots,n\}}\{d_j\}\Bigg);\\
&\max_{j\in\{1,2,\dots,n\}}\left\{\max_{h_1^j\in\xi^j}\{h_1^j\}\right\}\Bigg\rangle.
\end{aligned}$$

If

$$\text{“}a_j \leqslant a_i, b_j \leqslant b_i, c_j \leqslant c_i, d_j \leqslant d_i \text{ and } h_1^j \leqslant h_1^i \text{ for all } h_1^j \in \xi^j, h_1^i \in \xi^i\text{”}$$
$$\Rightarrow \text{“}\hbar_j = \langle (a_j, b_j, c_j, d_j); \xi^j \rangle \leqslant \hbar_i = \langle (a_i, b_i, c_i, d_i); \xi^i \rangle\text{”} \tag{13}$$

then

$$\hbar^- \leqslant \hbar_j \leqslant \hbar^+. \tag{14}$$

Proof. We have $\frac{\sum_{j=1}^n T_j}{\sum_{i=1}^n T_i} = 1$, since

$$\min_{j\in\{1,2,\dots,n\}}\{a_j\} \leqslant \{a_j\} \leqslant \max_{j\in\{1,2,\dots,n\}}\{a_j\}, \quad \min_{j\in\{1,2,\dots,n\}}\{b_j\} \leqslant \{b_j\} \leqslant \max_{j\in\{1,2,\dots,n\}}\{b_j\},$$
$$\min_{j\in\{1,2,\dots,n\}}\{c_j\} \leqslant \{c_j\} \leqslant \max_{j\in\{1,2,\dots,n\}}\{c_j\}, \quad \min_{j\in\{1,2,\dots,n\}}\{d_j\} \leqslant \{d_j\} \leqslant \max_{j\in\{1,2,\dots,n\}}\{d_j\},$$

and

$$\min_{h_1^j\in\xi^j}\{h_1^j\} \leqslant h_1^j \leqslant \max_{h_1^j\in\xi^j}\{h_1^j\}.$$

Let F_A be an operator. Then, by using the Theorem 3.2, it yields that

$$\hbar^- \leqslant \hbar_j \leqslant \hbar^+. \tag{15}$$

□

Theorem 5 (Monotonicity). *Let* $\hbar_j = \langle (a_j, b_j, c_j, d_j); \xi^j \rangle$ $(j = 1, 2, \dots, n)$ *and* $\acute{\hbar}_j = \langle (\acute{a}_j, \acute{b}_j, \acute{c}_j, \acute{d}_j); \acute{xi}^j \rangle$ $(j = 1, 2, \dots, n)$ *be two sets of GTHF-numbers on* $[0, 1] \subseteq \mathbb{R}$, $G = \{G_j : j \in \{1, 2, \dots, n\}\}$ *be a set of attributes such that there is a prioritization as in the Definition 5. If* $\hbar_j \leqslant \acute{\hbar}_j$ *for all* $j \in \{1, 2, \dots, n\}$ *based on equation (13), then*

$$F_A(\hbar_1, \hbar_2, \dots, \hbar_n) \leqslant F_A(\acute{\hbar}_1, \acute{\hbar}_2, \dots, \acute{\hbar}_n). \tag{16}$$

where

$$T_1 = 1, T_j = \prod_{k=1}^{j-1} \sum_{h_1^k\in\xi^k} \frac{h_1^k(d_k^2 - 2c_k^2 + 2b_k^2 - a_k^2 + d_kc_k - a_kb_k) + 3(c_k^2 - b_k^2)}{3h_1^k(d_k - c_k + b_k - a_k) + 6(c_k - b_k)}$$
$$(j = 2, \dots, n),$$
$$T_1 = 1, T_i = \prod_{k=1}^{i-1} \sum_{h_1^k\in\xi^k} \frac{h_1^k(d_k^2 - 2c_k^2 + 2b_k^2 - a_k^2 + d_kc_k - a_kb_k) + 3(c_k^2 - b_k^2)}{3h_1^k(d_k - c_k + b_k - a_k) + 6(c_k - b_k)}$$
$$(i = 2, \dots, n),$$

$$T_1 = 1, T_j = \prod_{k=1}^{j-1} \sum_{\acute{h}_1^k \in \acute{xi}^k} \frac{\acute{h}_1^k(d_k^2 - 2c_k^2 + 2b_k^2 - a_k^2 + d_k c_k - a_k b_k) + 3(c_k^2 - b_k^2)}{3\acute{h}_1^k(d_k - c_k + b_k - a_k) + 6(c_k - b_k)}$$
$$(j = 2, \dots, n)$$

and

$$T_1 = 1, T_i = \prod_{k=1}^{i-1} \sum_{\acute{h}_1^k \in \acute{xi}^k} \frac{\acute{h}_1^k(d_k^2 - 2c_k^2 + 2b_k^2 - a_k^2 + d_k c_k - a_k b_k) + 3(c_k^2 - b_k^2)}{3\acute{h}_1^k(d_k - c_k + b_k - a_k) + 6(c_k - b_k)}$$
$$(i = 2, \dots, n).$$

Definition 11. Let $\hbar_j = \langle (a_j, b_j, c_j, d_j); \xi^j \rangle$ $(j = 1, 2, \dots, n)$ be a collection of GTHF-numbers on $[0, 1] \subseteq \mathbb{R}$, $G = \{G_j : j \in \{1, 2, \dots, n\}\}$ be a set of attributes such that there is a prioritization as in the Definition 5. Then, the GTHFN-prioritized geometric operator, denoted by $\digamma_G(\hbar_1, \hbar_2, \dots, \hbar_n)$, is defined by

$$\digamma_G(\hbar_1, \hbar_2, \dots, \hbar_n) = \hbar_1^{\frac{T_1}{\sum_{i=1}^n T_i}} \otimes \hbar_2^{\frac{T_2}{\sum_{i=1}^n T_i}} \otimes \dots \otimes \hbar_1^{\frac{T_2}{\sum_{i=1}^n T_i}}, \tag{17}$$

where

$$T_1 = 1, T_j = \prod_{k=1}^{j-1} \sum_{h_1^k \in \xi^k} \frac{h_1^k(d_k^2 - 2c_k^2 + 2b_k^2 - a_k^2 + d_k c_k - a_k b_k) + 3(c_k^2 - b_k^2)}{3h_1^k(d_k - c_k + b_k - a_k) + 6(c_k - b_k)}$$
$$(j = 2, \dots, n)$$

and

$$T_1 = 1, T_i = \prod_{k=1}^{i-1} \sum_{h_1^k \in \xi^k} \frac{h_1^k(d_k^2 - 2c_k^2 + 2b_k^2 - a_k^2 + d_k c_k - a_k b_k) + 3(c_k^2 - b_k^2)}{3h_1^k(d_k - c_k + b_k - a_k) + 6(c_k - b_k)}$$
$$(i = 2, \dots, n).$$

Theorem 6. *Let* $\hbar_j = \langle (a_j, b_j, c_j, d_j); \xi^j \rangle$ $(j = 1, 2, \dots, n)$ *be a set of GTHF-numbers on* $[0, 1] \subseteq \mathbb{R}$, $G = \{G_j : j \in \{1, 2, \dots, n\}\}$ *be a set of attributes such that there is a prioritization as in the Definition 5. Then, their aggregated value by using the* $\digamma_G(\hbar_1, \hbar_2, \dots, \hbar_n)$ *operator is also a GTHF-number, and*

$$\begin{aligned}\digamma_G(\hbar_1, \hbar_2, \dots, \hbar_n) &= \hbar_1^{\frac{T_1}{\sum_{j=1}^n T_j}} \bigotimes \hbar_2^{\frac{T_2}{\sum_{j=1}^n T_j}} \bigotimes \dots \bigotimes \hbar_n^{\frac{T_n}{\sum_{j=1}^n T_j}} \\ &= \Bigg\langle \Big(a_j^{\frac{T_j}{\sum_{j=1}^n T_j}}, b_j^{\frac{T_j}{\sum_{j=1}^n T_j}}, c_j^{\frac{T_j}{\sum_{j=1}^n T_j}}, d_j^{\frac{T_j}{\sum_{j=1}^n T_j}}\Big); \\ &\quad \bigcup_{\gamma_1 \in \hbar_1, \gamma_2 \in \hbar_2, \dots, \gamma_n \in \hbar_n} \Bigg\{ \prod_{j=1}^n (\gamma_j)^{\frac{T_j}{\sum_{j=1}^n T_j}} \Bigg\} \Bigg\rangle,\end{aligned} \tag{18}$$

where

$$T_1 = 1, T_j = \prod_{k=1}^{j-1} \sum_{h_1^k \in \xi^k} \frac{h_1^k(d_k^2 - 2c_k^2 + 2b_k^2 - a_k^2 + d_k c_k - a_k b_k) + 3(c_k^2 - b_k^2)}{3h_1^k(d_k - c_k + b_k - a_k) + 6(c_k - b_k)}$$
$$(j = 2, \dots, n)$$

and

$$T_1 = 1, T_i = \prod_{k=1}^{i-1} \sum_{h_1^k \in \xi^k} \frac{h_1^k(d_k^2 - 2c_k^2 + 2b_k^2 - a_k^2 + d_k c_k - a_k b_k) + 3(c_k^2 - b_k^2)}{3h_1^k(d_k - c_k + b_k - a_k) + 6(c_k - b_k)}$$
$$(i = 2, \dots, n).$$

It can be easily proved that the $\digamma_G$ operator has the following properties.

Theorem 7 (Idempotency). *Let $\hbar_j = \langle (a_j, b_j, c_j, d_j); \xi^j \rangle$ $(j = 1, 2, \dots, n)$ be a set of GTHF-numbers on $[0, 1] \subseteq \mathbb{R}$, $G = \{G_j : j \in \{1, 2, \dots, n\}\}$ be a set of attributes such that there is a prioritization as in the Definition 5. If $\hbar_j = \hbar$ for all $j \in \{1, 2, \dots, n\}$, then*

$$\digamma_G(\hbar_1, \hbar_2, \dots, \hbar_n) = \hbar \tag{19}$$

where

$$T_1 = 1, T_j = \prod_{k=1}^{j-1} \sum_{h_1^k \in \xi^k} \frac{h_1^k(d_k^2 - 2c_k^2 + 2b_k^2 - a_k^2 + d_k c_k - a_k b_k) + 3(c_k^2 - b_k^2)}{3h_1^k(d_k - c_k + b_k - a_k) + 6(c_k - b_k)}$$
$$(j = 2, \dots, n)$$

and

$$T_1 = 1, T_i = \prod_{k=1}^{i-1} \sum_{h_1^k \in \xi^k} \frac{h_1^k(d_k^2 - 2c_k^2 + 2b_k^2 - a_k^2 + d_k c_k - a_k b_k) + 3(c_k^2 - b_k^2)}{3h_1^k(d_k - c_k + b_k - a_k) + 6(c_k - b_k)}$$
$$(i = 2, \dots, n).$$

Theorem 8 (Boundedness). *Let $\hbar_j = \langle (a_j, b_j, c_j, d_j); \xi^j \rangle$ $(j = 1, 2, \dots, n)$ be a set of GTHF-numbers on $[0, 1] \subseteq \mathbb{R}$, $G = \{G_j : j \in \{1, 2, \dots, n\}\}$ be a set of attributes such that there is a prioritization as in the Definition 5. Let*

$$\hbar^- = \Big\langle \Big(\min_{j \in \{1,2,\dots,n\}} \{a_j\}, \min_{j \in \{1,2,\dots,n\}} \{b_j\}, \min_{j \in \{1,2,\dots,n\}} \{c_j\}, \min_{j \in \{1,2,\dots,n\}} \{d_j\} \Big);$$
$$\min_{j \in \{1,2,\dots,n\}} \Big\{ \min_{h_1^j \in \xi^j} \{h_1^j\} \Big\} \Big\rangle$$

and

$$\hbar^{+} = \Big\langle \Big(\max_{j\in\{1,2,\dots,n\}}\{a_j\}, \max_{j\in\{1,2,\dots,n\}}\{b_j\}, \max_{j\in\{1,2,\dots,n\}}\{c_j\}, \max_{j\in\{1,2,\dots,n\}}\{d_j\} \Big); \max_{j\in\{1,2,\dots,n\}} \Big\{ \max_{h_1^j\in\xi^j} \{h_1^j\} \Big\} \Big\rangle.$$

If

$$\text{“}a_j \leqslant a_i, b_j \leqslant b_i, c_j \leqslant c_i, d_j \leqslant d_i \text{ and } h_1^j \leqslant h_1^i \text{ for all } h_1^j \in \xi^j, h_1^i \in \xi^i\text{”} \Rightarrow \text{”}\hbar_j = \big\langle (a_j, b_j, c_j, d_j); \xi^j \big\rangle \leqslant \hbar_i = \big\langle (a_i, b_i, c_i, d_i); \xi^i \big\rangle\text{”} \tag{20}$$

then

$$\hbar^{-} \leqslant \digamma_G(\hbar_1, \hbar_2, \dots, \hbar_n) \leqslant \hbar^{+}, \tag{21}$$

where

$$T_1 = 1, T_j = \prod_{k=1}^{j-1} \sum_{h_1^k\in\xi^k} \frac{h_1^k(d_k^2 - 2c_k^2 + 2b_k^2 - a_k^2 + d_k c_k - a_k b_k) + 3(c_k^2 - b_k^2)}{3h_1^k(d_k - c_k + b_k - a_k) + 6(c_k - b_k)} \quad (j = 2, \dots, n)$$

and

$$T_1 = 1, T_i = \prod_{k=1}^{i-1} \sum_{h_1^k\in\xi^k} \frac{h_1^k(d_k^2 - 2c_k^2 + 2b_k^2 - a_k^2 + d_k c_k - a_k b_k) + 3(c_k^2 - b_k^2)}{3h_1^k(d_k - c_k + b_k - a_k) + 6(c_k - b_k)} \quad (i = 2, \dots, n).$$

Theorem 9 (Monotonicity). *Let* $\hbar_j = \langle (a_j, b_j, c_j, d_j); \xi^j \rangle$ $(j = 1, 2, \dots, n)$ *and* $\acute{\hbar}_j = \langle (\acute{a}_j, \acute{b}_j, \acute{c}_j, \acute{d}_j); \acute{xi}^j \rangle$ $(j = 1, 2, \dots, n)$ *be two sets of GTHF-numbers on* $[0, 1] \subseteq \mathbb{R}$, $G = \{G_j : j \in \{1, 2, \dots, n\}\}$ *be a set of attributes such that there is a prioritization as in the Definition 5. If* $\hbar_j \leqslant \acute{\hbar}_j$ *for all* $j \in \{1, 2, \dots, n\}$ *based on equation (20), then*

$$\digamma_G(\hbar_1, \hbar_2, \dots, \hbar_n) \leqslant \digamma_G(\acute{\hbar}_1, \acute{\hbar}_2, \dots, \acute{\hbar}_n), \tag{22}$$

where

$$T_1 = 1, T_j = \prod_{k=1}^{j-1} \sum_{h_1^k\in\xi^k} \frac{h_1^k(d_k^2 - 2c_k^2 + 2b_k^2 - a_k^2 + d_k c_k - a_k b_k) + 3(c_k^2 - b_k^2)}{3h_1^k(d_k - c_k + b_k - a_k) + 6(c_k - b_k)} \quad (j = 2, \dots, n),$$

$$T_1 = 1, T_i = \prod_{k=1}^{i-1} \sum_{h_1^k \in \xi^k} \frac{h_1^k(d_k^2 - 2c_k^2 + 2b_k^2 - a_k^2 + d_k c_k - a_k b_k) + 3(c_k^2 - b_k^2)}{3h_1^k(d_k - c_k + b_k - a_k) + 6(c_k - b_k)}$$
$$(i = 2, \dots, n),$$
$$T_1 = 1, T_j = \prod_{k=1}^{j-1} \sum_{\acute{h}_1^k \in \acute{xi}^k} \frac{\acute{h}_1^k(d_k^2 - 2c_k^2 + 2b_k^2 - a_k^2 + d_k c_k - a_k b_k) + 3(c_k^2 - b_k^2)}{3\acute{h}_1^k(d_k - c_k + b_k - a_k) + 6(c_k - b_k)}$$
$$(j = 2, \dots, n)$$

and

$$T_1 = 1, T_i = \prod_{k=1}^{i-1} \sum_{\acute{h}_1^k \in \acute{xi}^k} \frac{\acute{h}_1^k(d_k^2 - 2c_k^2 + 2b_k^2 - a_k^2 + d_k c_k - a_k b_k) + 3(c_k^2 - b_k^2)}{3\acute{h}_1^k(d_k - c_k + b_k - a_k) + 6(c_k - b_k)}$$
$$(i = 2, \dots, n).$$

4. An Approach for GTHFPA Operator to MADM

4.1. *Decision-Making Steps*

In this section, we shall utilize the GTHFPA operators to MADM. To do this, we develop an algorithm which is presented in Fig. 1.

DEFINITION 12. Let $U = \{U_1, U_2, \dots, U_m\}$ be a set of alternatives, $E = \{E_1 = \bigcup_{r=1}^{k_1}\{e_{1r}\}, E_2 = \bigcup_{r=1}^{k_2}\{e_{1r}\}, \dots, E_n = \bigcup_{r=1}^{k_n}\{e_{1r}\} : k_1, k_2, \dots, k_n \in Z^+\}$ be a set of attributes. Here, there is a prioritization between the attributes expressed by the linear order $E_1 \succ E_2 \succ \cdots \succ E_n$ that indicates attribute E_j has a higher priority than E_k, if $j < k$. Suppose that GTHF sub-decision matrix for sub-attribute set $E_j = \bigcup_{r=1}^{k_j}\{e_{1r}\}$ be $H^j = (x_{ir}^j)_{m\times k_j}$ $(i = 1, 2, \dots, m, r = 1, 2, \dots, k_j)$, where x_{ir}^j is in the form of GTHF-number based on the alternative U_i and sub-attribute e_{jr}.

Now, we develop an approach for MADM problems under the GTHFPA operator (or GTHFPG operator) with GTHF-numbers as the following algorithm.

Algorithm:

Step 1. Construct the GTHF sub-decision matrix for sub-attribute set $E_j = \bigcup_{r=1}^{k_j}\{e_{jr}\}$ as $H^j = (x_{ir}^j)_{m\times k_j} = (\langle (a_{ir}^j, b_{ir}^j, c_{ir}^j, d_{ir}^j); \xi_{ir}^j \rangle)_{m\times k_j}$ $(i = 1, 2, \dots, m, r = 1, 2, \dots, k_j)$ for decision.

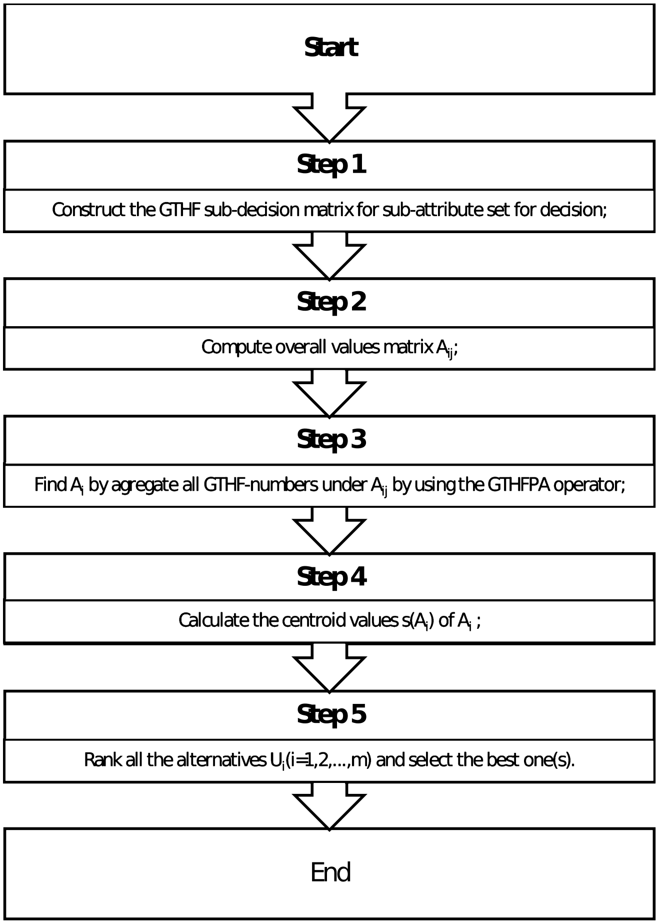

Fig. 1. Frame diagram for proposed work.

Step 2. Compute overall values matrix

$$\begin{aligned} A_{ij} &= \langle (a_{ij}, b_{ij}, c_{ij}, d_{ij}); \xi^{ij} \rangle = GTHFPA\big(x_{i1}^{j}, x_{i2}^{j}, \ldots, x_{ik_j}^{j}\big) \\ &= \frac{T_{i1}^{J}}{\sum_{r=1}^{k_j} T_{ir}^{J}} x_{i1}^{j} \bigoplus \frac{T_{i2}^{J}}{\sum_{r=1}^{k_j} T_{ir}^{J}} x_{i2}^{j} \bigoplus \cdots \bigoplus \frac{T_{ik_j}^{J}}{\sum_{r=1}^{k_j} T_{ir}^{J}} x_{ik_j}^{j} \end{aligned} \tag{23}$$

for $i \in \{1, 2, \ldots, m\}$ and $j \in \{1, 2, \ldots, n\}$ where A_{ij} denotes evaluation of the alternative U_i with respect to the attribute E_j, where

$$T_{ir}^{j} = \prod_{p=1}^{r-1} \sum_{h_{ip}\in\xi_{ir}^{j}} \frac{h_{ip}(d_{ip}^2 - 2c_{ip}^2 + 2b_{ip}^2 - a_{ip}^2 + d_{ip}c_{ip} - a_{ip}b_{ip}) + 3(c_{ip}^2 - b_{ip}^2)}{3h_{ip}(d_{ip} - c_{ip} + b_{ip} - a_{ip}) + 6(c_{ip} - b_{ip})}$$
$$(i = 1, 2, \dots, m, r = 2, \dots, k_j) \tag{24}$$

and

$$T_{i1} = 1, \quad (i = 1, 2, \dots, m). \tag{25}$$

Step 3. Find A_i $(i = 1, 2, \dots, m)$ by agregating all GTHF-numbers under A_{ij} $(j = 1, 2, \dots, n)$ using the GTHFPA operator as follows:

$$A_i = GTHFPA(A_{i1}, A_{i2}, \dots A_{in}) = \langle (a_i, b_i, c_i, d_i); \xi^i \rangle$$
$$= \frac{T_{i1}}{\sum_{j=1}^{n} T_{ij}} A_{i1} \bigoplus \frac{T_{i2}}{\sum_{j=1}^{n} T_{ij}} A_{i2} \bigoplus \dots \bigoplus \frac{T_{in}}{\sum_{j=1}^{n} T_{ij}} A_{in}, \tag{26}$$

where

$$T_{ij} = \prod_{p=1}^{j-1} \sum_{h_{ip}\in\xi^{ij}} \frac{h_{ip}(d_{ip}^2 - 2c_{ip}^2 + 2b_{ip}^2 - a_{ip}^2 + d_{ip}c_{ip} - a_{ip}b_{ip}) + 3(c_{ip}^2 - b_{ip}^2)}{3h_{ip}(d_{ip} - c_{ip} + b_{ip} - a_{ip}) + 6(c_{ip} - b_{ip})}$$
$$(i = 1, 2, \dots, m, \ j = 2, \dots, n) \tag{27}$$

and

$$T_{i1} = 1, \quad (i = 1, 2, \dots, m). \tag{28}$$

Step 4. Calculate the centroid values $s(A_i)$ of $A_i (i = 1, 2, \dots, m)$ using the Definition 3:

$$s(A_i) = \sum_{h_i\in\xi^i} \frac{h_i(d_i^2 - 2c_i^2 + 2b_i^2 - a_i^2 + d_i c_i - a_i b_i) + 3(c_i^2 - b_i^2)}{3h_i(d_i - c_i + b_i - a_i) + 6(c_i - b_i)}. \tag{29}$$

Step 5. Rank all the alternatives U_i $(i = 1, 2, , \dots, m)$ and select the best one(s). Here if $s(A_s) > s(A_t) \Rightarrow U_s > U_t$ $(s, t \in \{1, 2, \dots, m\})$.

5. An Application Case for Turkey

In this section, we present a MADM problem which is a software selection problem adapted/inspirated from Çelikbilek and Tüysüz (2016), Yuan *et al.* (2018). The application of the proposed approach is for evaluating renewable energy under subjective perspective with linguistic scales.

Table 1
Renewable energy resource alternatives.

Symbol	The renewable energy resource
U_1	Geothermal energy
U_2	Solar energy
U_3	Biomass energy
U_4	Wind energy

Table 2
GTHF-numbers for linguistic terms.

Linguistic terms	Linguistic values of GTHF-numbers	Score
Absolutely Low (AL)	$\langle(0.1, 0.2, 0.3, 0.5); \{0.1, 0.2, 0.4\}\rangle$	0, 0338
Very very Low (L)	$\langle(0.2, 0.3, 0.4, 0.6); \{0.1, 0.3, 0.5\}\rangle$	0, 0585
Very Low (VL)	$\langle(0.3, 0.4, 0.5, 0.6); \{0.2, 0.4, 0.7\}\rangle$	0, 0780
Fairly Low (FL)	$\langle(0.1, 0.4, 0.5, 0.6); \{0.2, 0.4, 0.6\}\rangle$	0, 0880
Low (L)	$\langle(0.4, 0.5, 0.7, 0.8); \{0.2, 0.4, 0.7\}\rangle$	0, 1560
Medium (M)	$\langle(0.5, 0.7, 0.8, 0.9); \{0.3, 0.5, 0.6\}\rangle$	0, 1657
Fairly High (FH)	$\langle(0.3, 0.5, 0.6, 0.8); \{0.2, 0.5, 0.9\}\rangle$	0, 1760
High (H)	$\langle(0.1, 0.4, 0.8, 0.9); \{0.1, 0.3, 0.5\}\rangle$	0, 1920
Very High (VH)	$\langle(0.2, 0.5, 0.6, 0.7); \{0.7, 0.8, 0.9\}\rangle$	0, 2240
Very Very High (VVH)	$\langle(0.6, 0.7, 0.8, 0.9); \{0.6, 0.8, 0.9\}\rangle$	0, 2300
Absolutely High (AH)	$\langle(0.4, 0.5, 0.7, 0.9); \{0.3, 0.7, 0.9\}\rangle$	0, 2818

5.1. *Case Study*

According to the plan of renewable energy development in Turkey, the Turkish government aims to reduce the country's dependence on imported energy. Having insufficient quantities of domestic oil and natural gas resources to support demand, the best guarantee of security of energy supply is clearly to maintain a diversity of energy sources and to focus on renewable energy sources, which the country has abundantly. There are five alternatives which are given in Table 1: Geothermal energy U_1, Solar energy U_2, Biomass energy U_3 and Wind energy U_4. The most common attributes for renewable energy evaluation involve E_1 economic, E_2 technical and E_3 environmental. While many national and international companies invest in these resources for a new source of income, states encourage these investments both to produce clean energy and to utilize their own resources in energy production. Therefore, our priority is environment E_3. Then we evaluate wind power plants as they will have a significant contribution to the country's economy since it is a new source of income E_1, and finally, E_2 technical effect. That is, in this case, there is a strict prioritization of parameters $E_3 > E_1 > E_2$, here $>$ indicates preference.

Also, description of subattributes for attributes E_1, E_2 and E_3 is presented in Table 3 as linguistic assessment of the renewable energy alternatives based on GTHF-numbers, is given as:

E_1: Economical attribute contains e_{11} = ervice life, e_{12} = investment cost and e_{13} = operation and maintenance cost. There is a strict prioritization between parameters $e_{11} > e_{12} > e_{13}$, here $>$ indicates preferred to.

Table 3
Linguistic assessment of the renewable energy alternatives based on GTHF-number.

Primary criteria	Secondary criteria	Alternatives	Evaluation values
E_1-Economical criterion	e_{11}-Service life	U_1	{(L)}
		U_2	{(FL)}
		U_3	{(H)}
		U_4	{(AH)}
	e_{12}-Investment cost	U_1	{(H)}
		U_2	{(FH)}
		U_3	{(VH)}
		U_4	{(AH)}
	e_{13}-Operation and maintenance cost	U_1	{(AH)}
		U_2	{(L)}
		U_3	{(VVH)}
		U_4	{(FL)}
E_2-Technical criterion	e_{21}-Availability	U_1	{(M)}
		U_2	{(VVH)}
		U_3	{(H)}
		U_4	{(L)}
	e_{22}-Capacity	U_1	{(M)}
		U_2	{(L)}
		U_3	{(H)}
		U_4	{(VH)}
	e_{23}-Resource density	U_1	{(FH)}
		U_2	{(H)}
		U_3	{(VH)}
		U_4	{(VVH)}
	e_{24}-Efficiency	U_1	{(AL) }
		U_2	{(VL)}
		U_3	{(FH)}
		U_4	{(VVH)}
E_3-Environmental criterion	e_{31}-Air pollution	U_1	{(AL)}
		U_2	{(AH)}
		U_3	{(H)}
		U_4	{(VH)}
	e_{32}-Noise pollution	U_1	{(M)}
		U_2	{(VL)}
		U_3	{(AL)}
		U_4	{(VVH)}

E_2: Technical attribute contains e_{21} = availability, e_{22} = capacity, e_{23} = resource density and e_{24} = efficiency. There is a strict prioritization between parameters $e_{21} > e_{22} > e_{23} > e_{24}$, here $>$ indicates preference.

E_3: Environmental attribute contains e_{31} = air pollution and e_{32} = noise pollution. There is a strict prioritization between parameters $e_{31} > e_{32}$, here $>$ indicates preference.

Moreover, experts are selected from a variety of departments of faculty of engineering to increase the objectivity of the results as much as possible. Experts give their evaluation information by GTHF-numbers for linguistic terms shown in Table 2.

Table 4
GTHF sub-decision matrices $(x^1_{ir})_{5\times 3}$.

Economical	Service life	Investment cost
u_1	$\langle(0.2, 0.3, 0.4, 0.6); \{0.1, 0.3, 0.5\}\rangle$	$\langle(0.1, 0.4, 0.8, 0.9); \{0.1, 0.3, 0.5\}\rangle$
u_2	$\langle(0.1, 0.4, 0.5, 0.6); \{0.2, 0.4, 0.6\}\rangle$	$\langle(0.3, 0.5, 0.6, 0.8); \{0.2, 0.5, 0.9\}\rangle$
u_3	$\langle(0.1, 0.4, 0.8, 0.9); \{0.1, 0.3, 0.5\}\rangle$	$\langle(0.2, 0.5, 0.6, 0.7); \{0.7, 0.8, 0.9\}\rangle$
u_4	$\langle(0.4, 0.5, 0.7, 0.9); \{0.3, 0.7, 0.9\}\rangle$	$\langle(0.4, 0.5, 0.7, 0.9); \{0.3, 0.7, 0.9\}\rangle$
	Maintenance cost	
u_1	$\langle(0.4, 0.5, 0.7, 0.9); \{0.3, 0.7, 0.9\}\rangle$	
u_2	$\langle(0.4, 0.5, 0.7, 0.8); \{0.2, 0.4, 0.7\}\rangle$	
u_3	$\langle(0.6, 0.7, 0.8, 0.9); \{0.6, 0.8, 0.9\}\rangle$	
u_4	$\langle(0.1, 0.4, 0.5, 0.6); \{0.2, 0.4, 0.6\}\rangle$	

Table 5
GTHF sub-decision matrices $(x^2_{ir})_{5\times 4}$.

Technical	Availability	Capacity
u_1	$\langle(0.5, 0.7, 0.8, 0.9); \{0.3, 0.5, 0.6\}\rangle$	$\langle(0.5, 0.7, 0.8, 0.9); \{0.3, 0.5, 0.6\}\rangle$
u_2	$\langle(0.6, 0.7, 0.8, 0.9); \{0.6, 0.8, 0.9\}\rangle$	$\langle(0.4, 0.5, 0.7, 0.8); \{0.2, 0.4, 0.7\}\rangle$
u_3	$\langle(0.1, 0.4, 0.8, 0.9); \{0.1, 0.3, 0.5\}\rangle$	$\langle(0.1, 0.4, 0.8, 0.9); \{0.1, 0.3, 0.5\}\rangle$
u_4	$\langle(0.4, 0.5, 0.7, 0.8); \{0.2, 0.4, 0.7\}\rangle$	$\langle(0.2, 0.5, 0.6, 0.7); \{0.7, 0.8, 0.9\}\rangle$
	Resource density	Efficiency
u_1	$\langle(0.3, 0.5, 0.6, 0.8); \{0.2, 0.5, 0.9\}\rangle$	$\langle(0.1, 0.2, 0.3, 0.5); \{0.1, 0.2, 0.4\}\rangle$
u_2	$\langle(0.1, 0.4, 0.8, 0.9); \{0.1, 0.3, 0.5\}\rangle$	$\langle(0.3, 0.4, 0.5, 0.6); \{0.2, 0.4, 0.7\}\rangle$
u_3	$\langle(0.2, 0.5, 0.6, 0.7); \{0.7, 0.8, 0.9\}\rangle$	$\langle(0.3, 0.5, 0.6, 0.8); \{0.2, 0.5, 0.9\}\rangle$
u_4	$\langle(0.6, 0.7, 0.8, 0.9); \{0.6, 0.8, 0.9\}\rangle$	$\langle(0.6, 0.7, 0.8, 0.9); \{0.6, 0.8, 0.9\}\rangle$

Table 6
GTHF sub-decision matrices $(x^3_{ir})_{5\times 2}$.

Environmental	Air pollution	Noise pollution
u_1	$\langle(0.1, 0.2, 0.3, 0.5); \{0.1, 0.2, 0.4\}\rangle$	$\langle(0.5, 0.7, 0.8, 0.9); \{0.3, 0.5, 0.6\}\rangle$
u_2	$\langle(0.4, 0.5, 0.7, 0.9); \{0.3, 0.7, 0.9\}\rangle$	$\langle(0.3, 0.4, 0.5, 0.6); \{0.2, 0.4, 0.7\}\rangle$
u_3	$\langle(0.1, 0.4, 0.8, 0.9); \{0.1, 0.3, 0.5\}\rangle$	$\langle(0.1, 0.2, 0.3, 0.5); \{0.1, 0.2, 0.4\}\rangle$
u_4	$\langle(0.2, 0.5, 0.6, 0.7); \{0.7, 0.8, 0.9\}\rangle$	$\langle(0.6, 0.7, 0.8, 0.9); \{0.6, 0.8, 0.9\}\rangle$

Then, in order to select/rank a renewable energy resource, we utilize the GTHFPA operator to develop an approach to multiple-attribute decision-making problems with GTHF information, which can be expressed by using the following algorithm:

Algorithm:

Step 1. We constructed three GTHF sub-decision matrices for sub-attribute set $E_j = \bigcup_{r=1}^{k_j}\{e_{jr}\}$ as $H^j = (x^j_{ir})_{4\times k_j}$ $(i = 1, 2, 3, 4;\ j = 1, 2, 3;\ r = k_1, k_2, k_3;\ k_1 = 1, 2, 3;\ k_2 = 1, 2, 3, 4;\ k_3 = 1, 2)$ for decision in Table 4–6.

Step 2. We computed overall values matrix based on Eqs. (23)–(24) in Table 7.

Step 3. We found A_i $(i = 1, 2, \dots, m)$ by aggregated all GTHF-numbers under A_{ij} $(j = 1, 2, \dots, n)$ by using Eqs. (26)–(28) in Table 8.

Table 7
The decision makers' evaluation of the alternatives with respect to criteria.

	Economical	Technical
u_1	$\langle(0.191, 0.321, 0.469, 0.654); \{0.107, 0.318, 0.524\}\rangle$	$\langle(0.459, 0.662, 0.770, 0.906); \{0.1, 0.3, 0.5\}\rangle$
u_2	$\langle(0.159, 0.426, 0.533, 0.651); \{0.2, 0.420, 0.697\}\rangle$	$\langle(0.497, 0.627, 0.801, 0.910); \{0.2, 0.5, 0.9\}\rangle$
u_3	$\langle(0.152, 0.437, 0.769, 0.868); \{0.286, 0.475, 0.655\}\rangle$	$\langle(0.112, 0.414, 0.801, 0.907); \{0.7, 0.8, 0.9\}\rangle$
u_4	$\langle(0.363, 0.488, 0.675, 0.863); \{0.288, 0.673, 0.881\}\rangle$	$\langle(0.409, 0.552, 0.720, 0.824); \{0.3, 0.7, 0.9\}\rangle$
	Environmental	
u_1	$\langle(0.141, 0.251, 0.351, 0.541); \{0.123, 0.238, 0.9\}\rangle$	
u_2	$\langle(0.369, 0.469, 0.637, 0.806); \{0.270, 0.627, 0.859\}\rangle$	
u_3	$\langle(0.1, 0.366, 0.715, 0.832); \{0.1, 0.284, 0.484\}\rangle$	
u_4	$\langle(0.324, 0.562, 0.662, 0.762); \{0.672, 0.8, 0.9\}\rangle$	

Table 8
$A_i (i = 1, 2, 3, 4)$ by aggregated all GTHF-numbers.

A_1	$\langle(0.2271, 0.3665, 0.5053, 0.6835); \{0.1468, 0.3409, 0.5417\}\rangle$
A_2	$\langle(0.2470, 0.4698, 0.5966, 0.7183); \{0.2610, 0.5001, 0.7479\}\rangle$
A_3	$\langle(0.1398, 0.4282, 0.7730, 0.8751); \{0.2450, 0.4363, 0.6229\}\rangle$
A_4	$\langle(0.3703, 0.5135, 0.6855, 0.8405); \{0.3812, 0.6727, 0.8656\}\rangle$

Table 9
Centroid values $s(A_i)$ of A_i.

$s(A_1) =$	1.3245
$s(A_2) =$	1.5383
$s(A_3) =$	1.7179
$s(A_4) =$	1.8062

Step 4. We calculated the centroid values $s(A_i)$ of A_i by using Eq. (29) in Table 9.

Step 5. All the alternatives U_i $(i = 1, 2, 3, 4)$ are ranked and the best one(s) is selected. Here, if $s(A_4) > s(A_3) > s(A_2) > s(A_1) \Rightarrow U_4 > U_3 > U_2 > U_1$. Thus, the most environmentally friendly renewable energy source is U_4.

6. Comparison and Analysis Discussion

When a decision analyst collects the data or information using GTHF-numbers, no prioritization operator can handle this information or data. The above defined GTHFN-prioritized aggregation operators are the only tool to solve this kind of information and help the decision analyst to make a decision. To demonstrate the effectiveness of the proposed method, a comparison of the decision-making results of the MADM methods based on prioritized aggregation operators of existing presented by Wei (2012), Wan *et al.* (2015) and Liang *et al.* (2017) is carried out, as shown in Table 10.

Table 10
The results from the different operators.

Methods	Ranking of alternatives	The optimal alternative
The method in Wei (2012)	$U_4 > U_2 > U_3 > U_1$,	U_4
The method in Wan *et al.* (2015)	$U_4 > U_3 > U_1 > U_2$,	U_4
The method in Liang *et al.* (2017)	$U_4 > U_3 > U_2 > U_1$,	U_4
Proposed (GTHFNPA)Method	$U_4 > U_3 > U_2 > U_1$,	U_4
Proposed (GTHFNPG) Method	$U_4 > U_3 > U_2 > U_1$,	U_4

7. Conclusion

GTHF-numbers are very useful for expressing ill-known quantities. Since aggregation operators play a vital role in decision-making, this paper investigates the prioritized MADM problems in which the attribute values are in the form of GTHF-numbers. Firstly, we introduced two aggregation techniques called generalized trapezoidal hesitant fuzzy prioritized weighted average operator and generalized trapezoidal hesitant fuzzy prioritized weighted geometric operator for aggregating the generalized trapezoidal hesitant fuzzy information. Next, we discussed some basic properties of the developed operators, namely idempotency, boundary and monotonicity. In addition, two approaches for multiple-attribute decision-making under the generalized trapezoidal hesitant fuzzy environments are developed. Finally, a practical study has been conducted to demonstrate the proposed MADM method in more practicality and effectiveness, since it considers prioritization relationships among attributes. Meanwhile, the prioritized operators of GTHF-numbers provide a new tool of information fusion for solving decision problems under hesitant environments. Further, the extensions of hesitant prioritized aggregation operators model to the MADM problems under other fuzzy environments would also be studied in the near future. In future, we plan to extend our research work to TOPSIS, ARAS, ELECTRE, WASPAS, MABAC, EDAS, QUALIFLEX, and so on. Obtaining the data and writing in this study as GTHF-numbers is the most common problem. Therefore, we will try to solve this problem in future.

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