# Annales Societatis Mathematicae Polonae Series IV: Fundamenta Informaticae III - 1 (1980)

# ON PROBABILISTIC AUTOMATA IN DETERMINISTIC ENVIRONMENT

## Jerren Gould, Edward J. Wegman

**Abstract.** We establish the basic notion of automata in environments and solve of the basic problems in the general theory of automata in deterministic environment (ADE). ADE are, in general, stronger than probabilistic automata. In an effort to find when an ADE and a PA have the same capability, we introduce the concept of *simulation* of an ADE by a PA. Every ADE with a finite environment set can be simulated by a PA (Theorem 1). There are sets which cannot be defined by PA but can be defined by ADE with finite environment sets and, hence, can be simulated by a PA. ADE for which the environment can be reduced to a finite set can be simulated a PA (Theorem 2). The set of stochastic matrices is a semi-group. We investigate the case where the environment set is also a semi-group and the transition function is a homomorphism between the semi-groups. If the semi-group environment set can be generated by a finite set, then the ADE can be simulated by a PA (Theorem 3). The continuous time PA of Knast [2] is seen to be a special case of the ADE.

**Keywords:** Automaton, probabilistic automaton, automaton in environments, probabilistic tables, cut-point, probabilistic cut-point event, environment.

## **ON METALINEAR ETOL SYSTEMS**

#### Grzegorz Rozenberg, Dirk Vermeir

Abstract. The concept of metalinearity in ETOL systems is investigated. Some structural characterizations, a pumping lemma and the closure properties of the resulting class of languages are established. Finally, some applications in the theory of L systems of finite index are provided.

Keywords: Formal languages, ETOL systems, finite index.

## FUNCTIONAL AUTOMATA

## Wojciech Rytter

**Abstract.** Functional automata are models of algorithms of a certain class. The following problems concerning functional automata are presented and solved in the paper:

1. The problem of minimization of functional automat.

2. The problem of the existence of the nondeterministic functional automaton nonequivalent to any deterministic one.

- 3. The problem of optimal control for the functional automaton.
- 4. The problem of the existence of a universal automaton.

**Keywords:** functional automaton, minimization, control, measurable functional automaton, non-determinism.

## ALGORITHMIC LOGIC WITH NONDETERMINISTIC PROGRAMS

#### Grażyna Mirkowska

Abstract. Algorithmic properties of nondeterministic programs are studied and axiomatized completely. Nondeterministic programs require two kinds of algorithmic formulas describing their behaviour:  $\Delta Ka$  – all computations of the program K are finite and all results satisfy a and  $\nabla Ka$ – there exists a finite computation such that its result satisfies the formula a.

Keywords: Nondeterministic program, algorithmic logic, completeness.

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# SOME REMARKS ON DETERMINISTIC MAZURKIEWICZ ALGORITHMS AND LANGUAGES ASSOCIATED WITH THEM

#### Ryszard Janicki

**Abstract.** The paper deals with a class of programs called deterministic Mazurkiewicz algorithms. It considers languages associated with such algorithms by using symbols instead of actual relations and concatenation of words rather than composition of actual relations. It is shown that the class of languages so generated is precisely the class of simple context-free languages of Korenjak and Hoperoft [8].

**Keywords:** Deterministic Mazurkiewicz algorithm, simple context-free language.

## A THEOREM ON THE CHARACTERISTIC OF NON-SEQUENTIAL PROCESSES

#### Eike Best

Abstract. Real non-sequential processes can be described in a consistent way if they are assumed to satisfy a certain density property. Density as defined in [1] can be interpreted as postulating that a "global state" consists of a "progress snapshot" of all single activities which constitute it.

The present report shows that postulating density amounts to postulating that (a) each single history of a process is either infinite or has a first cause (but not both), and (b) each single future of a process is either infinite or has a last effect (but not both). This result is interpreted and applied to the question of Turing-computability.

Keywords: Non-sequential processes, causal nets, computability.

# **NON-GENERABLE FORMAL LANGUAGES**

## Douglas Cenzer

**Abstract.** A set A of words on a finite alphabet  $\Sigma$  is said to be *generable* if it is the closure of a computable inductive operator; in particular, if S is a semi-Thue system then the set of words derivable in S is generable. An RE set of words (equivalently, a phrase-structure or type 0 language in the sense of Chomsky [Information and Control 2 (1959), 137-167]) which is non-generable is constructed by means of a finite injury priority argument. The construction is refined to obtain a non-generable set of degree 0' and, for any degree d, a non-generable set of degree  $\leq d$ .

Keywords: Formal language, semi-Thue system, recursive function, inductive definition, generable, degree of unsolvability.

## **COMPLETIONS OF ORDERED MAGMAS**

## Bruno Courcelle, Jean-Claude Raoult

Abstract. We give a completion theorem for ordered magmas (i.e. ordered algebras with monotone operations) in a general form. Particular instances of this theorem are already known, and new results follow. The semantics of programming languages is the motivation of such investigations.

**Keywords:** Complete partial orders, semantics of programming languages.

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## ON STABILITY OF PROBABILISTIC AUTOMATA IN ENVIRONMENTS

## Jerren Gould

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Abstract. Flachs [1], Rabin [7], and Paz [6] have considered topics in the stability of probabilistic automata. Here we extend these results to the more general forms, automata in deterministic environments (ADE). We shall be concerned with two types of stability problems that arise from small perturbations of the environment configurations for an ADE. By consideration of the asymptotic properties of long products of stochastic matrices whose entries are subject to small perturbations concomitant to the environment configuration perturbations, we arrive at sufficient conditions for the state distribution function to be stable (*s*-stability). In its acceptor formulation the behavior of an ADE is characterized by the sets  $T(\mathscr{A}, \mathbf{e}, \lambda)$ , the set of tapes accepted by ( $\mathscr{A}$  with environment sequence  $\mathbf{e}$  and cut-point  $\lambda$ . Sufficient conditions are given for tape-acceptance stability (*a*-stability) in terms of *s*-stability and in terms that do not require *s*-stability. These stability results are pointwise results that give the size of the perturbation of the environment configuration to avoid instability.

**Keywords:** Automaton, probabilistic automaton, automaton in environment, stability, *a*-stable, *s*-stable.

## A NOTE ON THE INTERSECTION OF CONTEXT-FREE LANGUAGES

#### Gheorge Păun

**Abstract.** It is shown that a) there is an equal matrix language which cannot be written as a finite intersection of context-free languages and b) any finite intersection of context-free languages can be generated by a regular conditional grammar with context-free control languages.

Keywords: Context-free languages, equal matrix languages, regular conditional grammars.

# SIMPLE EOL FORMS UNDER UNIFORM INTERPRETATION GENERATING CF LANGUAGES

#### Jürgen Albert, Hermann Maurer, Grzegorz Rozenberg

**Abstract.** In this paper we consider simple EOL forms (forms with a single terminal and single nonterminal) under uniform interpretations. We present a contribution to the analysis of generative power of simple EOL forms by establishing easily decidable necessary and sufficient conditions for simple EOL forms to generate (under uniform interpretations) CF languages only.

Keywords: Context-free languages, Lindenmayer systems, grammatical similarity.

# MODEL EXISTENCE THEOREM IN ALGORITHMIC LOGIC WITH NON-DETERMINISTIC PROGRAMS

#### Grażyna Mirkowska

**Abstract.** The paper is a continuation of the considerations connected with non-deterministic algorithmic logic. We will formulate a Hilbert style axiomatization basing on the analogous one defined for algorithmic logic. The main result is the theorem asserting that every consistent non-deterministic algorithmic theory possesses a model.

Keywords: Algorithmic logic, deterministic algorithmic logic, non-deterministic algorithmic logic.

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## **AXIOMS FOR MULTILEVEL OBJECTS**

J. A. Bergstra, H. J. M. Goeman, A. Ollongren, G. A. Terpstra, Th. P. Van Der Weide 171-180

**Abstract.** A set of axioms for structured objects of data is presented. In the structured objects components and levels are distinguished. Change of level is the result of a special application operator, components are accessible by successive selections. The set of access paths is also axiomatized. The set of axioms is uniform in the sense that features of various known classes of datastructures are combined.

Keywords: Multi-level datastructures, programming.

# COMPLEXITY OF INDEX SETS AND TRANSLATING FUNCTIONS

## Alexander Leitsch

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Abstract. The connection between index stes appearing in recursive enumerations of subrecursive classes and translating functions is investigated. Referring to a construction, which yields enumerations with low complexity for some index sets, we show that in such a case the complexity for some translating functions must be high. Furthermore a class of enumerations is discussed, which has elementary translating functions, but an important part of the index sets is not decidable by algorithms of the enumerated class (these index sets are of the form  $\{x : \psi_x(a) = f(x)\}$ ).

**Keywords:** Recursive enumerations of subrecursive classes, complexity of translating functions, complexity of index sets.

## GENERAL THEORY OF RELATIONAL AUTOMATA

#### Vera Trnková

**Abstract.** General theory of relational automata, including non-deterministic linear and bilinear machines, structured non-deterministic tree automata and automata in some primitive classes of algebras, is developed. Particular attention is paid to languages accepted by finite relational automata.

**Keywords:** Automata, relations, category, functor, functorial algebras, functorial machine, language.

# **RESOLUTION SYSTEMS AND THEIR APPLICATIONS I**

## Ewa Orłowska

**Abstract.** The central method employed today for theorem-proving is the resolution method introduced by J.A. Robinson in 1965 for the classical predicate calculus. Since then many improvements of the resolution method have been made. On the other hand, treatment of automated theorem-proving techniques for non-classical logics has been started, in connection with applications of these logics in computer science.

In this paper a generalization of a notion of the resolution principle is introduced and discussed. A certain class of first order logics is considered and deductive systems of these logics with a resolution principle as an inference rule are investigated. The necessary and sufficient conditions for the so-called resolution-completeness of such systems are given. A generalized Herbrand property for

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a logic is defined and its connections with the resolution-completeness are presented. A class of binary resolution systems is investigated and a kind of a normal form for derivations in such systems is given. On the ground of the methods developed the resolution system for the classical predicate calculus is described and the resolution systems for some non-classical logics are outlined. A method of program synthesis based on the resolution system for the classical predicate calculus is presented. A notion of a resolution-interpretability of a logic  $\mathcal{L}$  in another logic  $\mathcal{L}'$  is introduced. The method of resolution-interpretability consists in establishing a relation between formulas of the logic  $\mathcal{L}$  and some sets of formulas of the logic  $\mathcal{L}'$  with the intention of using the resolution system for  $\mathcal{L}'$  to prove theorems of  $\mathcal{L}$ . It is shown how the method of resolution-interpretability can be used to prove decidability of sets of unsatisfiable formulas of a given logic.

**Keywords:** Classical predicate calculus,  $\omega^+$ -valued logic, resolution principle, Herbrand's theorem, completeness, decid.ability, program synthesis.

# Annales Societatis Mathematicae Polonae Series IV: Fundamenta Informaticae III - 3 (1980)

# HAS-HIERARCHY; A NATURAL TOOL FOR LANGUAGE SPECIFICATION

# Teodor Rus

**Abstract.** This paper is an attempt to direct present-day research in programming language specification and their compiler construction towards a more natural approach. In order to do that, a language is considered for what it is, namely a communication device between systems. In view of this evidence, the first section of the paper develops a framework for the natural specification of language. Section two of the paper develops the HAS-Hierarchy as a device to be used in this natural language specification. Section three of the paper constructs a general model for programming language specification by the HAS-Hierarchy. Section four is devoted to the problem of compiler construction by using the HAS-Hierarchy as a natural tool for programming language specification.

**Keywords:** Syntax, semantics, heterogeneous algebra, contextfree grammar, normal algorithm, pattern matching process, compiler, compiler construction.

# A NOTE ON M-GROWTH FUNCTIONS OF FTOL SYSTEMS WITH RANK

Grzegorz Rozenberg, Dirk Vermeir

**Abstract.** A theorem is presented which characterizes the maximal growth in FTOL systems with rank.

Keywords: Formal language theory, FTOL systems, growth functions, rank.

# CONJUGATED MEASURES OF COMPUTATIONAL COMPLEXITY

## Fausto Adrianopoli

**Abstract.** It is shown here that using the Kleene's normal form for partial recursive functions (p. r. functions) it is possible to develop a different normal form which will give a relation between these p.r. functions and certain related step-counting ones. Also this new form suggests an interesting serial expansion in terms of minimal functions of these p.r. functions.

The notion of conjugated systems of computational complexity is eventually introduced.

Keywords: Measure of computational complexity, step-counting function.

# ON THE ALGORITHMIC THEORY OF STACKS

## Andrzej Salwicki

Abstract. The algorithmic theory of stacks, ATS, formalizes properties of relational systems of stacks. It turns out that apart from previously known axioms a new axiom of algorithmic nature,

while  $\neg$  empty (s) do s : = pop (s) true

is in place. The representation theorem stating that every relational system of stacks is isomorphic to a system of finite sequences of elements is proved. The connections between ATS and a type STACKS declaration (written in LOGLAN programming language) are shown.

**Keywords:** Abstract data types, algorithmic theory, – of stacks, – of dictionaires, of stacks and links, interpretability relation between theories, model, representation theorem, type declaration.

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# **RESOLUTION SYSTEMS AND THEIR APPLICATIONS II**

#### Ewa Orłowska

## INCOMPLETE INFORMATION IN A RELATIONAL DATABASE

#### John Grant

**Abstract.** In this paper we investigate the inclusion of incomplete information in the relational database model. This is done by allowing nonatomic entries, i.e. sets, as elements in the database. A nonatomic entry is interpreted as a set of possible elements, one of which is the correct one. We deal primarily with numerical entries where an allowed set is an interval, and character string entries. We discuss the various operations of the relational algebra as well as the notion of functional dependency for the database model.

**Keywords:** Relational database, incomplete information, relational algebra, functional dependency.

# AN ALGORITHM OF FINDING AN ACYCLIC *f*-GRAPH FOR A FAMILY OF SETS

## Mirosław Truszczyński

Abstract. Acyclic families of sets are investigated. A theorem giving necessary and sufficient conditions for the family of sets to be acyclic is formulated. and proved. Then an algorithm is described of finding the acyclic f-graph for a given family of sets whenever this family is acyclic. Its computational complexity is equal to  $\mathcal{O}(n^3 + n^2k)$ , so it is better than the other algorithms known so far.

Keywords: Consecutive retrieval, storage, records, f-graph, acyclic family.

# *NP*-COMPLETENESS OF COMBINATORIAL PROBLEMS WITH UNARY NOTATION FOR INTEGERS

## Martti Penttonen

Abstract. Most NP-complete problems remain NP-complete even though the notation for integers is changed to unary. The knapsack problem is an exception, it becomes provably polynomial time recognizable. However, we present a modified knapsack problem that remains NP-complete also in unary notation.

Keywords: NP-completeness, satisfiability, chromatic number, exact cover knapsack, unary notation.

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# TREE-CODIFICATIONS AND CONVOLUTIONAL CODES

# Adrian Atanasiu

Abstract. The paper deals with some C-grammars and tree-codification properties extending the result in [1]. We try to exhibit new aspects and important results, e.g. that any algebraic convolutional code is a particular case of a tree-code, as well as the finite forms of the code-sets. They can result in new methods of decodification and error correction in the case of convolutional coding.

Keywords: Formal languages, algebraic coding theory, automata theory.

# PROGRAMS AND TERM TRANSFORMERS

# Stefan Sokołowski

**Abstract.** Predicates describing the states of computation may be regarded as functions into the Boolean algebra false, true and programs as transformers of those functions. If we do not restrict ourselves to this algebra, we get instead terms describing the states of computation and programs transforming the terms. In many cases this approach turns out to be more natural. This paper is a mathematical study of partial correctness and termination of programs in the language of term transformations.

**Keywords:** Semantics of programs, partial correctness, termination, calculus of relations, test relations, term transformers.

# SETS GENERATED BY STOCHASTIC AUTOMATA OF MARKOV'S CHAIN TYPE

#### Sławomir Janicki

**Abstract.** In the earlier paper of the author [2] it has been introduced the concept of the generability for stochastic automaton. Here we give new necessary and sufficient conditions for the generability of the set of infinite sequences of automaton states. Moreover, we consider the generability of subset, complement, union, intersection and difference of generable sets.

**Keywords:** Stochastic automaton, probability measure, carrier, word, language, generability, generable set.

# THE METRIC SPACE OF INFINITE TREES. ALGEBRAIC AND TOPOLOGICAL PROPERTIES

#### Andre Arnold, Maurice Nivat

**Abstract.** The set of infinite trees is considered as an ultrametric compact space. The composition of trees is investigated. Applications to non deterministic recursive program schemes are given.

Keywords: Infinite trees, ultrametric spaces, algebraic semantics, nondeterministic programs.

## CONTINUOUSLY GENERATED FIXED POINTS IN $P\omega$

#### Felipe Bracho

Abstract. In this paper we investigate the fixed. points of functions that can be obtained by continuous fixed point operators in  $P\omega$ . We also introduce an operator Cgn that generates all of

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these fixed points for any continuous function. In the last two sections of the paper we study the fixed points of  $\lambda a.a \circ a$  (retracts) and of  $\lambda y \lambda f.f(y)(f)$  (fixed point operators).

**Keywords:** Programming language semantics, lattice, algebraic lattice, retract, continuous function, fixed points.

# **ON INTERSECTIONS OF CONTEXT-FREE LANGUAGES**

Irina Gorun

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# DECISION PROBLEMS IN GENERALIZED VECTOR ADDITION SYSTEMS

#### Hans Kleine Büning

**Abstract.** This paper concerns with the reachability and equality problem of generalized vector addition systems, where the systems work in the cartesian-product of the integers with finite (resp. infinite) exception set.

**Keywords:** Generalized vector addition system, vector addition system, reachability problem, equality problem.