Computational Aspects of Tomographic and Neuroscientific Problems

Preface

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1. Introduction

This special issue of Fundamenta Informaticae is based on the Meeting on Tomography and Applications that took place from May 15 to May 17, 2017 at the Department of Mathematics of Politecnico di Milano. The five selected papers went through a thorough refereeing process. They provide a good insight into both the theory and applications of the current research in tomography and neuroscience.

In the framework of discrete tomography, where one considers volumes composed of several materials, a new segmentation method is presented in the paper by H.Der Sarkissian, N. Viganò and K.J. Batenburg. The proposed algorithm requires only two FBP operations and can work with moderately low number of projections.

The second paper, by E. Duchi, V. Guerrini, and S. Rinaldi, deals with patterns avoiding permutations, which are motivated by data structuring problems in computer science and lead to challenging combinatorial problems.

A new algorithm for super-resolution reconstruction in the context of deep brain stimulation surgery is presented in the paper by F. Feschet and J.J. Lemaire. In particular, the denoising method, that exploits and adapts known approaches, leads to promising quality reconstruction which allows unambiguous segmentation of the deep brain to be used in surgery.

The two remaining papers deal with neuroimaging and network models. Indeed, for the first time, the 11th edition of the event hosted talks in the field of Neuroscience which exploits methodologies such as diffusion MRI and PET for imaging and benefits from various techniques of the multidisciplinary area of tomography.

The first of these papers, by M. Mancini and M. Cercignani, is a review of the current use of network modelling in neuroimaging, clarifying the underlying mathematical concepts and the related methodological choices, outlining the current issues, and discussing perspectives for the near future.

The second paper, by P. Dulio, P. Finotelli, A. Frosini, E. Pergola, and A. Presenti, deals with a recent mathematical model from the neuroscience literature, which advocates the use of a weighted graph in the study of the functional activity involving brain regions. The model, which exploits different parameters related to the brain network, is applied to and tested on a synthetic data set simulating the active cerebral areas at the resting state.

The Meeting on Tomography and Applications, taking place since 2007, represents by now an established international meeting forum for the tomographic research community. It attracts researchers from many countries and provides a friendly platform for direct contacts which promote scientific collaboration. The meeting is genuinely interdisciplinary and provides an opportunity of sharing ideas and discussions from various viewpoints, thanks to the participants representing a broad range of expertise ranging from mathematics, computer science, engineering, physics to applied aspects of medicine and biology. In this context, it was very natural to widen the scope of this edition of the meeting to include also the talks from neuroscience.

The remainder of this preface consists of two parts: the first one presents the contents of the invited lectures, and the second one provides an overview of the contributions to the 11th edition of this meeting.

2. Outline of the talks presented during the Meeting on Tomography and Applications, May 15-17, 2017, Mathematics Department, Politecnico di Milano.

The program of the meeting contained the following lectures.

INVITED LECTURES

- Mara Cercignani, Brighton and Sussex Medical School

  Dreams and Reality of measuring brain connectivity in vivo with MRI.

  ABSTRACT The human brain can be described as a complex network, formed by spatially distributed, but functionally linked regions that continuously share information with each other. This arrangement ensures efficiency and resilience to damage. In this view, the characterization of brain connectivity is necessary to increase our understanding of how functional brain states emerge from their underlying structural substrate and how neurons and neural networks process information. Magnetic Resonance Imaging (MRI) offers a range of technique that enables the measurement of both, functional and structural connectivity. In this context, functional connectivity is defined as a correlation between remote neurophysiological events in temporal domain, while structural or anatomical connectivity refers to the physical pathways that connect the "nodes of the network", i.e., the main white matter tracts of the brain. Characterising these properties non-invasively and defining the so-called "human connectome" has been the target of numerous efforts. This talk will provide an overview of the methods used for mapping brain connections, with a specific focus on the combination of structural and functional information. Moreover, we will discuss some of the limitations and the pitfalls of these techniques.

- Bernadette Hahn, University of Würzburg

  Challenges in dynamic tomography.

  ABSTRACT Image reconstruction in standard tomography (CT, MRI, etc.) is well understood if the object under investigation is stationary during the data acquisition. However, this assumption is violated in many medical and industrial applications, e.g., due to patient and organ motion or while imaging fluid flow. Consequently, standard reconstruction techniques lead to motion artefacts in the computed images, e.g., blurring, ghosting, etc., which can significantly impede a

reliable diagnostics. For this reason, the reconstruction method has to take the time-dependency into account. The mathematical model of this dynamic problem reveals that additional information, for example about the motion, is required. In this talk, we present the reconstruction algorithms that compensate for the objects deformation. Further, the effects of the dynamic behavior on the quality of the reconstructed images are discussed.

- Ville Kolehmainen, University of Eastern, Finland

Image reconstruction in limited data X-ray tomography.

ABSTRACT Image reconstruction in x-ray tomography with limited measurement data is considered. The Bayesian formulation of the problem is discussed and a few case studies are considered from medical imaging applications. A systematic approach, called S-curve, for the selection of the prior parameter for sparsity promoting prior models is also discussed.

CONTRIBUTED TALKS

- Mauro Bracconi, Politecnico di Milano

Micro-computed tomography for the development of virtual models of open-cell foams.

ABSTRACT Structured reactors are widely acknowledged to be at the hearth of process intensification, one of the most challenging and trending topics in chemical engineering [1]. In this view, honeycomb monoliths, periodic open cellular structures (POCs), open-cell foams are candidates for next generation packing in fixed bed reactors. Open-cell foams cellular materials made of interconnected solid struts which enclose void regions, are recently receiving growing attention as innovative catalyst supports, particularly for processes strongly limited by mass and heat transfer. The description of the geometrical and transport properties in such structures is not trivial due to their complex tridimensional structure, resulting in a poor description of those features by means of the current engineering correlations. Hence, a fundamental analysis of open-cell foams is needed to investigate their properties. In this respect, Computational Fluid Dynamics (CFD) is a valuable tool to improve the understanding of open cell foams, being able to offer a deep insight in the complex flow field within the random tridimensional foam matrix. A CFD analysis requires however the accurate description of the foam microstructural geometry. Currently, digital reconstruction of the foam geometry is carried out by means of micro-computed tomography (CT) or magnetic resonance imaging (MRI). Those methodologies generate a faithful reproduction of the microstructure enabling further analysis of the geometrical and transport properties. However, these techniques are expensive and time-consuming preventing large scale screening and CT-aided design of these structures. To overcome this limitation, we have proposed a Voronoi-based methodology for the virtual reconstruction of these structures based on a few pieces of geometrical information [2]. In this respect, tomographic reconstructions of real foams played a key role in the development of the model. The deep insight in the structure helped to define both the topological requirements of the virtual reconstruction and the meaningful geometrical parameters adopted as input of the procedure. The geometrical features predicted by means of the Voronoi-based reconstruction have been validated against the results of the tomographic reconstruction resulting in an accurate reproduction of the topology and specific surface area. The transport properties evaluated through CFD simulations on both the virtual and tomographic reconstruction showed the capability of our procedure in properly

modeling the interaction between the fluid flow and the structure. In particular, a good agreement with experimental pressure drops ensures the capability of the reconstructed sample to reproduce the real foams behavior. Micro-computed tomography has been a valuable tool for the development of a virtual methodology able to faithfully reconstruct the complex structure of open-cell foams by allowing a deep investigation of the real foam geometry and by enabling the validation of the reconstructed structure by the comparison with tomographic measurements.

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• Chiara Carminati, Paul Scherrer Institut, Switzerland

MuhRec – a reconstruction tool for neutron and x-ray tomography.

ABSTRACT The aim of this contribution is to present the MuhRec software, which is a framework for computed tomography reconstruction based on filtered back projection. MuhRec is a multi platform non-commercial software for computed tomography (CT) reconstruction that has been developed at the Paul Scherrer Institut to support research and users at the neutron imaging beamlines [1]. Its development is ongoing and it was chosen in the framework of the European project SINE2020 (www.sine2020.eu, [Accessed 27 April 2017]), to provide to the neutron imaging community a general tool for tomographic image reconstruction. In this framework, open source release of the software is also foreseen during 2017. The software is able to load raw projection data and reference images of different formats (e.g. series of fits, tiff and NeXus file). From these, the reconstruction engine is designed to allow for a flexible configuration of an arbitrary set of pre-processing modules followed by one back projection module. For neutron imaging experiments, the most common configuration includes: i) computation of attenuation images from raw data with normalization to reference images, ii) spot cleanings, iii) ring cleanings, iv) projection filters, and v) back projections for parallel beam geometry. Its modular structure enables the extension with new reconstruction algorithms or pre-processing modules. Among these, implementations of the wavelet-based ring cleaning filter [2] and the adaptive filter [3] have been added. Recent efforts were also focused on supporting cone-beam geometry. This is due to the fact that the neutron beam is slightly divergent resulting in non negligible cone beam effect in some experiments. Furthermore, there is an increasing number of second modality installation using X-rays on neutron imaging beamlines world-wide. The cone beam reconstructor is thus required to reconstruct the data from both modalities. The CBCT reconstruction option uses the Feldkamp-Davis-Kress (FDK) algorithm [4] that was derived from the open source library plastimatch (www.plastimatch.org, [Accessed 27 April 2017]) implementation and adapted to work within the MuhRec environment. The GUI was accordingly modified to include the CBCT parameters that describe the geometrical relationship between the source, the detector, and the sample: source to object distance (SOD), source to detector distance (SDD) and the piercing point, i.e., the point on the detector closest to the source. Methods for automatic computation of projection matrices for circular orbit and to estimate the piercing

point position within the image position by processing open beam images were implemented. Both command line and GUI modes are available to facilitate users to set up the reconstructor and execute the processing. Examples of reconstruction with typical pre-processing pipeline will be shown for both neutron and x-ray tomography.

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- Estefanía Cuenca, Politecnico di Milano

  Characterization of self-healing concrete by X-ray microtomography.

  ABSTRACT Self-healing strategies are regarded as a promising solution to reduce the high maintenance and repair cost of concrete infrastructures. In the present research, focus is posed on autogenous and engineered self-healing in steel fiber reinforced concrete (SFRC). The tensile behavior of the two different mixes, differing by the presence of the crystalline admixtures, has been investigated after a series of precracking and a water immersion conditioning cycle. Specimens were first pre-cracked up to a crack width of 0.25mm and then they were healed for three months in water. To assess microstructure evolution during healing, recourse is made to high resolution X-ray computed microtomography (X-ray microCT), to be repeated before and after water immersion. During recent years, X-ray microCT has been widely applied as a nondestructive technique allowing for a micrometer resolution within the bulk [1, 2, 3, 4, 5, 6]. As a novelty, herein the CT scan has been used to determine the position and orientation of the steel fibers in SFRC specimens, to analyze diffused damage and discrete fractures within the SFRC specimen after the pre-crack test. Further developments will concern the assessment of porous distribution within the concrete matrix and their connection degree.

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- Arjan den Dekker, University of Antwerp

Super-resolution reconstruction of magnetic resonance images.

ABSTRACT Magnetic resonance imaging (MRI) suffers from an inherent trade-off between spatial resolution, acquisition time, and signal-to-noise ratio (SNR). In clinical practice, the direct acquisition of 3-D isotropic high-resolution (HR) images is not feasible since it results either in unacceptably long acquisition times or a poor SNR. As a consequence, most MRI studies settle for either anisotropic or isotropic low resolution (LR) images. Fortunately, recent work has shown that super-resolution reconstruction (SRR) methods are able to improve the trade-off mentioned above. SRR methods reconstruct an HR 3-D isotropic image from a set of multi-slice 3-D anisotropic images with a low through-plane resolution, where each LR image is sampled in a different fashion. Previously, our group (Vision Lab, University of Antwerp) proposed a generic SRR method for anatomical MRI. More recently, this methodology was successfully extended to diffusion MRI and relaxometry. It is demonstrated in this talk that experimental results show a significant increase in spatial resolution while preserving a high SNR for a fixed acquisition time.

- Fabien Feschet, Université d'Auvergne Clermont

On some computational questions raised by Deep Brain Stimulation.

ABSTRACT Deep Brain Stimulation (DBS) is a surgery consisting in the implantation of electrodes in the deep nuclei of the pathological brain. To be performed, it requires the targeting of the deep nuclei and structures. This localization procedure could be either indirect with the use of atlases and co-registration or direct using dedicated MRI sequence. In Clermont-Ferrand (France), we have developed a dedicated acquisition called WAIR (White Matter Inversion Recovery) which permits direct targeting. However, the acquisition is quite long in clinical practices and the SNR is high but irregular. We introduce the denoising problem on WAIR imaging and the build of a reference atlas for the non pathological brain. This problem is shown to correspond to a reconstruction problem in irregular space acquisitions with several angles for which a geometric solution is presented. The focus of the second part of the talk is on the measurement of the effect of DBS in the specific case of brain trauma with severely damaged brain. This question is solved using a voxel approach in PET acquisitions. It is also shown that a statistical characterization of PET variations is a key point to obtain more standardized results than an atlas based method currently used.

- Rodolfo A. Fiorini, Politecnico di Milano

Universal Lossless data compression computational effectiveness comparison.

ABSTRACT Advanced instrumentation, dealing with nano-scale technology at the current edge of human scientific enquiry, like X-Ray CT, generates an enormous quantity of data from single experiment. The very best modern lossless data compression algorithms use standard approaches and are unable to match high end requirements for mission critical application with

full information conservation (a few pixels may vary by com/decom processing). The universal modular arithmetic approach to achieve true lossless com/decom and to overcome those constraints is presented. To test practical implementation performance and effectiveness, an example on computational imaging is presented, benchmarked to standard well-known lossless compression techniques, and critically discussed.

- Yan Gerard, Université d'Auvergne Clermont

About the complexity of reconstructing convex lattice sets from horizontal and vertical X-rays.

ABSTRACT We consider the problem of reconstructing a lattice set from its horizontal and vertical X-rays. Without any topological or geometrical constraint, it is known since the works of Gale and Ryser in 1957, that the problem can be solved in polynomial time. With constraints of connectivity (such sets are usually called polyominoes) or horizontal + vertical convexity, the problem becomes NP-hard (by Gerhard Woeginger in 2001). A first polynomial algorithm has been presented in 1996 by Elena Barcucci, Alberto Del Lungo, Maurice Nivat and Renzo Pinzani for the reconstruction of HV-convex polyominoes from their orthogonal projections. Similar result has been obtained for the so-called Q-convex sets under some assumptions on the sets of directions of Q-convexity. This very nice result from 2003 due to Sara Brunetti and Alain Daurat benefits from Darboux theorem and Richard Gardner and Peter Gritzmann's works about the reconstruction of convex sets. Recall that the directions of the Q-convexity should be the directions of the X-rays and satisfy the condition of unicity provided by Gardner and Gritzmann in 1999. It excludes the pair of horizontal and vertical directions. We could use this algorithm to try to reconstruct convex lattice sets - in the classical meaning of digital or discrete geometry, a convex lattice set is equal to the intersection of its convex hull with the lattice - but there would be no guarantee to obtain a convex solution, just a Q-convex one which is a weaker constraint. It turns out that a basic question remains open : determine the complexity of the reconstruction of a convex lattice set from only its horizontal and vertical X-rays. The purpose of the talk is to deal with this quite old challenge.

- Sara El Hadji, Politecnico di Milano

ART 3.5D a novel algorithm, to label arteries and veins from 3D angiography.

ABSTRACT Several neurosurgical procedures, such as Artero Venous Malformations (AVMs), aneurysm embolizations, and StereoElectroEncephaloGraphy (SEEG) require accurate reconstruction of the cerebral vascular tree, as well as the classification of arteries and veins, to increase the safety of the intervention. We propose a novel approach attempting to recover the dynamic information from standard Contrast Enhanced Cone Beam Computed Tomography (CE-CBCT) scans. The algorithm proposed by our team is called ART 3.5 D. It is a novel algorithm based on the post- processing of both the angiogram and the raw data of a standard Digital Subtraction Angiography from a CBCT (DSA- CBCT) allowing arteries and veins segmentation and labeling.

- Lajos Hajdu, University of Debrecen

Correction of noise in discrete tomography.

ABSTRACT In this talk, a new approach is presented to correct the noisy measurement in discrete tomography. Since the method relies on the structure of dependencies among the line sums, related results of Van Dalen, Batenburg, Stolk, Tijdeman and Hajdu are briefly recalled.

A comparison between the new approach and an earlier method of Tijdeman and Hajdu is also done. The presented results are joint with R. Tijdeman.

- Jan Kuske, Heidelberg University

  Performance bounds for co-/sparse box constrained signal recovery.

  ABSTRACT The recovery of structured signals from few linear measurements is a central point in both compressed sensing (CS) and discrete tomography. In CS the signal structure is described by means of a low complexity model like, e.g., co-sparsity. The CS theory shows that any signal/image can be undersampled at a rate dependent on its intrinsic complexity. Moreover, in such undersampling regimes the signal can be recovered by sparsity promoting convex regularization like, e.g., ℓ_1 or total variation (TV) minimization. Precise relations between many low complexity measures and the sufficient number of random measurements are known for many sparsity promoting norms. However, a rigorous description of the undersampling rate for TV seminorms is still lacking. We address this issue by: a) providing dual certificates for testing uniqueness of a given cosparse signal with bounded signal values, b) approximating the undersampling rates via the statistical dimension of the TV descent cone, and c) showing empirically that the provided rates also hold for tomographic measurements.

- Viktor Nikitin, Lund University, Sweden

  Fast log-polar based reconstruction of parallel-beam tomography data.

  ABSTRACT A fast log-polar-based method is proposed for parallel-beam tomography data reconstruction with reduced computational complexity of $O(N^2 log N)$. The method is based on the fact that the projection and the back-projection operators, i.e., the main computational parts for constructing reconstruction schemes, can both be expressed as convolutions in log-polar coordinates. Hence they can be rapidly evaluated in terms of FFT if data is re-sampled at log-polar coordinates. Sinograms are typically given on an equally spaced grid in polar coordinates, and reconstructions are represented in Cartesian coordinates. Therefore, in addition to FFT, several steps of interpolation have to be conducted in order to apply the transforms by means of convolutions. However, in comparison to the interpolation conducted in Fourier-based gridding methods, the interpolation performed in the polar and image domains will typically deal with data that are substantially less oscillatory. Reasonable reconstruction results can thus be expected using interpolation schemes of moderate order. It also provides better control over the artifacts that can appear due to measurement errors, since the interpolation errors are kept local. The log-polar based reconstruction method is implemented at the MAX IV Laboratory on GPUs with the processing times faster than analogues. Therefore, we foresee a high impact in the reconstructing of large (time-resolved) tomographic datasets.

- Luca Ratti, Politecnico di Milano

  Reconstruction of discontinuous coefficients in a semilinear parabolic equation: an inverse problem motivated by cardiac electrophysiology.

  ABSTRACT In this talk we tackle the problem of identifying the location of small regions in which the coefficients of a semilinear parabolic equation are altered with respect to a reference case, using the knowledge of the solution of the equation on the boundary of the domain. The problem is motivated by the long-term task of identifying ischemic regions inside the cardiac tissue via non-invasive measurements of the electrical potential. The formulation shares

some features with the Electrical Impedance Tomography problem, but shows many further difficulties, like the non-linearity of the direct problem and the small number of measurement at disposal. The well-posedness of the direct problem is discussed and a one-shot reconstruction strategy for the inverse problem is reported - it is based on the topological gradient of a suitable cost functional and exploits an asymptotic expansion of the boundary data in presence of small inclusions. Numerical results obtained in several test cases are shown and the feasibility and the stability of the technique is discussed. This is a joint work with E. Beretta, C. Cavaterra, M.C. Cerutti and A. Manzoni

- Lama Tarsissi, Universit Savoie Mont Blanc Chambéry

Properties of a convex path.

ABSTRACT In this talk, first steps for the reconstruction of digitally convex polyomino P are presented. The boundary of a digitally convex polyomino P is a convex path that is a word whose Lyndòn factorization is composed by Christoffel words. In another words, the factorization is made of Christoffel words with a decreasing order of slopes. Based on this idea, some characteristics of Lyndon and Christoffel words are recalled and some of their properties explored. Those properties allow us to perturb the path in order to maintain the convexity of the polyomino P.

- Vincent Van Nieuwenhove, University of Antwerp

A 4D reconstruction algorithm for cerebral perfusion CT.

ABSTRACT Cerebral perfusion x-ray computed tomography (PCT) is a powerful tool for non-invasive imaging of hemodynamic information throughout the brain. Conventional PCT requires the brain to be imaged multiple times during the perfusion process, and hence radiation dose is a major concern. A PCT reconstruction algorithm is proporsed that allows for lowering the dose while maintaining a high quality of the perfusion maps. It relies on an accurate estimation of the arterial input function (AIF), which in turn depends on the quality of the attenuation curves in the arterial region. The method accurately models the attenuation curves inside the vessel and arterial regions and optimizes its shape directly based on the acquired x-ray projection data. The proposed algorithm is validated with simulation and real clinical experiments. Quantitative and qualitative results show that vessel and arterial attenuation curves can be accurately estimated from only few x-ray projections.

- Nicola Viganò, ESRF Grenoble

Light-field imaging from a tomography perspective.

ABSTRACT During the last decade, "Light-field Imaging" has been an emerging technology that allows to record directional information about the captured light through what is called "Plenoptic Photography". As a result, it is possible to use light-field images (acquired with plenoptic cameras) to perform image refocusing after acquisition, and create extended depth-of-field images, which present different scenes from different distances from the camera, all well in focus. Thanks to these properties, plenoptic imaging is being investigated for the possibility to retrieve depth information from one single acquisition. In this talk we introduce and develop the interesting similarity between plenoptic imaging and Computed Tomography, with consequent use of the traditional formalism and tools derived from tomography, in order to extract higher quality information from light-field images.

- Laurent Vuillon, Université de Savoie

  Reconstruction of digitally convex polyominoes.

ABSTRACT In this talk, the tomographical aspects of digitally convex polyminoes are studied. A polyomino P is said digitally convex if its convex hull contains no integer point outside P. A nice result by Brlek, Lachaud, Provençal and Reutenauer provides a link between digitally convex notion and combinatorics on words. Indeed, a polyomino P is described by its boundary word b. The boundary word b could be divided in 4 monotone paths and one can compute the Lyndon factorization of each path. If each of these factorizations contains only Christoffel words, then one has a digitally convex polyomino. These notions are used to propose an algorithm to reconstruct (if possible) a digitally convex polyomino from horizontal and vertical vectors of projection.

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