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FROM CONCEPT LATTICES TO APPROXIMATION SPACES: ALGEBRAIC STRUCTURES OF SOME SPACES OF PARTIAL OBJECTS

Piero Pagliani

Abstract. The paper presents general logic-algebraic structures related to Wille's Concept Lattices, Pawlak's theory of Information Systems and Approximation Spaces. These structures are studied in order to analyse increasing concepts definitions and decreasing objects identifications. Here they are investigated starting from the notions of *polarity* and *quantum at a point*, applying the approach used in Bell's representation theorem for ortholattices and the mathematical tools provided by pointless topology. In this way we intend to connect this analysis to the notion of *topology of positive information* introduced in Scott's Information Systems and Scott's Domains. In particular, notions connected to the concept of 'objects definition improvement' (via attributes, or parameters) are algebraically investigated in order to analyse general and particular characteristics of Information Systems and a related classification for them is proposed.

GENERALIZED DERIVATIVES

Lila Kari

Abstract. The customary language-theoretic derivative of a word u with respect to a word v means the deletion of v from the beginning or end of u. We investigate the natural generalization, where v can be deleted from an arbitrary position in u. Apart from general closure and decidability properties, we pay special attention to regular languages, obtaining an exhaustive characterization.

A PROLEGOMENON TO PARTIAL DEDUCTION

Jan Komorowski

Abstract. Partial deduction is a specialization principle related to the law of syllogism. It has several computational applications in logic programming but it has been recently also used in deductive databases, machine learning, software synthesis and other areas of computing. This article is a systematic introduction to partial deduction, its applications and open problems. Starting from an informal and intuitive presentation, the fundamental notions such as correctness and completeness are discussed. A selection of applications is presented to illustrate partial deduction in different contexts.

LOGICAL CHARACTERIZATIONS OF BOUNDED QUERY CLASSES I: LOGSPACE ORACLE MACHINES

Iain A. Stewart

Abstract. We consider three sub-logics of the logic $(\pm HP)^*[FO_s]$ and show that these sub-logics capture the complexity classes obtained by considering logspace deterministic oracle Turing machines with oracles in **NP** where the number of oracle calls is unrestricted and constant, respectively; that is, the classes $\mathbf{L}^{\mathbf{NP}}$ and $\mathbf{L}^{\mathbf{NP}}[O(1)]$. We conclude that if certain logics are of the same expressibility then the Polynomial Hierarchy collapses. We also exhibit some new complete problems for the complexity class $\mathbf{L}^{\mathbf{NP}}$ via projection translations (the fnst to be discovered: projection translations are extremely weak logical reductions between problems) and characterize the complexity class $\mathbf{L}^{\mathbf{NP}}[O(1)]$ as the closure of NP under a new, extremely strict truth-table reduction (which we introduce in this paper).

LOGICAL CHARACTERIZATIONS OF BOUNDED QUERY CLASSES II: POLYNOMIAL-TIME ORACLE MACHINES

Iain A. Stewart *93-105*

Abstract. This paper continues an investigation into the expressibility of the logic $(\pm HP)^*[FO_s]$. In particular, we show that the logic $(\pm HP)^*[FO_S]$ has the same expressibility as its sub-logic $(\pm HP)^1[FO_s]$ which is known to capture the complexity class $\mathbf{L}^{\mathbf{NP}}$ (this class being those sets of strings accepted by some logspace deterministic oracle Turing machine with an oracle in **NP**). We consequently show that a naturally defined hierarchy within $\mathbf{P}^{\mathbf{NP}}$ collapses.

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Abstract. In [13] we have presented the ideas underlying an automated theorem prover based on tableaux extended with unification under restrictions. In [6] a full description of an implementation of this theorem prover in PROLOG is, given. In this paper we first shortly repeat the main ideas, referring to [13] for more details. Next we present the test results of our theorem prover mainly with respect to Pelletier's 75 problems for testing automatic theorem provers ([7]). We also give a comparison of our results with the results obtained by, the resolution-based theorem provers *PCPROVE* and *OTTER* and by the tableau-based theorem provers of M. Fitting and S. Reeves. Short discussions of these theorem provers accompany the test results. For more elaborate discussions the reader is referred to [6].

Abstract. Representation systems based on inheritance networks are founded on the hierarchical structure of knowledge. Such representation is composed of a set of objects and a set of *is-a* links between nodes. Objects are generally defined by means of a set of properties. An inheritance mechanism enables us to share properties across the hierarchy, called an inheritance graph. It is often difficult, even impossible to define classes by means of a set of necessary and sufficient conditions. For this reason, exceptions must be allowed and they induce nonmonotonic reasoning. Many researchers have used default logic to give them formal semantics and to define sound inferences. In this paper, we propose a survey of the different models of nonmonotonic inheritance systems by means of default logic. A comparison between default theories and inheritance mechanisms is made. In conclusion, the ability of default logic to take some inheritance mechanisms into account is discussed.

Abstract. Ulam asked what is the minimum number of yes-no questions necessary to find an unknown number in the search space $[1, ..., 2^n]$, if up to l of the answers may be erroneous. The solutions to this problem provide optimal *adaptive 1* error correcting codes. Traditional, non-adaptive 1 error correcting codes correspond to the particular case when all questions are formulated before all answers, We show that answers in Ulam's game obey the $(1 + 2)$-valued logic of Łukasiewicz. Since approximately finite-dimensional (AF) C^*-algebras can be interpreted in the infinite-valued sentential calculus, we discuss the relationship between game-theoretic notions

and their C^*-algebritic counterparts. We describe the correspondence between continuous trace AF C^*-algebras, and Ulam games with separable Boolean search space S, whose questions are the clopen subspaces of S. We also show that these games correspond to finite products of countable Post MV algebras, as well as to countable lattice-ordered Specker groups with strong unit.

DECIDING CLAUSE CLASSES BY SEMANTIC CLASH RESOLUTION

Alexander Leitsch *163-182*

Abstract. It is investigated, how semantic clash resolution can be used to decide some classes of clause sets. Because semantic clash resolution is complete, the termination of the resolution procedure on a class Γ gives a decision procedure for Γ. Besides generalizing earlier results we investigate the relation between termination and clause complexity. For this purpose we define the general concept of atom complexity measure and show some general results about termination In terms of such measures. Moreover, rather than using fixed resolution refinements we define an algorithmic generator for decision procedures, which constructs appropriate semantic refinements out of the syntactical structure of the clause sets. This method is applied to the Bernays - Schönfinkel class, where It gives an efficient (resolution) decision procedure.

SPATIAL MENTAL MODELS IN COGNITIVE SYSTEMS

Su-Shing Chen *183-192*

Abstract. This paper is concerned with a mathematical foundation of representation issues and reasoning schemes of spatial information and knowledge. Spatial mental models in cognitive systems using a hybrid symbolic AI and highly parallel (e.g., neural networks or connectionism) framework of deductive and inductive reasoning in spatial domains will be constructed. The framework for "induction" of Holland, Holyoak, Nisbett, and Thagard was proposed as the classifer system approach to inferential and learning processes in biological cognitive systems which remedies the inadequacies of the classical logical approaches to induction: formal inductive logics. In this paper, we use a hybrid symbolic and highly parallel approach instead of the classifier system only.

SELECTED ALGORITHMS OF MACHINE LEARNING FROM EXAMPLES

Jerzy W. Grzymala-Busse *193-207*

Abstract. This paper presents and compares two algorithms of machine learning from examples, ID3 and AQ, and one recent algorithm from the same class, called LEM2. All three algorithms are illustrated using the same example. Production rules induced by these algorithms from the well-known Small Soybean Database are presented. Finally, some advantages and disadvantages of these algorithms are shown.

APPLICATION OF A ROUGH SET-BASED INDUCTIVE LEARNING SYSTEM

Michael Hadjimichael, Anita Wasilewska *209-220*

Abstract. We present here an application of Rough Set formalism to Machine Learning. The resulting Inductive Learning algorithm is described, and its application to a set of real data is

examined. The data consists of a survey of voter preferences taken during the 1988 presidential election in the U.S.A. Results include an analysis of the predictive accuracy of the generated rules, and an analysis of the semantic content of the rules.

LEARNING RELATIONS: AN EVALUATION OF SEARCH STRATEGIES

Cristina Baroglio, Marco Botta, Attilio Giordana

Abstract. Inducing concept descriptions in first order logic is inherently a complex task; then, heuristics are needed to keep the problem to manageable size. In this paper we explore the effect of alternative search strategies, including the use of information gain and of a-priori knowledge, on the quality of the acquired relations, intended as the ability to reconstruct the rule used to generate the examples. To this aim, an artificial domain has been created, in which the experimental conditions can be kept under control, the ”solution” of the learning problem is known and a perfect theory is available. Another investigated aspect is the impact of more complex description languages, such as, for instance, including numerical quantifiers. The results show that the information gain criterion is too greedy to be useful when the concepts have a complex internal structure; however, this drawback is more or less shared with any purely statistical evaluation criterion. The addition of parts of the available domain theory increases the obtained performance level. Similar results have been previously obtained on a number of real applications and of test-cases taken from standard machine learning data bases.

SOME RECENT DEVELOPMENTS IN THE REPRESENTATION AND PROCESSING OF KNOWLEDGE

Michael M. Richter

RECOGNIZABLE AND RATIONAL SUBSETS OF ALGEBRAS

Magnus Steinby

Abstract. In this paper recognizable and rational subsets of general algebras of finite type are studied. These are natural generalizations of the familiar notions originally defined for monoids, but they also include as special cases recognizable and rational tree languages, for example. Some closure properties of the families of recognizable and rational subsets are studied. The operations considered seem to be intimately connected with the recognizability and rationality properties of subsets, and they can also be defined for subsets of abstract algebras, regardless of the nature of the elements. The last part of the paper is devoted to generalized forms of Kleene’s theorem and related results.

AN ABSTRACT STRATEGY FOR TRANSFORMING LOGIC PROGRAMS

Maurizio Proietti, Alberto Pettorossi

Abstract. We study the problem of automating some development techniques for logic programs. These techniques are based on the application of semantics preserving transformation rules which are driven by strategies. We propose an *abstract strategy* which is parametrized by three mathematical functions called *definition-folding, selection*, and *replacement*.

Once these three functions are supplied, our abstract strategy becomes a concrete one which can be used during program development for driving the application of the Defmition, Folding, Unfolding, and Goal Replacement Rules.

We show that the definition-folding function can be determined in an automatic way from the description of the syntactic properties of the program we wish to derive.

We also show through some examples that many program derivation strategies described in the literature, such as the methodology for eliminating unnecessary variables, the tupling strategy, the partial deduction techniques, and the promotion strategy, can be viewed as particular instances of ow abstract strategy.

DEFINABILITY OF ARITHMETIC OPERATIONS FROM THE ORDER AND A RANDOM RELATION

Ivan Korec *287-296*

Abstract. For almost all binary relations $\mathbf{R} \subseteq \mathrm{N}^2$ Na the addition and multiplication on the set N of nonnegative integers (and hence all arithmetical relations) are first order definable in the structure (N; $\leq$, **R**). The defining formulae can be chosen independently on **R** and the words "for almost all" mean "with probability 1" by a very natural probability measure.

LOGIC AND ARTIFICIAL INTELLIGENCE: A NEW SYNTHESIS?

Donald Perlis *297-305*

Abstract. There have been two major efforts at a synthesis of logic and artificial intelligence, one building on the other. I will review these and argue that yet another addition is needed.

MODAL FRAME CLASSES REVISITED

Johan Van Benthem *307-317*

Abstract. We re-analyze the original algebraic proof of the Goldblatt-Thomason theorem characterizing modally definable frame classes, providing an alternative model-theoretic argument. The analysis also provides a more general perspective on the use of algebraic versus model-theoretic methods in Modal Logic.

ON THE ALGEBRAIZATION OF SOME GENTZEN SYSTEMS

Jordi Rebagliato and Ventura Verdú *319-338*

Abstract. In this paper we study the algebraization of two Gentzen systems, both of them generating the implication-less fragment of the intuitionistic propositional calculus. We prove that they are algebraizable, the variety of pseudocomplemented distributive lattices being an equivalent algebraic semantics for them, in the sense that their Gentzen deduction and the equational deduction over this variety are interpretable in one another, these interpretations being essentially inverse to one another. As a consequence, the consistent deductive systems that satisfy the properties of Conjunction, Disjunction and Pseudo-Reductio ad Absurdum are described by giving apropriate Gentzen systems for them. All these Gentzen systems are algebraizable, the subvarieties of

the variety of pseudocomplemented distributive lattices being their equivalent algebraic semantics respectively.

Finally we give a Gentzen system for the conjunction and disjunction fragment of the classical propositional calculus, prove that the variety of distributive lattices is an equivalent algebraic semantics fore it and give a Gentzen system, weaker than the latter, the variety of lattices being an equivalent algebraic semantics for it.

INITIAL AND QUASIINITIAL MODELS OF THEORIES

Hugo Volger *339-362*

Abstract. We have shown earlier that the theories which uniformly admit initial families of models resp. disjoint initial families of models reap. initial models are exactly those which are closed under equalizers reap. equalizers and pullbacics (= connected limits) reap. equalizers and products (= arbitrary limits). In addition, syntactical characterizations had been given.

To obtain an analogous result for pullbacks we have to weaken the initiality notions. Weakening the uniqueness of the morphism to a uniqueness up to an isomorphism we arrive at the corresponding quasiinitiality notions. As a new result we shall show that the theories which uniformly admit disjoint quasi-initial families of models reap. quasiinitial models are exactly those which are closed under pullbacks reap. pullbacks and products (= arbitrary limits). As a tool we use a localized version of initiality which was suggested by J. Adamek. Using our result Hébert recently obtained a syntactical characterization for the case of disjoint quasiinitial families. In the case of quasiinitial families the characterization by means of certain class of limit constructions is still an open problem.