NOTES

KARPOV AND KASPAROV: THE END IS PERFECTION

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[Below, this Journal reports on an investigation prompted by a burning question: did Karpov and Kasparov play correctly the endgame when they met in October 1991 in Tilburg? The short answer is that they did. The interest of the report to our readers may be centered on the mechanics of consulting and as it happens even reconstructing a database when it suddenly is urgently necessary for consultation. Lewis Stiller must be regarded as the prime author of this contribution which has been given a somewhat unusual form for reasons which will become clear as the story unfolds. - Eds.]

The problem

It happened when Karpov first met Kasparov in the Interpolis tournament in Tilburg 1991 (they were due to meet again in a second round robin). Excitingly, the game ended in a pawnless KBNNKR configuration and it dawned upon some of the cognoscenti that they henceforth were treading on possibly known ground. Had not Stiller (1991a) let it be known that nowadays databases are available for pawnless six-piece endgames? And was the KBNNKR database among them?

As it happened, Karpov quite easily drove Kasparov’s King into a corner and thereafter confined the BK to that area. Somewhat surprisingly the outcome still was a draw, achieved by a stalemate forced by Kasparov sacrificing his Rook. To chess commentators all over the globe, this gave rise to a burning question: could Karpov have achieved a win, as would seem not unlikely by virtue of the superiority of his forces?

Specifically IGM Raymond Keene, The Times’ chess editor, wished to determine, once for all, whether and if so where the contestants had deviated from the optimal path.

A joint forensic operation

At this point, the search for the truth of the matter became a joint forensic operation with Keene in the lead, Stiller owner of the relevant database as the ultimate arbiter and your Editors as intermediaries, with Victor Allis serving as an indispensable liaison officer.

When asked by Email to analyze the Karpov-Kasparov endgame, Stiller replied that the database would have to be recomputed, because it had not been saved. What was still at hand was a statistical summary of results, insufficient to answer questions about the game theoretical results of any particular manifestation of KBNNKR. From what was retained, one could conclude that generally but not invariably, the value of the game was drawn, and that the maximin consisted of 49 moves, – tantalizing but insufficient information.

Lewis Stiller undertook to settle the specific issue by requesting time for his computer to reconstruct the database in question. The CM-2, however, was temporarily shut down due to a water leak in the building in which it is housed; and Stiller did not know when it would again be accessible. Fortunately, Stiller’s lab was greatly intrigued by the request and the reasons behind it, so much so that Stiller was given the assurance that his request would be considered for eventual scheduling. Cautiously he stipulated that both the decision to schedule and the scheduling itself might take some time.

Conclusions

When time was obtained – at a shorter notice than anyone dared hope – on the original computer, it took some two hours on the CM-2 (7 MHz SIMD bit-serial 8 gigabyte RAM 16-cube with SUN front end) to prove that, given K-K’s initial position of KBNNKR, a draw would be the outcome of perfect play.
Moreover, it was shown that neither contestant ever departed from optimality where we note, of course, that in a game as complex as this there is a multitude of equi-optimal paths.

Karpov-Kasparov

In Diagram 1, the beginning position of the Karpov-Kasparov KBNNKR endgame (Tilburg, 1991) has been depicted. The game proceeded as follows.


A maximin

In Diagram 2 a maximin position is presented. In the optimal line of play, equi-optimal moves are given in parentheses.

1. Na3-c4 Ra6-e6 2. Ne4-f2 Re6-e8 3. Bd8-g5 Re8-f8 4. Nf2-d1 Rb-g8 5. Bg5-c7 Rg8-e8 6. Be7-b4 Re8-b8 7. Bb4-a5 Kd1-b1 8. Nd1-c3 Kb1-a1 9. Nc3-e4 Kd1-b1 10. Ba5-c7 Rb-b8 11. Be7-b6 Kb1-a2 12. Kd3-c3 Rb-b8 13. Bb6-d4 Rd-b8 14. Ne4-d6 Rd8-b8 15. Ne4-c5 Rb8-g8 16. Nd6-c4 Rg8-g4 17. Bd4-e5 Rg4-g5 18. Be5-d6 Rg5-g6 19. Bd6-e7 Rg6-g7 20. Be7-h4 Rg7-h7 (Rg7-g4) 21. Bh4-e1 Rh7-h1 22. Be1-d2 Rh1-d1 23. Bd2-f4 Rd1-f1 24. Bf4-c7 Rf1-c1 25. Kc3-d3 Ka2-b1 26. Be7-f4 Rc1-g1 (Rc1-h1) 27. Kg3-c3 Kg1-g4 28. Bf4-c7 Rg4-g7 29. Be7-a5 Kb1-c1 30. Nc4-b2 Kc1-b1 (Rg7-g5) 31. Nc5-d3 Rg7-g5 (Rg7-a7) 32. Ba5-b4 Rg5-b5 33. Bh4-d6 Rh5-b6 (Rh5-d5) 34. Bd6-f4 Rb6-f6 35. Bd4-d2 Rf6-f7 (Rf6-f8 Rd6-f6) 36. Nh2-c4 Rf7-d7 37. Bd2-c3 (Bd2-f4 Bb2-g5 Bd2-h6) Rd7-e7 38. Be3-c5 (Be3-c1) Re7-c7 39. Be5-a3 Re7-e8 40. Kc3-b3 (Nd3-b4) Re8-b8 41. Nd3-b4 Rb8-d8 42. Nc4-b6 Rd8-f8 (Rd8-g8 Rd8-h8 Rd8-d2) 43. Nb6-d5 (Nb6-a4) Rf8-f2 44. Nd5-c3 Kbd1-a1 45. Nb4-c6 Rf2-g2 (Rf2-h2 Rf2-c2) 46. Nc6-d4 Rg2-d2 47. Ba3-c1 Rd2-f2 (Rd2-g2 Rd2-h2) 48. Kb3-a3 (Nd4-e2) Rf2-a2 (Rf2-f3 Rf2-f4 Rf2-f5 Rf2-f6 Rf2-f7 Rf2-f8 Rf2-g2 Rf2-h2 Rf2-f1 Rf2-e2 Rf2-d2 Rf2-c2 Rf2-b2) 49. Nc3xa2
Some Statistics

In Table 1 the distribution of WTM win-in-\(n\) positions is exhibited.

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Table 1: The distribution of WTM win-in-\(n\) positions.
* This figure includes Black in check.

Zugzwangs

Noam Elkies, a Harvard University mathematician and talented endgame composer, had been a collaborator of Stiller’s throughout the quest to solve 6-piece endgames. Dr. Elkies designed the max-to-mate algorithm, suggested numerous testing procedures, determined the most promising 6-piece endgames to solve, and helped Stiller understand and implement aspects of the program modules that rely on abstract algebra (Stiller, 1991b), among other contributions. He also designed the module that computes mutual zugzwangs, positions from which White can win if and only if Black is to move. This module found 1270 such mutual zugzwangs, three of which are listed in Diagrams 3a-c. (Note that 3b is particularly elegant.)

Diagrams 3a-c: Three mutual zugzwang positions of different type.
References


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**HOW TO WIN WITH A KNIGHT AHEAD**

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In a regular chess game, being a Knight ahead is normally sufficient to win a game. In the endgame, by contrast, a draw may be the result. However, there are some exceptions. The KNNK endgame is considered a draw, but giving the weak side an extra Knight increases the winning chances. The maximum number of moves to win is 7 (see Diagram 1) and is given on Ken Thompson’s (1990) CD-ROM.


![Diagram 1: White to move (mate in 7).](image1)

Most of the 5-piece endgames with a Knight ahead are on the CD-ROM. These endgames are normally a draw, but some interesting wins are documented in the chess literature. For instance, in Euwe (1950, p. 30) a win in 7 is given for the following KQNKQ endgame (see Diagram 2).


![Diagram 2: White to move (a win in 7).](image2)