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TOPSIS Methods for Probabilistic Hesitant Fuzzy MAGDM and Application to Performance Evaluation of Public Charging Service Quality

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Abstract. The performance evaluation of public charging service quality is frequently viewed as the multiple attribute group decision-making (MAGDM) issue. In this paper, an extended TOPSIS model is established to provide new means to solve the performance evaluation of public charging service quality. The TOPSIS method integrated with FUCOM method in probabilistic hesitant fuzzy circumstance is applied to rank the optional alternatives and a numerical example for performance evaluation of public charging service quality is used to test the newly proposed method's practicability in comparison with other methods. The results display that the approach is uncomplicated, valid and simple to compute. The main results of this paper: (1) a novel PHF-TOPSIS method is proposed; (2) the extended TOPSIS method is developed in the probabilistic hesitant fuzzy environment; (3) the FUCOM method is used to obtain the attribute weight; (4) the normalization process of the original data has adapted the latest method to verify the precision; (5) The built models and methods are useful for other selection issues and evaluation issues.
Key words: multiple attributes group decision making (MAGDM), probabilistic hesitant fuzzy sets (PHFS), FUCOM method, TOPSIS method, performance evaluation, public charging service quality.

1. Introduction

Many management decision-making problems in the real world, such as logistics park location, supplier selection, medical service evaluation, fault diagnosis, etc., can be considered from the perspective of MADM (Garg *et al.*, 2018; Akram *et al.*, 2021; Waseem *et al.*, 2019; Lu *et al.*, 2021; Wei *et al.*, 2022). As an important branch of management science and modern decision science, MADM theory and methods have been widely used in many practical decision-making problems (Yang and Pang, 2019; Xu and Zhang, 2019; Zavadskas *et al.*, 2013; Ning *et al.*, 2022). In the actual MAGDM process, due to the complexity and uncertainty of objective things, the limitations of human cognition and the ambiguity of thinking, it is difficult to use quantitative and accurate information to describe decision objects (Wang *et al.*, 2022; Zhang H. *et al.*, 2022; Liu *et al.*, 2019; Li *et al.*, 2021). In 1965, Zadeh (1965) was the first to define a novel fuzzy sets (FSs) to cope with information in the fuzzy new domain (Garg and Kumar, 2018; Garg, 2018b; Zhang and Xu, 2015; Su *et al.*, 2022; Jiang *et al.*, 2022; Lei *et al.*, 2022). To extend the FSs, the intuitionistic fuzzy sets (IFSs) (Atanassov, 1989) were also defined. Subsequently, FSs and its related extension knowledges were predominantly exploited in decision analysis domains (Yu *et al.*, 2017; Wan and Li, 2014; Zhang D. *et al.*, 2022; Zhang *et al.*, 2022a). Su *et al.* (2011) proposed the interactive method for dynamic IF-MAGDM. Arya and Yadav (2018) defined the intuitionistic fuzzy super-efficiency slack-based measure. Tian *et al.* (2017) studied the partial derivative and complete differential of binary IF-mathematical functions. Garg (2018a) proposed the improved cosine similarity measure for IFSs. Tan (2011) constructed the Choquet integral-based TOPSIS method for IF-MADM. Zhao *et al.* (2017) defined the Interactive intuitionistic fuzzy algorithms for multilevel programming problems. Li (2011) built the GOWA operator to MADM using IFSs. Buyukozkan *et al.* (2018) selected the transportation schemes with integrated intuitionistic fuzzy Choquet integral method. Joshi *et al.* (2018) defined the Jensen-alpha-Norm dissimilarity measure for IFSs. De and Sana (2018) defined the The (p, q, r, l) method for random demand with Bonferroni mean under IFSs. Li *et al.* (2018) defined the time-preference and VIKOR-based dynamic method for IF-MADM. Niroomand (2018) defined the multi-objective based direct solution method for linear programming along with intuitionistic fuzzy parameters. Zhao *et al.* (2021) perfected TODIM for IF-MAGDM on the strength of cumulative prospect theory. Yu S. *et al.* (2017) defined the derivatives and differentials for multiplicative IFSs. Yu (2012) defined the generalized prioritized geometric operators under IFSs. Xiao *et al.* (2020) built the intuitionistic fuzzy taxonomy method. Wu and Zhang (2011) built the IF-MADM based on weighted entropy. Verma and Sharma (2014) defined the measure of inaccuracy IF-MADM. Iakovidis and Papageorgiou (2011) defined the cognitive maps for medical decision making under IFSs.

Then the hesitant fuzzy element (HFE) proposed by Xia and Xu (2011) is to solve the problem of determining the element's membership to a set on account of the uncertainty between different numbers and then prove the intuitionistic fuzzy set and hesitant fuzzy set. With the proposition of the HFE, the idea of correspondent operators to aggregate hesitant fuzzy information was obtained. Not long after this, Xu and Xia (2011) raised the idea of the score function, deviation function and the comparison rule, and set the basis on the calculation. Xu and Cai (2010) provided the aggregating operators to integrate the hesitant fuzzy information. Nevertheless, HFE can be regarded as a particular equivalent form whose occurring probabilities of the possible value are equal. The probabilistic hesitant fuzzy set and the corresponding score function, deviation function and its comparison law were proposed by Xu and Zhou (2017). Moreover, the probabilistic hesitant fuzzy weighted averaging geometric operators were introduced by Xu and Zhou (2017) to process PHFE information. Then the improved PHFS was introduced by Zhang *et al.* (2017) to give more space for hesitation, the integrations of the improved PHES can be calculated by the improved operators. Farhadinia and Xu (2021) gave the comparison techniques of PHFEs. Krishankumar *et al.* (2021a) built the COPRAS approach to PHFSs. Krishankumar *et al.* (2021b) proposed to extend a well-known VIKOR method to the PHFS context. Lin *et al.* (2021) put forward a novel probabilistic hesitant fuzzy

MULTIMOORA method. Liu *et al.* (2021) defined the DEA cross-efficiency with probabilistic hesitant fuzzy preference relations. Yang and Xu (2021) defined the measure of probabilistic hesitant I-fuzzy sets and decision making for strategy choice. Song and Chen (2021) extended the COPRAS method to solve MADM problems under probabilistic hesitant fuzzy environment. Liu and Guan (2021) devised a new PHFE comparison method and then defined the comprehensive characteristic distance measure based on four characteristics.

Technique for order performance by similarity to ideal solution (TOPSIS) was originally developed by Hwang and Yoon (1981) for the sake of addressing a MADM problem. Lai *et al.* (1994) expanded TOPSIS to deal with a diverse objective decision making problem for Bow River Valley water quality management. Chen (2000) proposed the TOPSIS approach for group decision-making within the fuzzy environment. Wang and Elhag (2006) employed the fuzzy TOPSIS approach on the basis of alpha level sets for bridge risk assessment. Taleizadeh *et al.* (2009) designed a novel method which combined Pareto, TOPSIS and genetic algorithm to solve the multi-product multi-constraint inventory control systems with random fuzzy replenishments. Zhang *et al.* (2022b) defined the TOPSIS method for spherical fuzzy MAGDM based on cumulative prospect theory. Wei (2010) developed the TOPSIS method to cope with 2-tuple linguistic MAGDM with incomplete weight information. Nilashi *et al.* (2019) used two MADM techniques, Decision Making Trial and Evaluation Laboratory (DEMATEL) and Fuzzy TOPSIS, to reveal the interrelationships among the factors and to find the relative importance of these factors in the decision making model. In this paper, we extend the TOPSIS method to probabilistic hesitant fuzzy (PHF) environment based on the FUCOM method to deal with the flexible and complicated decision-making circumstance. The following is the innovation of this paper: (1) a novel PHF-TOPSIS method is proposed; (2) the extended TOPSIS method is developed in the probabilistic hesitant fuzzy environment; (3) the FUCOM method is used to obtain the attribute weight; (4) the normalization process of the original data has adapted the latest method to verify the precision.

The whole thread of the article is as follows: Section 2 gives a simple introduction of the PHF information, Section 3 structures the model of TOPSIS and Section 4 illustrates an example for performance evaluation of public charging service quality to prove the practicability of this new method. Section 5 gives a sensitivity analysis and comparison analysis with other existing models.

2. Preliminaries

Definition 1 (Xu and Zhou, 2017). Assume q is a fixed set, and probabilistic hesitant fuzzy sets on E, which range from 0 to 1, and the probabilistic hesitant fuzzy element (PHFE) is described as follows:

$$V_q = \{v_q(t_q^a \mid g_q^a) \mid t_q^a, g_q^a\}, \tag{1}$$

where $t_q^a \in R$, $0 \leqslant t_q^a \leqslant 1$, $a = 1, 2, \dots, \#t$, and $\#t$ represents the total number of elements, t_q^a shows the degree of membership, while g_q^a is the probability of the membership degree, $\sum_{a=1}^{\#t} t_q^a = 1$.

The first and significant step is the normalization process and we adapt the normalization approach proposed by Li *et al.* (2020) to break the limitation when processing multiplication of the sets which include different probabilities.

Definition 2 (Li *et al.*, 2020). Let $v(t_i|g_i) = \{t^i(g^i)\}$, $v_1(t_a|g_a) = \{t_1^a(g_1^a)\}$ and $v_2(t_b|g_b) = \{t_2^b(g_2^b)\}$ be three PHFEs, $i = 1, 2, \dots, \#t$, $a = 1, 2, \dots, \#t_1$, $b = 1, 2, \dots, \#t_2$.

Step 1. Define the first element. If $g_1^1 < g_2^1$, then $t_1^1(g_1^1) = t_1^1(g_1^1)$ and $t_2^1(g_2^1) = t_2^1(g_2^1)$, otherwise, $t_1^1(g_1^1) = t_1^1(g_2^1)$ and $t_2^1(g_2^1) = t_2^1(g_2^1)$.

Step 2. Determine the second element. If $g_1^1 < g_2^1$ and $g_2^1 - g_1^1 \leqslant g_1^2$, then $t_1^2(g_1^2) = t_1^2(g_2^1 - g_1^1)$ and $t_2^2(g_2^2) = t_2^1(g_2^1 - g_1^1)$. If $g_1^1 < g_2^1$ and $g_2^1 - g_1^1 > g_1^2$, then $t_1^2(g_1^2) = t_1^2(g_1^2)$ and $t_2^2(g_2^2) = t_2^1(g_1^2)$. If $g_1^1 \geqslant g_2^1$ and $g_1^1 - g_2^1 \leqslant g_2^2$, then $t_1^2(g_1^2) = t_1^1(g_1^1 - g_2^1)$ and $t_2^2(g_2^2) = t_2^2(g_1^1 - g_2^1)$. If $g_1^1 \geqslant g_2^1$ and $g_1^1 - g_2^1 \geqslant g_2^2$, then $t_1^2(g_1^2) = t_1^1(g_2^2)$ and $t_2^2(g_2^2) = t_2^2(g_2^2)$.

Step 3. Determine the third element. If $g_1^1 \geqslant g_2^1$, $g_1^1 - g_2^1 \leqslant g_2^2$ and $g_2^1 \leqslant g_2^2 - g_1^1 + g_2^1$, then $t_1^3(g_1^3) = t_1^2(g_1^2)$ and $t_2^3(g_2^3) = t_2^2(g_1^2)$. If $g_1^1 \geqslant g_2^1$, $g_1^1 - g_2^1 \leqslant g_2^2$ and $g_2^1 > g_2^2 - g_1^1 + g_2^1$, then $t_1^3(g_1^3) = t_1^2(g_2^2 + g_2^1 - g_1^1)$ and $t_2^3(g_2^3) = t_2^2(g_2^2 + g_2^1 - g_1^1)$. If $g_1^1 \geqslant g_2^1$, $g_1^1 - g_2^1 > g_2^2$ and $g_1^2 \geqslant g_2^2 + g_2^3$, then $t_1^3(g_1^3) = t_1^2(g_2^3)$ and $t_2^3(g_2^3) = t_2^3(g_2^3)$. If $g_1^1 \geqslant g_2^1$, $g_1^1 - g_2^1 > g_2^2$ and $g_1^2 < g_2^2 + g_2^3$, then $t_1^3(g_1^3) = t_1^2(g_1^2 - g_2^2)$ and $t_2^3(g_2^3) = t_2^3(g_1^2 - g_2^2)$. If $g_1^1 < g_2^1$, $g_2^1 - g_1^1 \leqslant g_1^2$ and $g_1^2 + g_1^1 \leqslant g_2^2 + g_2^1$, then $t_1^3(g_1^3) = t_1^2(g_1^2 - g_2^1 + g_1^1)$ and $t_2^3(g_2^3) = t_2^2(g_1^2 - g_2^1 + g_1^1)$. If $g_1^1 < g_2^1$, $g_2^1 - g_1^1 > g_1^2$ and $g_2^1 + g_1^2 \leqslant g_1^3 + g_1^1$, then $t_1^3(g_1^3) = t_1^3(g_2^1 - g_1^1 - g_1^2)$ and $t_2^3(g_2^3) = t_2^1(g_2^1 - g_1^1 - g_1^2)$. If $g_1^1 < g_2^1$, $g_2^1 - g_1^1 > g_1^2$ and $g_2^1 + g_1^2 > g_1^3 + g_1^1$, then $t_1^3(g_1^3) = t_1^3(g_1^3)$ and $t_2^3(g_2^3) = t_2^1(g_1^3)$, where $g_1^1 + g_1^2 + \dots + g_1^{\#t_1} = 1$ and $g_2^1 + g_2^2 + \dots + g_2^{\#t_2} = 1$, $\#t_1 = \#t_2$.

Definition 3 (Xu and Zhou, 2017). Calculate the score function = by Eq. (2):

$$s\big(\bar{v}(\bar{g})\big) = \sum_{i=1}^{\#t} \bar{t}^i \bar{g}^i, \tag{2}$$

where $\bar{t}^i$ shows the i-th largest elements of normalized PHFE, and $\bar{g}^i$ is the probability of occurrence of the corresponding element.

Definition 4 (Sha *et al.*, 2021). Compare $\bar{v}_1(\bar{g}_1) = \{\bar{t}_1^a(\bar{g}_1^a)\}$ and $\bar{v}_2(\bar{g}_2) = \{\bar{t}_2^b(\bar{g}_2^b)\}$ by the following laws:

(1) $\bar{v}_1(\bar{g}_1) > \bar{v}_2(\bar{g}_2)$, if $s(\bar{v}_1(\bar{g}_1)) > s(\bar{v}_2(\bar{g}_2))$;
(2) $\bar{v}_1(\bar{g}_1) > \bar{v}_2(\bar{g}_2)$, if $s(\bar{v}_1(\bar{g}_1)) > s(\bar{v}_2(\bar{g}_2))$ and $d(\bar{k}_1(\bar{j}_1)) < d(\bar{k}_2(\bar{j}_2))$;

(3) $\bar{v}_1(\bar{g}_1) = \bar{v}_2(\bar{g}_2)$, if $s(\bar{v}_1(\bar{g}_1)) = s(\bar{v}_2(\bar{g}_2))$ and $d(\bar{k}_1(\bar{j}_1)) = d(\bar{k}_2(\bar{j}_2))$;
(4) $\bar{v}_1(\bar{g}_1) < \bar{v}_2(\bar{g}_2)$, if $s(\bar{v}_1(\bar{g}_1)) = s(\bar{v}_2(\bar{g}_2))$ and $d(\bar{k}_1(\bar{j}_1)) > d(\bar{k}_2(\bar{j}_2))$.

Definition 5 (Sha *et al.*, 2021). $\bar{v}_1(\bar{g}_1) = \{\bar{t}_1^a(\bar{g}_1^a)\}$ and $\bar{v}_2(\bar{g}_2) = \{\bar{t}_2^b(\bar{g}_2^b)\}$ are normalized PHFEs, where $\#t_1 = \#t_2 = \#t$ and $g_1^a = g_2^b = g^i$. The Lance distance between them is given as Eq. (3):

$$d\big(\bar{v}_1(\bar{g}_1), \bar{v}_2(\bar{g}_2)\big) = \frac{1}{\#t} \sum_{a=b=1}^{\#t} \frac{|\bar{t}_1^a \bar{g}_1^a - \bar{t}_2^b \bar{g}_2^b|}{\bar{t}_1^a \bar{g}_1^a + \bar{t}_2^b \bar{g}_2^b}. \tag{3}$$

Definition 6 (Li *et al.*, 2020). The $\bar{k}_1(\bar{j}_1) = \{\bar{f}_1^a(\bar{j}_1^a)\}$ and $\bar{k}_2(\bar{j}_2) = \{\bar{f}_2^b(\bar{j}_2^b)\}$ are normalized PHFEs and the algorithms about them are as follows:

$$(1)\ \bar{v}_1(\bar{g}_1) \oplus \bar{v}_2(\bar{g}_2) = \bigcup_{a=1,\dots,\#\bar{t}_1,\, b=1,\dots,\bar{t}_2} \big\{\big(\bar{t}_1^a + \bar{t}_2^b - \bar{t}_1^a \bar{t}_2^b\big)(\bar{g}_i)\big\}; \tag{4}$$

$$(2)\ \bar{v}_1(\bar{g}_1) \otimes \bar{v}_2(\bar{g}_2) = \bigcup_{a=1,\dots,\#\bar{t}_1,\, b=1,\dots,\bar{t}_2} \big\{\bar{t}_1^a \bar{t}_2^b(\bar{g}_i)\big\}. \tag{5}$$

Definition 7 (Liao *et al.*, 2021, 2022). Let f_c $(c = 1, 2, \dots, l)$ be a non-empty collection, and the PHF weighted averaging (PHFWA) operator is calculated by Eq. (6):

$$\begin{aligned} &PHFWA\big(\bar{f}_1(\bar{g}_1), \bar{f}_2(\bar{g}_2), \dots, \bar{f}_l(\bar{g}_l)\big) \\ &\quad = \bigoplus_{c=1}^{l} (\bar{f}_c \bar{g}_c) = \bigcup_{\bar{t}_1 \in \bar{f}_1, \bar{t}_2 \in \bar{f}_2, \dots, \bar{t}_l \in \bar{f}_l} \left\{1 - \prod_{c=1}^{l} (1 - \bar{t}_c)^{u_c} \big(\bar{g}^i\big)\right\}, \end{aligned} \tag{6}$$

where $u_c = (u_1, u_2, \dots, u_l)$ represents the weight between the PHFEs and $\sum_{c=1}^{l} u_c = 1$, $u_c \in [0, 1]$.

Definition 8 (Liao *et al.*, 2021, 2022). The laws of PHF weighted geometric (PHFWG) operator are shown as follows:

$$\begin{aligned} &PHFWG = \big(\bar{f}_1(\bar{g}_1), \bar{f}_2(\bar{g}_2), \dots, \bar{f}_l(\bar{g}_l)\big) \\ &\quad = \bigoplus_{c=1}^{l} (\bar{f}_c)^{u_c} = \bigcup_{\bar{t}_1 \in \bar{f}_1, \bar{t}_2 \in \bar{f}_2, \dots, \bar{t}_l \in \bar{f}_l} \left\{\prod_{c=1}^{l} (\bar{t}_c)^{u_c} \big(\bar{g}^i\big)\right\}. \end{aligned} \tag{7}$$

3. PHF-TOPSIS Method for MAGDM

The MAGDM decision matrix is $V^c = [v_{rx}^c(g_{rx})]_{s \times y}$, and the optional alternatives are defined as $Q_r = \{Q_1, Q_2, \dots, Q_s\}$ and the attribute is shown as $I_x = \{I_1, I_2, \dots, I_y\}$ and the decision makers (DMs) are defined as $c = \{c_1, c_2, \dots, c_l\}$, while the weighting

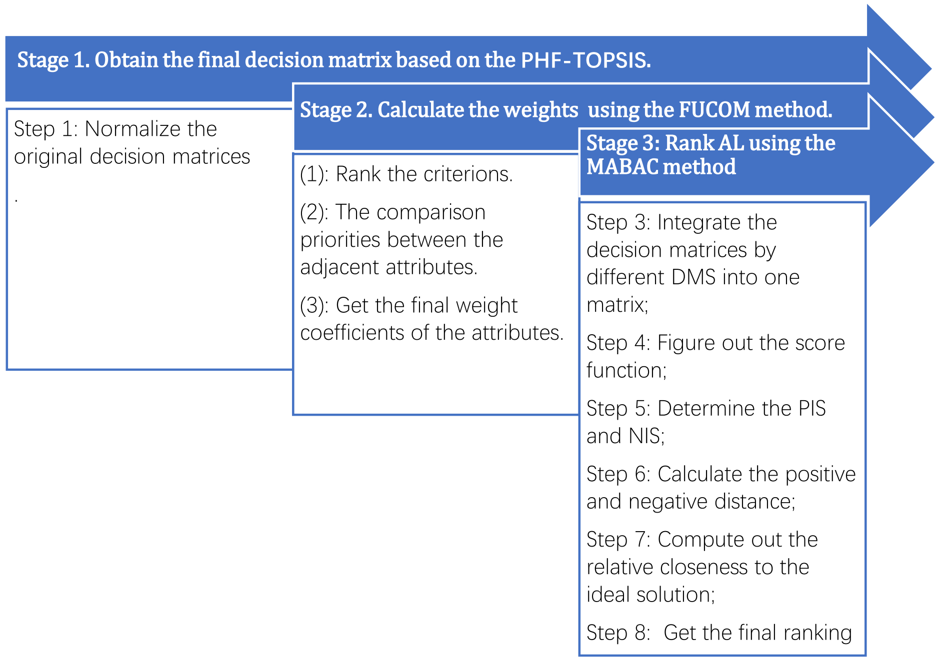

Fig. 1. Framework of the proposed PHF-TOPSIS.

vector between the DMs is defined as $u_c = \{0.3, 0.3, 0.4\}$ and the weighting vector among the criterions is j_x which is unknown, $\sum_{c=1}^{l} u_c = 1$, $\sum_{x=1}^{y} j_x = 1$ $(c = i, 2, \dots, l)$.

$$V^c = \begin{bmatrix} v_{11}^c & \cdots & v_{1x}^c & \cdots & v_{1y}^c \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ v_{r1}^c & \cdots & v_{rx}^c & \cdots & v_{ry}^c \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ v_{s1}^c & \cdots & v_{sx}^c & \cdots & v_{sy}^c \end{bmatrix}_{s\times y}; \quad r = 1, \dots, s, \ x = 1, \dots, y, \ c = 1, \dots, l.$$

With the above conventions, the operation of the PHF-TOPSIS is as follows: the whole operation flow chart is shown in Fig. 1.

Step 1. Normalize the original decision matrices through Eq. (8).

$$\begin{cases} \bar{v}_{rx}(\bar{g}_{rx}) = \{t_{er}(g_{er})\}, & \text{if the attribute is positive attribute,} \\ \bar{v}_{rx}(\bar{g}_{rx}) = \{(1 - t_{rx})(g_{rx})\}, & \text{if the attribute is negative attribute.} \end{cases} \tag{8}$$

Then using the introduction in Definition 2 to get normalized matrices.

Step 2. Acquire the criterion weights $\hat{p}_x$ using FUCOM method.

The full consistency (FUCOM) method proposed by Pamučar *et al.* (2018) is the latest model for weighting the coefficients of attributes. Compared to other methods, such as the best worst method (BWM) and the analytic hierarchy process (AHP) method, the FUCOM method can give more appropriate results when considering the relation between the criterions and the number of comparisons (only $y-1$). The application of this new model for criterion weight has been used in numerous fields, such as assessment of alternative fuel vehicles for sustainable road transportation (Pamucar *et al.*, 2021), safety evaluation of road sections (Simić *et al.*, 2020), sustained academic quality assurance and ABET accreditation (Ahmad and Qahmash, 2020) and so on. Bozanic *et al.* (2020) built the MADM with Z-numbers based on FUCOM and MABAC. Durmić *et al.* (2020) combined FUCOM-Rough SAW model. Bozanić *et al.* (2021) built the FUCOM-Fuzzy RAFSI model for selecting the group of construction machines for enabling mobility with D-numbers. Pamucar and Ecer (2020) prioritized the weights of the evaluation criteria under fuzziness through the fuzzy full consistency method-FUCOM-F. Stević and Brković (2020) built novel integrated FUCOM-MARCOS model for evaluation of human resources in a transport company. Simić *et al.* (2020) built the CRITIC-fuzzy FUCOM-DEA-fuzzy MARCOS model for safety evaluation of road sections based on geometric parameters of road. Fazlollahtabar *et al.* (2019) defined the FUCOM method in group decision-making for selection of forklift in a warehouse. Durmić (2019) evaluated the criteria for sustainable supplier selection using FUCOM method. Baig *et al.* (2022) used FUCOM and FQFD for prioritizing the vulnerabilities and identifying those capabilities that can ensure protection against these vulnerabilities.

The specific process to get the weight is as follows:

(i) Rank the criterions and get the set from $I_x^c = \{I_1^c, I_2^c, \dots, I_y^c\}$ which is according to the relative importance of the criteria. Thus, the parameters rank is obtained by the values of the weight coefficient:

$$I_{y(1)}^c > I_{y(2)}^c > \cdots > I_{y(k)}^c,$$

where k denotes the order of the criterions.

(ii) The comparison priorities between the adjacent attributes $\psi_{\frac{x-1}{x}}^c$, $x = 1, 2, \dots, y$, which denotes the value of the $N_{y(x-1)}^c$ relative to $N_{y(x)}^c$, then we get the set of the criterion comparative preference:

$$\psi^c = \left\{\psi_{\frac{1}{2}}^c, \psi_{\frac{2}{3}}^c, \dots, \psi_{\frac{x-1}{x}}^c\right\}.$$

(iii) Get the final weight coefficients of the attributes $p_x^c = \{p_1^c, p_2^c, \dots, p_y^c\}$, which should meet the conditions showing as follows:

- The comparison priorities $\psi_{\frac{x-1}{x}}^c$ calculated in (ii) are supposed to be equal to the ratio of the weight coefficient through Eq. (9):

$$\psi_{\frac{x-1}{x}}^c = \frac{p_{x-1}^c}{p_x^c}. \tag{9}$$

- The second condition is about the weight coefficients which should satisfy the following rule by using Eq. (10):

$$\frac{p^c_{x-2}}{p^c_x} = \psi^c_{\frac{x-2}{x-1}} \otimes \psi^c_{\frac{x-1}{x}}. \tag{10}$$

Thus, the inequality constraints for this model are shown in Eq. (11):

$$\begin{aligned} &\min \chi^c, \\ &\text{s.t.} \left|\psi^c_{\frac{x-1}{x}} - \frac{p^c_{x-1}}{p^c_x}\right| \leqslant \chi^c, \left|\frac{p^c_{x-2}}{p^c_x} - \psi^c_{\frac{x-2}{x-1}} \otimes \psi^c_{\frac{x-1}{x}}\right| \leqslant \chi^c, \\ &\sum_{x=1}^{y} p^c_x = 1, \quad p^c_x \geqslant 0. \end{aligned} \tag{11}$$

With the help of the MATLAB software, we get the final result of the weighting vector of the evaluation criterions for each DM. Then the integrated weight $\hat{p}_x$ is finally obtained by geometric means.

Step 3. Integrate the decision matrices by different DMS into one matrix $\hat{v}_{rx}(\hat{g}_{rx}) = \{\hat{t}_{rx}(\hat{g}_{rx})\}_{s\times y}$, using Eq. (12):

$$\begin{aligned} &PHFWA(\bar{v}^l_{rx}, \bar{v}^2_{rx}, \dots, \bar{v}^l_{rx}) \\ &= \bigoplus_{c=1}^{l} (\bar{v}^c_{rx} u_c) = \bigcup_{\bar{t}^1_{rs}\in\bar{v}^1_{rs}, \bar{t}^2_{rs}\in\bar{v}^2_{rs}, \dots, \bar{t}^l_{rs}\in\bar{v}^l_{rs}} \left\{1 - \prod_{c=1}^{l} (1 - \bar{t}^c_{rs})^{u_c} (\bar{g})\right\}. \end{aligned} \tag{12}$$

Step 4. Figure out the score function of the integrated decision matrix by using Eq. (13)

$$s(\hat{v}_{rs}(\hat{g}_{rs})) = \sum_{i=1}^{\#t} \hat{t}^i_{rs} \hat{g}^i_{rs}. \tag{13}$$

Step 5. Determine the $\hat{v}^*_x$ and $\hat{v}^-_x$ indexes by the following equation by using Eq. (14):

$$\begin{cases} \hat{v}^*_x(\hat{g}^*_x) = \max_r s(\hat{v}_{rx}), \\ \hat{v}^-_x(\hat{g}^-_x) = \min_r s(\hat{v}_{rx}). \end{cases} \tag{14}$$

Step 6. Calculate the positive and negative distance by using Eqs. (15)–(16)

$$d^*_r(\hat{v}_{rx}(\hat{g}_{rx}), \hat{v}^*_x(\hat{g}^*_x)) = \sum_{x=1}^{y} \hat{p}_x \frac{1}{\#t} \sum_{a=b=1}^{\#t} \frac{|\hat{t}^{a^*}_x \hat{g}^{a^*}_x - \hat{t}^b_{rx} \hat{g}^b_{rx}|}{\hat{t}^{a^*}_x \hat{g}^{a^*}_x + \hat{t}^b_{rx} \hat{g}^b_{rx}}, \tag{15}$$

$$d^-_r(\hat{v}_{rx}(\hat{g}_{rx}), \hat{v}^-_x(\hat{g}^-_x)) = \sum_{x=1}^{y} \hat{p}_x \frac{1}{\#t} \sum_{a=b=1}^{\#t} \frac{|\hat{t}^{a^-}_x \hat{g}^{a^-}_x - \hat{t}^b_{rx} \hat{g}^b_{rx}|}{\hat{t}^{a^-}_x \hat{g}^{a^-}_x + \hat{t}^b_{rx} \hat{g}^b_{rx}}. \tag{16}$$

Step 7. Compute out the relative closeness to the ideal solution.

The relative closeness of alternative Q_r with the probabilistic hesitant fuzzy positive ideal solution Q^* is described by using Eq. (17):

$$H_r = \frac{d_r^-(\hat{v}_{rx}(\hat{g}_{rx}), \hat{v}_x^-(\hat{g}_x^-))}{d_r^*(\hat{v}_{rx}(\hat{g}_{rx}), \hat{v}_x^*(\hat{g}_x^*)) + d_r^-(\hat{v}_{rx}(\hat{g}_{rx}), \hat{v}_x^-(\hat{g}_x^-))}. \tag{17}$$

Step 8. Get the final ranking.

The final rank is determined by the value of relative closeness, and the sort rule is: the bigger the relative closeness is, the more appropriate the scheme is.

4. Case Study

Public service is provided by the public sector to the public at a professional level according to its own social responsibility. In my country, public service is provided to the external public by government departments and institutions with public management functions according to the law. If there are public services, the problem of public service charges will inevitably arise. The public service charges mentioned here do not include taxes, but include administrative management fees, which mainly refer to the specific fees charged by the departments and institutions that provide public services to the public in need of services, so that they can enjoy the right to and benefits of public services. The logic of public service charging is roughly as follows: first, to provide public services, costs will inevitably occur, and fees can compensate for the cost input of public services; second, public services are public goods, and public goods are noncompetitive and non-exclusive, and their external effect is obvious, and it is easy to generate "free-rider" behaviour. Charging fees can curb waste in public service consumption and improve the quality and efficiency of public services. Social investment can increase funding sources for public services. However, since currently in our country public services are mainly provided by government departments, and the government is also responsible for the examination and approval of charging items and charging standards for public services, as well as supervising and inspecting charging behaviours, public services are basically monopoly industries. With low efficiency and poor quality, people feel that public service charges and public service utility are asymmetrical, so there are many criticisms about public service charges. The performance evaluation of public charging service quality is frequently viewed as the multiple attribute group decision-making (MAGDM) issue. In this paper, an extended TOPSIS model is established to provide new means to solve the performance evaluation of public charging service quality. The TOPSIS method integrated with FUCOM method in probabilistic hesitant fuzzy circumstance is applied to rank the optional alternatives and a numerical example for performance evaluation of public charging service quality is used to test the newly proposed method's practicability in comparison with other methods. Therefore, to illustrate the method presented in this paper, we will give a numeric-based example for performance evaluation of public charging service quality using the PHFSs in this part. Five applicable new public charging service sections A_i $(i = 1, 2, 3, 4, 5)$

Table 1
Decision matrix c_1 given by the first DM.

Alternative	I_1	I_2	I_3	I_4
Q_1	{0.4(0.1), 0.2(0.5), 0.3(0.4)}	{0.6(0.3), 0.2(0.7)}	{0.3(0.4), 0.6(0.1), 0.2(0.5)}	{0.3(0.2), 0.7(0.2), 0.4(0.6)}
Q_2	{0.3(0.2), 0.6(0.4), 0.7(0.4)}	{0.5(0.3), 0.7(0.4), 0.4(0.3)}	{0.7(0.5), 0.5(0.3), 0.2(0.2)}	{0.6(0.3), 0.5(0.5), 0.4(0.2)}
Q_3	{0.5(0.3), 0.2(0.6), 0.3(0.1)}	{0.3(0.4), 0.2(0.5), 0.4(0.1)}	{0.4(0.3), 0.5(0.3), 0.2(0.4)}	{0.4(0.2), 0.6(0.3), 0.3(0.5)}
Q_4	{0.6(0.3), 0.4(0.1), 0.2(0.6)}	{0.2(0.5), 0.3(0.5)}	{0.3(1)}	{0.5(1)}

Table 2
Decision matrix c_2 given by the second DM.

Alternative	I_1	I_2	I_3	I_4
Q_1	{0.5(0.2), 0.3(0.5), 0.2(0.3)}	{0.5(0.4), 0.4(0.2), 0.2(0.4)}	{0.2(0.8), 0.5(0.1), 0.6(0.1)}	{0.4(0.5), 0.3(0.2), 0.2(0.3)}
Q_2	{0.4(0.1), 0.7(0.7), 0.5(0.2)}	{0.3(0.2), 0.7(0.4), 0.5(0.4)}	{0.5(0.2), 0.4(0.6), 0.7(0.2)}	{0.2(0.1), 0.6(0.4), 0.7(0.5)}
Q_3	{0.3(0.4), 0.6(0.3), 0.4(0.3)}	{0.5(0.2), 0.3(0.5), 0.1(0.3)}	{0.4(0.3), 0.3(0.7)}	{0.3(0.4), 0.2(0.6)}
Q_4	{0.4(0.3), 0.2(0.4), 0.3(0.3)}	{0.4(0.4), 0.6(0.2), 0.2(0.4)}	{0.1(0.7), 0.5(0.3)}	{0.5(0.4), 0.2(0.6)}

Table 3
Decision matrix c_3 given by the third DM.

Alternative	I_1	I_2	I_3	I_4
Q_1	{0.4(0.5), 0.3(0.2), 0.6(0.3)}	{0.3(0.8), 0.5(0.2)}	{0.6(0.3), 0.2(0.7)}	{0.4(0.2), 0.8(0.1), 0.3(0.7)}
Q_2	{0.5(0.4), 0.4(0.1), 0.7(0.5)}	{0.2(0.2), 0.7(0.6)0.6(0.2)}	{0.5(0.3), 0.4(0.2), 0.7(0.5)}	{0.5(0.3), 0.6(0.6), 0.7(0.1)}
Q_3	{0.1(0.3), 0.4(0.7)}	{0.5(0.3), 0.3(0.7)}	{0.3(0.3), 0.2(0.6), 0.5(0.1)}	{0.3(0.5), 0.4(0.5)}
Q_4	{0.2(0.6), 0.3(0.4)}	{0.4(0.4), 0.2(0.5), 0.6(0.1)}	{0.3(0.2), 0.4(0.4), 0.2(0.4)}	{0.2(0.1), 0.4(0.7), 0.5(0.2)}

are considered. To evaluate the five applicable new public charging service sections by three experts d_λ ($\lambda = 1, 2, 3$), four attributes are given: ①G_1 is the infrastructure; ②G_2 is the urban-rural integration; ③G_3 is the economic development; ④G_4 is the resources and environment. For the performance evaluation of public charging service quality, there are four attributes to be chosen, in which three DMs select the suppliers whose expert weighting is $u_c = \{0.3, 0.3, 0.4\}$. The following is the process of numerical example applying this model. The evaluation result is listed in Tables 1–3.

Then, the PHF-TOPSIS method is used to deal with the performance evaluation of public charging service quality with PHFNs.

Step 1. Obtain the normalized matrices (see Tables 4–6).

Step 2. Use FUCOM method to calculate the criterion weight.

- Calculate the criterion weight for DM 1.

(i) The DM 1 gives the ranking of significance of different attributes:

$$I_1^1 > I_2^1 > I_3^1 > I_4^1.$$

(ii) Table 7 shows the priorities of each attribute which is range from 1 to 4, based on the comparison in the former step. According to the data in Table 7, we get the comparative priorities as follows.

Table 4
The standardized decision matrix by the first DM.

Alternative	I_1	I_2
Q_1	{0.4(0.1), 0.2(0.1), 0.2(0.4), 0.3(0.2), 0.3(0.2)}	{0.6(0.1), 0.2(0.1), 0.2(0.4), 0.2(0.2), 0.6(0.2)}
Q_2	{0.3(0.1), 0.3(0.1), 0.6(0.4), 0.7(0.2), 0.7(0.2)}	{0.5(0.1), 0.4(0.1), 0.7(0.4), 0.5(0.2), 0.4(0.2)}
Q_3	{0.3(0.1), 0.5(0.1), 0.2(0.4), 0.5(0.2), 0.2(0.2)}	{0.4(0.1), 0.2(0.1), 0.2(0.4), 0.3(0.2), 0.3(0.2)}
Q_4	{0.4(0.1), 0.6(0.1), 0.2(0.4), 0.6(0.2), 0.2(0.2)}	{0.2(0.1), 0.3(0.1), 0.3(0.4), 0.2(0.2), 0.2(0.2)}
Alternative	I_3	I_4
Q_1	{0.6(0.1), 0.2(0.1), 0.2(0.4), 0.3(0.2), 0.3(0.2)}	{0.3(0.1), 0.3(0.1), 0.4(0.4), 0.4(0.2), 0.7(0.2)}
Q_2	{0.5(0.1), 0.7(0.1), 0.7(0.4), 0.2(0.2), 0.5(0.2)}	{0.6(0.1), 0.5(0.1), 0.5(0.4), 0.6(0.2), 0.4(0.2)}
Q_3	{0.4(0.1), 0.5(0.1), 0.2(0.4), 0.4(0.2), 0.5(0.2)}	{0.6(0.1), 0.3(0.1), 0.3(0.4), 0.6(0.2), 0.4(0.2)}
Q_4	{0.3(0.1), 0.3(0.1), 0.3(0.4), 0.3(0.2), 0.3(0.2)}	{0.2(0.1), 0.2(0.1), 0.2(0.4), 0.2(0.2), 0.2(0.2)}

Table 5
The standardized decision matrix by the second DM.

Alternative	I_1	I_2
Q_1	{0.2(0.1), 0.3(0.1), 0.3(0.4), 0.2(0.2), 0.5(0.2)}	{0.4(0.1), 0.4(0.1), 0.5(0.4), 0.2(0.2), 0.2(0.2)}
Q_2	{0.4(0.1), 0.7(0.1), 0.7(0.4), 0.7(0.2), 0.5(0.2)}	{0.3(0.1), 0.3(0.1), 0.7(0.4), 0.5(0.2), 0.5(0.2)}
Q_3	{0.4(0.1), 0.6(0.1), 0.3(0.4), 0.4(0.2), 0.6(0.2)}	{0.1(0.1), 0.3(0.1), 0.3(0.4), 0.1(0.2), 0.5(0.2)}
Q_4	{0.4(0.1), 0.3(0.1), 0.2(0.4), 0.4(0.2), 0.3(0.2)}	{0.6(0.1), 0.6(0.1), 0.4(0.4), 0.2(0.2), 0.2(0.2)}
Alternative	I_3	I_4
Q_1	{0.5(0.1), 0.6(0.1), 0.2(0.4), 0.2(0.2), 0.2(0.2)}	{0.2(0.1), 0.4(0.1), 0.4(0.4), 0.2(0.2), 0.3(0.2)}
Q_2	{0.5(0.1), 0.5(0.1), 0.4(0.4), 0.4(0.2), 0.7(0.2)}	{0.2(0.1), 0.7(0.1), 0.7(0.4), 0.6(0.2), 0.6(0.2)}
Q_3	{0.4(0.1), 0.3(0.1), 0.3(0.4), 0.4(0.2), 0.3(0.2)}	{0.2(0.1), 0.2(0.1), 0.2(0.4), 0.3(0.2), 0.3(0.2)}
Q_4	{0.5(0.1), 0.1(0.1), 0.1(0.4), 0.5(0.2), 0.1(0.2)}	{0.2(0.1), 0.2(0.1), 0.2(0.4), 0.5(0.2), 0.5(0.2)}

Table 6
The standardized decision matrix by the third DM.

Alternative	I_1	I_2
Q_1	{0.6(0.1), 0.4(0.1), 0.4(0.4), 0.6(0.2), 0.3(0.2)}	{0.5(0.1), 0.5(0.1), 0.3(0.4), 0.3(0.2), 0.3(0.2)}
Q_2	{0.4(0.1), 0.7(0.1), 0.5(0.4), 0.7(0.2), 0.7(0.2)}	{0.2(0.1), 0.3(0.1), 0.7(0.4), 0.7(0.2), 0.6(0.2)}
Q_3	{0.1(0.1), 0.4(0.1), 0.4(0.4), 0.4(0.2), 0.1(0.2)}	{0.5(0.1), 0.3(0.1), 0.3(0.4), 0.5(0.2), 0.3(0.2)}
Q_4	{0.2(0.1), 0.2(0.1), 0.3(0.4), 0.2(0.2), 0.2(0.2)}	{0.6(0.1), 0.2(0.1), 0.2(0.4), 0.4(0.2), 0.4(0.2)}
Alternative	I_3	I_4
Q_1	{0.6(0.1), 0.2(0.1), 0.2(0.4), 0.2(0.2), 0.6(0.2)}	{0.8(0.1), 0.3(0.1), 0.3(0.4), 0.3(0.2), 0.4(0.2)}
Q_2	{0.5(0.1), 0.7(0.1), 0.7(0.4), 0.5(0.2), 0.4(0.2)}	{0.7(0.1), 0.5(0.1), 0.6(0.4), 0.6(0.2), 0.5(0.2)}
Q_3	{0.3(0.1), 0.5(0.1), 0.2(0.4), 0.2(0.2), 0.3(0.2)}	{0.3(0.1), 0.4(0.1), 0.3(0.4), 0.4(0.2), 0.4(0.2)}
Q_4	{0.3(0.1), 0.3(0.1), 0.4(0.4), 0.2(0.2), 0.2(0.2)}	{0.2(0.1), 0.4(0.1), 0.4(0.4), 0.4(0.2), 0.5(0.2)}

Table 7
The priorities of criteria of DM 1.

Criteria	I_1^1	I_2^1	I_3^1	I_4^1
$N_{y(x-1)}^1$	1	2.7	3.8	4

Table 8
The priorities of criteria of DM 2.

Criteria	I_1^2	I_2^2	I_3^2	I_4^2
$N_{y(x-1)}^1$	3.2	4.8	2.3	1

(iii) A finite model for criterion weight coefficient meeting the condition which is introduced in the above:

$$\min \chi^1$$
$$\text{s.t.} \begin{cases} \left|\frac{p_1^1}{p_2^1} - 2.70\right| <= \chi^1, \ \left|\frac{p_2^1}{p_3^1} - 1.41\right| <= \chi^1, \ \left|\frac{p_3^1}{p_4^1} - 1.05\right| <= \chi^1, \\ \left|\frac{p_1^1}{p_3^1} - 3.80\right| <= \chi^1, \ \left|\frac{p_1^1}{p_4^1} - 1.48\right| <= \chi^1, \\ \sum_{x=1}^{y} p_x^1 = 1, \quad p_x^1 \geqslant 0. \end{cases} \tag{18}$$

The weight can be calculated by the software Lingo, and the result is $p_x^i = \{0.531, 0.197, 0.140, 0.133\}$, and the result of χ^1 is 0.00.

- Calculate the criterion weight for DM 2 (see Table 8).

(i) The DM 1 gives the ranking of significance of different attributes

$$I_4^2 > I_3^2 > I_1^2 > I_2^2.$$

(ii) According to the data in Table 8, we get the comparative priorities as follows:

$$\min \chi^2$$
$$\text{s.t.} \begin{cases} \left|\frac{p_4^2}{p_3^2} - 2.30\right| <= \chi^2, \ \left|\frac{p_3^2}{p_1^2} - 1.39\right| <= \chi^2, \ \left|\frac{p_1^2}{p_2^2} - 1.50\right| <= \chi^2, \\ \left|\frac{p_4^2}{p_1^2} - 3.20\right| <= \chi^2, \ \left|\frac{p_3^2}{p_2^2} - 2.09\right| <= \chi^2, \\ \sum_{x=1}^{y} p_x^2 = 1, \quad p_x^2 \geqslant 0. \end{cases} \tag{19}$$

The weight can be calculated by the software LINGO, and the result is $p_x^2 = \{0.160, 0.106, 0.222, 0.511\}$, and the result of χ^2 is 0.00.

- Calculate the criterion weight for DM 3 (see Table 9).

(i) The DM 1 gives the ranking of significance of different attributes

$$I_4^3 > I_3^3 > I_1^3 > I_2^3.$$

Table 9
The priorities of criteria of DM 3.

Criteria	I_1^3	I_2^3	I_3^3	I_4^3
$N_{y(x-1)}^1$	2.6	1	4.2	3.3

Table 10
The integrated decision matrix.

Alternative	I_1	I_2
Q_1	{0.444(0.1), 0.315(0.1), 0.315(0.4), 0.418(0.2), 0.367(0.2)}	{0.506(0.1), 0.392(0.1), 0.341(0.4), 0.242(0.2), 0.384(0.2)}
Q_2	{0.372(0.1), 0.613(0.1), 0.599(0.4), 0.700(0.2), 0.650(0.2)}	{0.332(0.1), 0.295(0.1), 0.700(0.4), 0.592(0.2), 0.517(0.2)}
Q_3	{0.261(0.1), 0.497(0.1), 0.315(0.4), 0.432(0.2), 0.319(0.2)}	{0.370(0.1), 0.271(0.1), 0.271(0.4), 0.340(0.2), 0.367(0.2)}
Q_4	{0.327(0.1), 0.376(0.1), 0.242(0.4), 0.404(0.2), 0.231(0.2)}	{0.508(0.1), 0.376(0.1), 0.295(0.4), 0.287(0.2), 0.287(0.2)}
Alternative	I_3	I_4
Q_1	{0.572(0.1), 0.350(0.1), 0.200(0.4), 0.231(0.2), 0.418(0.2)}	{0.559(0.1), 0.332(0.1), 0.362(0.4), 0.304(0.2), 0.490(0.2)}
Q_2	{0.500(0.1), 0.650(0.1), 0.631(0.4), 0.392(0.2), 0.539(0.2)}	{0.561(0.1), 0.571(0.1), 0.608(0.4), 0.600(0.2), 0.506(0.2)}
Q_3	{0.362(0.1), 0.447(0.1), 0.231(0.4), 0.327(0.2), 0.367(0.2)}	{0.384(0.1), 0.315(0.1), 0.271(0.4), 0.444(0.2), 0.372(0.2)}
Q_4	{0.367(0.1), 0.245(0.1), 0.290(0.4), 0.332(0.2), 0.204(0.2)}	{0.200(0.1), 0.287(0.1), 0.287(0.4), 0.381(0.2), 0.424(0.2)}

Table 11
The score of the integrated decision matrix.

Alternative	I_1	I_2	I_3	I_4
Q_1	0.359	0.351	0.302	0.393
Q_2	0.608	0.565	0.553	0.577
Q_3	0.352	0.314	0.312	0.341
Q_4	0.294	0.321	0.285	0.324

(ii) According to data in Table 9, we get the comparative priorities as follows:

$$\min \chi^3$$
$$\text{s.t.}\begin{cases} \left|\frac{p_2^3}{p_1^3}-2.60\right| <= \chi^3, \quad \left|\frac{p_1^3}{p_4^3}-1.27\right| <= \chi^3, \quad \left|\frac{p_4^3}{p_3^3}-1.27\right| <= \chi^3, \\ \left|\frac{p_2^3}{p_4^3}-3.30\right| <= \chi^3, \quad \left|\frac{p_1^3}{p_3^3}-1.62\right| <= \chi^3, \\ \sum_{x=1}^{y} p_x^3 = 1, \quad p_x^3 \geqslant 0. \end{cases} \tag{20}$$

The weight can be calculated by the software LINGO, and the result is $p_x^3 = \{0.200, 0.519, 0.124, 0.157\}$, and the result of χ^3 is 0.00.

The final criterion weight is obtained by the integration weight combined with experts' decision weight, and the result is $\hat{p}_x = \{0.287, 0.299, 0.158, 0.256\}$.

Step 3. Integrate the decision matrices by different DMS into one matrix (see Table 10).

Step 4. Figure out the score function of the integrated decision matrix (see Table 11).

Step 5. Determine the $\hat{\upsilon}_x^*$ and $\hat{\upsilon}_x^-$ indexes (see Tables 12–13).

Table 12
The positive index $\hat{\upsilon}_x^*$.

I_1	I_2
{0.372(0.1), 0.613(0.1), 0.599(0.4), 0.700(0.2), 0.650(0.2)}	{0.332(0.1), 0.295(0.1), 0.700(0.4), 0.592(0.2), 0.517(0.2)}
I_3	I_4
{0.500(0.1), 0.650(0.1), 0.631(0.4), 0.392(0.2), 0.539(0.2)}	{0.561(0.1), 0.571(0.1), 0.608(0.4), 0.600(0.2), 0.506(0.2)}

Table 13
The positive index $\hat{\upsilon}_x^-$.

I_1	I_2
{0.327(0.1), 0.376(0.1), 0.242(0.4), 0.404(0.2), 0.231(0.2)}	{0.370(0.1), 0.271(0.1), 0.271(0.4), 0.340(0.2), 0.367(0.2)}
I_3	I_4
{0.367(0.1), 0.245(0.1), 0.290(0.4), 0.332(0.2), 0.204(0.2)}	{0.200(0.1), 0.287(0.1), 0.287(0.4), 0.381(0.2), 0.424(0.2)}

Table 14
The positive and negative distance.

$d_1^*(\hat{\upsilon}_{rx}(\hat{g}_{rx}), \hat{\upsilon}_x^*(\hat{g}_x^*))$	0.232	$d_1^-(\hat{\upsilon}_{rx}(\hat{g}_{rx}), \hat{\upsilon}_x^-(\hat{g}_x^-))$	0.152
$d_2^*(\hat{\upsilon}_{rx}(\hat{g}_{rx}), \hat{\upsilon}_x^*(\hat{g}_x^*))$	0.000	$d_2^-(\hat{\upsilon}_{rx}(\hat{g}_{rx}), \hat{\upsilon}_x^-(\hat{g}_x^-))$	0.266
$d_3^*(\hat{\upsilon}_{rx}(\hat{g}_{rx}), \hat{\upsilon}_x^*(\hat{g}_x^*))$	0.219	$d_3^-(\hat{\upsilon}_{rx}(\hat{g}_{rx}), \hat{\upsilon}_x^-(\hat{g}_x^-))$	0.083
$d_4^*(\hat{\upsilon}_{rx}(\hat{g}_{rx}), \hat{\upsilon}_x^*(\hat{g}_x^*))$	0.290	$d_4^-(\hat{\upsilon}_{rx}(\hat{g}_{rx}), \hat{\upsilon}_x^-(\hat{g}_x^-))$	0.034

Step 6. Calculate the positive and negative distance (see Table 14).

Step 7. Compute out the relative closeness to the ideal solution.

Then the result is $H_r = \{0.396, 1.000, 0.274, 0.105\}$, and we a get rank $Q_2 > Q_1 > Q_3 > Q_4$.

5. Comparison and Discussion

In this section, TODIM (PHF-TODIM) method (Zhang *et al.*, 2018), PHFWA operator (Xu and Zhou, 2017), PHFWG operator (Xu and Zhou, 2017) are utilized to compare with the PHF-FUCOM-TOPSIS method to test its feasibility and practicability. In order to compare the results more intuitively, we represent the result as a line chart in Fig. 2 and Table 15 where the original result is processed by the same manner in range 0 to 1.

From the above detailed analysis, it could be seen that these four given models have the same optimal choice Q_2 and the order of these four methods is the same. This verifies that the PHF-FUCOM-TOPSIS is reasonable & effective. These four given models have their given advantages: (1) PHFWA operator emphasizes group decision influences; (2) PHFWG operator emphasizes individual decision influences; (3) The PHF-TODIM method is an interactive multi-criteria decision-making method. The method is based on the value function of prospect theory, establishes the relative dominance function of a certain plan compared with other plans according to the psychological behaviour of decision makers, and selects the best plan according to the size of the dominance, so as to determine the optimal plan. At the moment, the TODIM method is continuously improved and

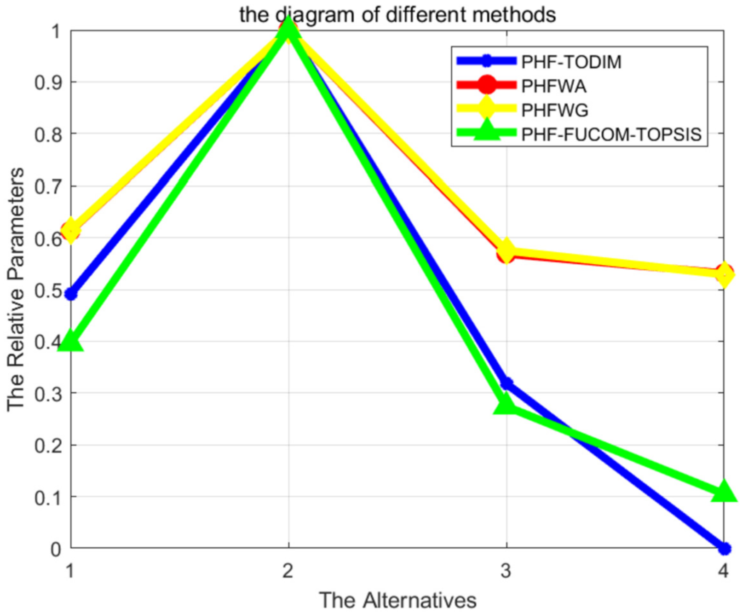

Fig. 2. The comparison of different methods.

Table 15
The sequence from different methods.

Method	The sequence	The best alternative
PHF-TODIM (Zhang *et al.*, 2018)	$Q_2 > Q_1 > Q_3 > Q_4$	Q_2
PHFWA operator (Xu and Zhou, 2017)	$Q_2 > Q_1 > Q_3 > Q_4$	Q_2
PHFWG operator (Xu and Zhou, 2017)	$Q_2 > Q_1 > Q_3 > Q_4$	Q_2
PHF-FUCOM-TOPSIS	$Q_2 > Q_1 > Q_3 > Q_4$	Q_2

widely used in decision-making in various fields. (4) The "ideal solution" and "negative ideal solution" in the PHF-FUCOM-TOPSIS method are two basic concepts of the TOPSIS method. The so-called ideal solution is an assumed optimal solution (scheme), and its various attribute values reach the best value among the alternative schemes; while the negative ideal solution is the assumed worst solution (scheme), and each of its attribute values achieve the worst value among the alternatives. The rule for sorting the schemes is to compare the alternatives with the ideal solution and the negative ideal solution. If one of the alternatives is closest to the ideal solution while far from the negative ideal solution, then this scheme is the best one among the alternatives.

6. Conclusions

In this study, we propose a new PHF-FUCOM-TOPSIS model for performance evaluation of public charging service quality and apply it in the probabilistic hesitant fuzzy environment. A novel extended TOPSIS model integrated with FUCOM method was proposed to evaluate green selection supplier. Finally, we apply this method in a numerical study for performance evaluation of public charging service quality and compare the results with other methods to test its validity. The specific contributions of it are as follows:

(1) It integrates classical TOPSIS method and FUCOM method in the probabilistic hesitant fuzzy environment including more information to make the decision-making process more reasonable.
(2) It extends the FUCOM method to calculate criterion weight in the probabilistic hesitant fuzzy environment.

In the future, we firmly believe that PHF-FUCOM-TOPSIS method will be applied in a larger number of fields. Meanwhile, we should consider the attributes of the actual situation when solving the performance evaluation of public charging service quality and apply this new model in more fields.

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