

## GUEST EDITOR'S FORWARD

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This special issue of *Fundamenta Informaticae* contains seven papers on modal logics and their applications in knowledge representation. Most of the papers discuss nonmonotonic modal logics, especially those related to autoepistemic logic of Moore and Levesque. Boutilier studies a generalization of autoepistemic logic which allows him to introduce a gradation of beliefs and talk about the degree of entrenchment. Schwarz introduces an attractive alternative to autoepistemic logic, the logic of reflexive knowledge. Lakemeyer investigates a variant of first-order autoepistemic logic in which objective formulas uniquely determine the set of agents beliefs. Marek and Truszczyński study connections between autoepistemic and default logics. Finally, the papers by Eiter and Gottlob, and by Niemelä contain a discussion of complexity issues in autoepistemic reasoning. The only paper not directly related to autoepistemic reasoning is the paper by Fitting, where a certain multivalued logic is proposed to formalize reasonings of several experts some of whom may dominate others.

The papers provide an ample evidence that modal logics are quickly becoming a standard tool to study problems in Artificial Intelligence and often offer elegant solutions to a wide range of problems from belief revision to multi-agent reasoning. We hope that this issue will raise reader's interest in the area of modal logics and their applications, and will contribute to its further development.

I will now briefly discuss each of the papers in this issue. The first of them, by Boutilier, deals with the notion of entrenchment of beliefs in logics of self-knowledge. Default and autoepistemic logics do not have any means for gradation of beliefs. Hence, all beliefs in belief sets constructed in these logics are regarded as equally plausible. Boutilier argues that this is a serious drawback of "classical" nonmonotonic logics. He proposes a formalism which alleviates this problem. The logic of Boutilier is based on a class of Kripke models. The worlds in the models are arranged into totally ordered collections of clusters (hence, a close relation to the logic S4.3). In addition, it is required that (1) the frame of a model has a terminal cluster, and that (2) each propositional valuation appears in a model. Formulas true in the terminal cluster of a model are exactly those the agent believes in. The difference between Boutilier's approach and autoepistemic logic of "only believing" by Levesque is that while Levesque allows only two clusters (hence, a formula is either believed or disbelieved) Boutilier allows arbitrarily many. This richer structure of models allows him to specify the hierarchy on beliefs according to agent's willingness to give them up. The paper contains a thorough study of the properties of this hierarchy.

Eiter and Gottlob study the complexity of brave and cautious reasoning with parsimonious and moderately grounded expansions. They prove that the problem

to decide whether a formula belongs to all parsimonious expansions is  $\Pi_3^P$ -complete and the problems to decide whether a formula belongs to at least one parsimonious expansion is  $\Sigma_3^P$ -complete. The corresponding problems for moderately grounded expansions are also proved to reside on the third level of the polynomial hierarchy. Eiter and Gottlob prove that under some syntactic restrictions on input theories the complexity of reasoning with moderately grounded expansions moves down to the second level of the polynomial hierarchy, while the same restrictions have no effect on the complexity in the case of reasoning with parsimonious expansions.

The paper by Fitting is concerned with the following problem. Imagine a collection of experts each with its own Kripke model for establishing the truth value of a formula in a world of a model. Assume, in addition, that there is a binary dominance relation defined on the set of experts which forces an expert to regard a formula as true if at least one of the experts that dominates it regards the formula as true. Can such situation be described in terms of a single Kripke model? Fitting shows it is possible. The idea is to use multivalued Kripke models rather than two-valued ones. In fact, the truth values in the model have to form a Heyting algebra. Fitting introduces the notion of a Heyting-valued Kripke model and shows that it is equivalent to a system with several Kripke models (experts) ordered by a dominance relation. For every Heyting algebra  $\mathcal{T}$  he also provides an axiomatization of the set of all formulas true in all  $\mathcal{T}$ -valued Kripke models.

Lakemeyer discusses in his paper issues related to reasoning by agents with perfect introspection. It is noted that such agents may have incomplete information about the real world but that they must have complete knowledge about their own beliefs. Consequently, it follows that beliefs of agents with perfect introspection should be completely determined by objective truths about the world. Propositional logics of belief, such as autoepistemic logic, have this property. Situation gets complicated in the first-order case. For example, in the well-known logic  $KL$  by Levesque beliefs are not uniquely determined by the objective truths. Lakemeyer proposes a weakening of the logic  $KL$ , the logic  $KL^-$  and shows that it indeed has the desired property.

Marek and Truszczynski present results on the relationship between default logic and two modal logics: the nonmonotonic modal logic  $KD45$  (which is equivalent to autoepistemic logic) and modal logic  $S5$ . They introduce the notion of a weak extension of a default theory and show that under a natural interpretation default logic with weak extensions can be regarded as a fragment of autoepistemic logic. A converse interpretation allowing us to embed autoepistemic logic in default logic is also given. The same interpretation which relates weak extensions and stable expansions is shown to link sets closed under a default theory with modal logic  $S5$ . In the paper this connection is exploited to provide some interesting complexity results on reasoning with sets closed under default theories.

The paper by Niemelä is the second in this issue, besides the paper by Eiter and Gottlob, which focuses on the complexity of reasoning with propositional modal non-monotonic logics. Niemelä considers three versions of propositional autoepistemic logic. First, he proves that it is  $\Sigma_2^P$ -complete to decide whether a formula belongs to some stable expansion of a finite theory, and that it is  $\Pi_2^P$ -complete to determine if a formula is in all stable expansions. A modification of autoepistemic logic can be obtained if the notion of a stable expansion is replaced by another one. In his earlier papers Niemelä proposed two new classes of expansions: enumeration based expansions and  $L$ -hierarchical expansions. In the present paper Niemelä shows that the complexity of autoepistemic reasoning with enumeration based and  $L$ -hierarchical expansions

remains the same as in the case of stable expansions. It is also worth-mentioning that Niemelä's paper is one of few that deals with decidability and complexity issues not only in the propositional case but also in the first-order setting.

Schwarz modifies in his paper semantical considerations of Moore, which led the latter to his beautiful and highly influential definition of autoepistemic logic. Schwarz notes that autoepistemic logic is well-suited to formalize introspective reasoning about beliefs but has drawbacks as a formalism to reason about knowledge. Reinterpreting the modality as "is known", and tailoring Moore's argument to this situation, Schwarz arrives at a nonmonotonic logic with several elegant properties. First of all, his logic of *reflexive knowledge*, despite being a modal formalism, can be described entirely in terms of the propositional consequence operator. Secondly, the logic of reflexive knowledge turns out to belong to the family of modal nonmonotonic logic introduced by McDermott and Doyle. Hence, it possesses an elegant syntactic characterization. Schwarz points out that in many aspects autoepistemic and reflexive knowledge logics share similar properties. These similarities are not coincidental. Schwarz shows that each logic can be embedded faithfully in the other.

To conclude this short introduction to this special issue, let me use this opportunity to express again my gratitude to all the authors who contributed their papers and to all the referees who reviewed the submissions.