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Theory and Applications of Fractional Fourier Transform and its Variants

Preface

The Fourier transform (FT) is a mathematical operator enjoying many features and applications in almost all sciences. In particular, it enables to decompose a time signal into its fundamental frequencies similarly to a musical chord expressed by the amplitude of its constituent notes. However, FT was reported to have some shortcomings.In particular, it cannot give the local time-frequency property, which is essential for processing nonstationary signals. Therefore, a novel analysis method – Fractional Fourier Transform (FRFT) was proposed and can be thought of as a generalization of FT. The FRFT can transform a function to any intermediate domain between time and frequency, i.e., the unified time-frequency transform. In the last decade, the FRFT has been applied to various fields, including communication, encryption, optimal engineering, radiology, remote sensing, fractional calculus, fractional wavelet transform, pseudo-differential operator, pattern recognition, and image processing.

This special issue provides a forum for the dissemination of new methods and results in the field of FRFT, with special emphasis on efforts related to the engineering applications.It also accepts manuscripts related to the variants and extensions of FRFT.

We received 173 manuscripts, and 46 papers were rejected during the initial review. The rest 127 manuscripts underwent strict independent review with at least two reviewers. Finally, 32 papers are accepted. The acceptance ratio of this special issue is 18.50%.

In the paper entitled "A Comprehensive Survey on Fractional Fourier Transform", Zhang Y.D., et al. presents a comprehensive investigation of FRFT. Firstly, they provided definition of FRFT and its three discrete versions (weighted-type, sampling-type, and eigendecomposition-type). Secondly, they offered a comprehensive theoretical research and technological studies that consisted of hardware implementation, software implementation, and optimal order selection. Thirdly, they presented a survey on applications of FRFT to following fields: communication, encryption, optimal engineering, radiology, remote sensing, fractional calculus, fractional wavelet transform, pseudo-differential operator, pattern recognition, and image processing. This survey was expected to be beneficial for the researchers studying on FRFT.

In the paper entitled "Coefficient Estimate of bi-Bazileviuc Functions Associated with Fractional q-calculus Operators", Murugusundaramoorthy G., et al. introduced and investigate two new subclasses of the function class $\Sigma$of bi-univalent functions defined in the open unit disk, which are associated with fractional q-calculus operators, satisfying subordinate conditions. Furthermore, they found estimates on the Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ for functions in these new subclasses. Several (known or new) consequences of the results were also pointed out.

In the paper entitled "A New Family of the Local Fractional PDEs", Yang X.J., et al. investigated a new family of the local fractional PDEs. The linear, quasilinear, semilinear and nonlinear local fractional PDEs were presented. Furthermore, three types of the local fractional PDEs were discussed, namely, parabolic, hyperbolic and elliptic. Several examples illustrated the corresponding models in nonlinear mathematical physics.

In the paper entitled "Two-Dimensional Chebyshev Wavelet Method for Camassa-Holm Equation with Riesz", Ray S.S., et al. presented a new wavelet based method viz. Chebyshev wavelet method to compute the numerical solution of Riesz time-fractional Camassa-Holm equation. The approximate solutions of time-fractional Camassa-Holm equation thus obtained by two-dimensional Chebyshev wavelet method were compared with those obtained by analytical methods such as homotopy analysis method and variational iteration method. The present scheme was quite simple, effective and appropriate for obtaining the numerical solution of the Riesz time-fractional Camassa-Holm equation.

In the paper entitled "Solvability of a Coupled System of Fractional Differential Equations with Nonlocal and Integral Boundary Conditions", Ahmad B., et al. concerned the existence and uniqueness of solutions for a coupled system of fractional differential equations with nonlocal and integral boundary conditions. The existence and uniqueness of solutions was established by Banach's contraction principle, while the existence of solutions was derived by using Leray-Schauder's alternative. The results were explained with the aid of examples. The case of nonlocal strip conditions was also discussed.

In the paper entitled "On Fractional Neutral Integro-differential Systems with State-dependent Delay in Banach Spaces", Kailasavalli S., et al. principally involved with existence results for fractional neutral integro-differential systems with state-dependent delay in Banach spaces, based on concepts for semigroups, fractional calculus, resolvent operator and Banach contraction principle. To acquire the main results, their working concepts were that the functions deciding the equation fulfill certain Lipschitz conditions of local , which was similar to the hypotheses in Reference "Andrade F, 2016". Finally, an example was additionally offered to exhibit the achieved hypotheses.

In the paper entitled "On Analytical Approximate Solution of the Fractional Type Rosenau-Hyman Equation", Babaei A., et al. obtained an approximate solution of the fractional type Rosenau-Hyman equation according to an appropriate initial condition with the help of the reduced differential transform method (RDTM). The fractional derivatives were described in the Caputo sense. They presented the comparison of the methodology with the homotopy perturbation method and the variational iteration method. The results showed that solutions obtained by the RDTM were reliable and this method was effective for this type of nonlinear fractional partial differential equations.

In the paper entitled "Model of Thin Viscous Uid Sheet ow within the Scope of Fractional Calculus: Fractional Derivative with and No Singular Kernel", Atangana A., et al. performed a comparative analysis of a model of thin viscous fluid sheet flow between Caputo and Caputo-Fabrizio derivative with fractional order. They presented some properties of both derivatives sides by sides, and then examined the existence of the exact solution of both nonlinear equations via the fixed-point theorem.

A detailed study of the uniqueness of analysis for both models was presented. Numerical simulations were presented to access the difference between both models.

In the paper entitled "Analysis of Riccati Differential Equations within a New Fractional Derivative without Singular Kernel", Jafari H., et al. reported an analytical method for solving Riccati differential equation with a new fractional derivative. They presented numerical results of solving the fractional Riccati differential equations by using the variational iteration method and its modification. The obtained results of two methods demonstrated the efficiency and simplicity of the MVIM that gave good approximations for a larger interval.

In the paper entitled "Exp(- $\phi\eta$))-Expansion Method and Shifted Chebyshev Wavelets for Generalized Sawada-Kotera of Fractional Order", Mohyud-Din S.T., et al. studied a nonlinear generalized Sawada-Kotera equation of fractional order via the exp(-$\phi(\eta)$)–expansion method and Shifted modified Chebyshev Wavelet technique. They obtained abundant exact solutions and approximate solution of the equation. The results of the study showed that the exp(-$\phi(\eta)$)–expansion method was very effective and proficient for solving nonlinear fractional partial differential equations. The solitary wave solutions were obtained through the hyperbolic, trigonometric, exponential and rational functions. Graphical representations along with the numerical data reinforced the efficacy of the used procedure. The specified idea was very expedient for fractional PDEs, and could be extended to other physical problems. Results of the proposed methods showed an excellent conformity with the exact solution of the considered problem.

In the paper entitled "Abnormal Breast Detection in Mammogram Images by Feed-forward Neural Network trained by Jaya Algorithm", Wang S., et al. proposed a novel computer-aided diagnosis system for detecting abnormal breasts in mammogram images. First, they segmented the region-of-interest. Next, the weighted-type fractional Fourier transform (WFRFT) was employed to obtain the unified time-frequency spectrum. Third, principal component analysis (PCA) was introduced and used to reduce the spectrum to only 18 principal components. Fourth, feed-forward neural network (FNN) was utilized to generate the classifier. Finally, a novel algorithm-specific parameter free approach, Jaya, was employed to train the classifier. Their proposed "WFRFT + PCA + Jaya-FNN" achieved sensitivity of 92.26%$\pm$3.44%, specificity of 92.28%$\pm$3.58%, and accuracy of 92.27%$\pm$3.49%. The proposed CAD system was effective in detecting abnormal breasts and performs better than 5 state-of-the-art systems. Besides, Jaya was more effective in training FNN than BP, MBP, GA, SA, and PSO.

In the paper entitled "Residual Power Series Method for Fractional Diffusion Equations", Kumar S., et al. implemented a modified form of residual power series method (RPSM) to find the approximate analytical solution of a time fractional diffusion equations. The proposed method was an analytic technique based on the generalized Taylor's series formula which constructed an analytical solution in the form of a convergent series. In order to illustrate the advantages and the accuracy of the RPSM, the authors had applied the method to two different examples. In case of first example, different cases of initial conditions were considered. Finally, the solutions of the time fractional diffusion equations were investigated through graphical representation, which interpreted simplicity, accuracy and practical usefulness of the present method.

In the paper entitled "Nonoverlapping Schwarz Waveform Relaxation Algorithm for A Class of Time-Fractional Heat Equations", Wu S.L., et al. analyzed the convergence properties of the Schwarz waveform relaxation (SWR) algorithm with Robin transmission conditions (TCs) for a class of heat equations with Riemann-Liouville fractional derivative. The Robin TCs contained a free parameter, which had a significant effect on the convergence rate of the SWR algorithm, and optimizing this parameter was an important step for the convergence analysis of the SWR algorithm. By studying the monotonic properties of the convergence factor obtained by applying the Fourier transform to the error functions, they provided a reliable choice of the Robin parameter in the nonoverlapping case. Numerical results were provided, which showed that the analyzed Robin parameter resulted in satisfactory convergence rate.

In the paper entitled "On the Monotone Convergence of General Iterative Methods with Applications in Fractional Calculus", Anastassiou G.A. presented monotone convergence results for general iterative methods in order to approximate a solution of a nonlinear equation defined on a partially ordered linear topological space. The main novelty of this paper was that the operators appearing in the iterative method were not necessarily linear. This way the authors expanded of the applicability of iterative methods. Some applications were also provided from fractional calculus using Caputo and Canavati type fractional derivatives and other areas.

In the paper entitled "A New Optimization Method Based on Generalized Polynomials for Fractional Differential Equations", Heydari H., et al. presented a new optimization method based on a new class of functions, namely generalized polynomials (GPs) for solving linear and nonlinear fractional differential equations (FDEs). In the proposed method, the solution of the problem under study was expanded in terms of the GPs with fixed coefficients, free coefficients, and control parameters. The initial conditions were employed to compute the fixed coefficients. The residual function and its $||.||2$ were employed for converting the problem under consideration to an optimization one and then choosing the unknown free coefficients and control parameters optimally. As a useful result, the necessary conditions of optimality were derived as a system of nonlinear algebraic equations with unknown free coefficients and control parameters. The validity and accuracy of the approach were illustrated by some numerical examples. The obtained results showed that the proposed method was very efficient and accurate.

In the paper entitled "Pathological Brain Detection via Wavelet Packet Tsallis Entropy and Real-Coded Biogeography-based Optimization", Wang S.H., et al. proposed a novel system of pathological brain detection (PBD) system that combined wavelet packet Tsallis entropy (WPTE), feedforward neural network (FNN), and real-coded biogeography-based optimization (RCBBO). Their experiments showed the proposed "WPTE + FNN + RCBBO" approach yielded an average accuracy of 99.49% over a 255-image dataset. Thus, the WPTE + FNN + RCBBO performed better than 10 state-of-the-art approaches.

In the paper entitled "SM-Algorithms for Approximating the Variable-Order Fractional Derivative of High Order", Moghaddam B.P., et al. discussed different definitions of variable-order derivatives of high order, and they proposed accurate and robust algorithms for the approximate calculation. The proposed algorithms were based on finite difference approximations and B-spline interpolation. They compared the performance of the algorithms by experimental convergence order. Numerical examples were presented demonstrating the efficiency and accuracy of the proposed algorithms.

In the paper entitled "A New Approach for Modeling with Discrete Fractional Equations", Atici F., et al. introduced a new class of nonlinear discrete fractional equations to model tumor growth rates in mice. For the data fitting purpose, they developed a new method, which can be considered as an improved version of the partial sum method for parameter estimations. They demonstrated the goodness of fit by comparing the models with three statistical measures.

In the paper entitled "Tea Category Identification Using Wavelet Packet Entropy and Generalized Eigenvalue Proximal SVM", Wang S.H., et al. aimed to increase classification accuracy of tea-category identification (TCI) system. Their proposed methods first extracted 64 color histogram to obtain color information, and 16 wavelet packet entropy to obtain the texture information. With the aim of reducing the 80 features, principal component analysis was harnessed. The reduced features were used as input to generalized eigenvalue proximal support vector machine (GEPSVM). Winner-takes-all (WTA) was used to handle the multiclass problem. Two kernels were tested, linear kernel and Radial basis function (RBF) kernel. Ten repetitions of 10-fold stratified cross validation technique were used to estimate the out-of-sample errors. They named their method as GEPSVM + RBF + WTA and GEPSVM + WTA. The results showed that PCA reduced the 80 features to merely five with explaining 99.90% of total variance. The recall rate of GEPSVM + RBF + WTA achieved the highest overall recall rate of 97.9%. This was higher than the result of GEPSVM + WTA and other 9 state-of-the-art algorithms: back propagation neural network, RBF support vector machine, genetic neural-network, linear discriminant analysis, and fitness-scaling chaoticartificial bee colony artificial neural network.

In the paper entitled "Fractional Diffusion Equation with Spherical Symmetry and Reactive Boundary Conditions", Lenzi M.K., et al. analyzed the behavior of a system governed by a fractional diffusion equation with spherical symmetry and subjected to integro–differential boundary conditions, which can simulate sorption, desorption and reaction processes. They considered the processes defined in terms of kinetic equations that coupled the surface processes with the bulk dynamic enabling to describe scenarios where the surface modified the bulk dynamics and this may changed the behavior on surface. This problem was presented in terms of a general formulation satisfying the mass balance, and a particular application characterized by a reversible process on the surface was analyzed. For this application, they obtained exact solutions in terms of the Green function approach and evaluated the concentrations on the spherical surface and in the bulk for different processes. These results led to a rich class of scenarios, which can be related to an anomalous diffusion.

In the paper entitled "On General Solution for Fractional Dierential Equations with Not Instantaneous Impulses", Zhang X.M., et al. mainly studied a kind of fractional differential equations with not instantaneous impulses, and found the equivalent equations of the impulsive system. The obtained result discovered that there existed general solution for the impulsive system. Next, an example was given to illustrate the obtained result.

In the paper entitled "Transient Space-fractional Diffusion with a Power-law Superdiffusivity: Approximate Integral-balance Approach", Hristov J. focused on an approximate analytical solution of an initial-boundary value problem of spatial-fractional partial differential diffusion equation with Riemann Liouville fractional derivative in space. The spatial correlation of the superdiffusion coefficient as a power-law had been discussed in cases of fast and slow spatial superdiffusion. Approximate closed form solutions in terms of non-linear similarity variable were based on the integral-balance method and series expansion of the assumed parabolic profile with undefined exponent. The law of the spatial and temporal propagation of the solution was the primary issue and discussed in two cases: fast and slow superdiffussion.

In the paper entitled “Dynamics of Commodities Prices: Integer and Fractional Models”, David S., et al. examined the time series of four important agricultural commodities, namely the soybean, corn, coffee and sugar prices. Time series could exhibit long-range dependence and persistence in their observation. The long memory feature of data was a documented fact and there had been an increasing interest in studying such concepts in the perspective of economics and finance. In their work, they started by analyzing the time series of the four commodities by means of the Fractional Fourier Transform (FrFT) to unveil time-frequency patterns in the data. In a second phase, they applied Auto Regressive Integrated Moving Average (ARIMA) and Auto Regressive Fractionally Integrated Moving Average (ARFIMA) models for obtaining the spot price composition and predict future price. The ARFIMA process was a known class of long memory model, representing a generalization of the ARIMA algorithm. They compared the performances of the ARIMA and the ARFIMA models, and they showed that the ARFIMA had a superior performance for future price forecasting.

In the paper entitled “Non-differentiable Solutions for Local Fractional Nonlinear Riccati Differential Equations”, Yang X.J., et al. investigated local fractional nonlinear Riccati differential equations (LFNRDE) by transforming them into local fractional linear ordinary differential equations. The case of LFNRDE with constant coefficients was considered and non-differentiable solutions for special cases obtained.

In the paper entitled “The RC Circuit Described by Local Fractional Differential Equations”, Zhao X.H., et al. first proposed a non-differentiable resistor-capacitor circuit comprised of the capacitor and resistor in the fractal-time domain. The solution behavior of the corresponding local fractional ordinary differential equation was presented for the Mittag-Leffler decay defined on Cantor sets. The obtained results revealed the sufficiency of the local fractional calculus in the analysis of the fractal electrical systems.

In the paper entitled “Interpretation of Fractional Derivatives as Reconstruction from Sequence of Integer Derivatives”, Tarasov V.E. proposed an ”informatic” interpretation of the Riemann-Liouville and Caputo derivatives of non-integer orders as reconstruction from infinite sequence of standard derivatives of integer orders. The reconstruction was considered with respect to orders of derivatives.

In the paper entitled “An Optimization Method Based on Wavelets for Solving Multi Variable-order Fractional Differential Equations”, Heydari H. derived a new operational matrix of variable-order fractional derivative (OMV-FD) for the second kind Chebyshev wavelets (SKCWs). Moreover, a new optimization wavelet method based on SKCWs was proposed to solve multi variable-order fractional differential equations (MV-FDEs). In the proposed method, the solution of the problem under consideration was expanded in terms of SKCWs. Then, the residual function and its errors in 2-norm were employed for converting the problem under study to an optimization one, which optimally chose the unknown coefficients. Finally, the method of constrained extremum was applied, which consisted of adjoining the constraint equations obtained from the given initial conditions to the object function obtained from residual function by a set of unknown Lagrange multipliers. The main advantage of this approach was that it reduced such problems to those optimization problems, which greatly simplified them and also led to obtain a good approximate solution for them.

In the paper entitled “On the Critical Strip of the Riemann zeta Fractional Derivative”, Guariglia E., et al. believed that the α-order fractional derivative of the Dirichlet eta function $\eta^{(\alpha)}(s)$ was computed in order to investigate the behavior of the fractional derivative of the Riemann zeta function $\eta^{(\alpha)}(s)$ on the critical strip. The convergence of $\eta^{(\alpha)}(s)$ was studied. In particular, its half-plane of convergence gave the possibility to better understand the $\eta^{(\alpha)}(s)$ and its critical strip. As an application, two signal processing networks, corresponding to $\eta^{(\alpha)}(s)$ and to its Fourier transform respectively, were shortly described.

In the paper entitled "Nonlinear Degenerate Fractional Evolution Equations with Nonlocal Conditions", Derdar N., et al. investigated the unique solvability of a class of nonlinear nonlocal differential equations associated with degenerate linear operator at the fractional Caputo derivative. For the main results, they used the theory of fractional calculus and (L, p)-boundedness technique that based on the analysis of both strongly (L, p)-sectorial operators and strongly (L, p)-radial operators. The obtained results were applicable to degenerate fractional Cauchy and Showalter–Sidorov problems in Banach spaces. Finally, they gave an application described by time-fractional order Oskolkov system.

In the paper entitled "Existence Results for Fractional Evolution Systems with Riemann-Liouville Fractional Derivatives and Nonlocal Conditions", Kalamani P., et al. principally involved with existence results of Riemann-Liouville (RL) fractional neutral integro-differential systems (FNIDS) with nonlocal conditions (NLCs) in Banach spaces, based on concepts for semigroup theory, fractional calculus, Banach contraction principle and Krasnoselskii fixed point theorem (FPT). An example was offered to demonstrate the theoretical concepts.

In the paper entitled "Texture Analysis Method Based on Fractional Fourier Entropy and Fitness-scaling Adaptive Genetic Algorithm for Detecting Left-sided and Right-sided Sensorineural Hearing Loss", Wang S.H., et al. used magnetic resonance imaging to obtain the imaging data, and then proposed a new computer-aided diagnosis (CAD) system, on the basis of texture analysis method. In the first, they extracted 12-element feature from each brain image via fractional Fourier entropy. Afterwards, multilayer perceptron was employed as the classifier, which was trained by a novel fitness-scaling adaptive genetic algorithm. The statistical analysis over 49 subjects showed the overall accuracy of their method yielded 95.51%. Experimental results performed better than four state-of-the-art weight optimization methods, and this CAD system gave significantly better performance than manual interpretation.

In the paper entitled "Hybrid Watermarking Algorithm using Finite Radon and Fractional Fourier Transform", Sharma J.B. proposed a watermarking scheme based on finite radon transform (FRAT), fractional Fourier Transform (FRFT) and singular value decomposition. In the proposed scheme, image to be watermarked was first transformed by finite radon transform, the radon transformed image was further transformed by FRFT, and singular values of FRFT transformed image were modified to embed the watermark. Inverse transformation was applied to obtain watermarked image. Simulations were performed under various test conditions with different FRFT transform angles for improved robustness and visual transparence of watermarked image. Results of the proposed scheme were better in comparison to the existing schemes for most of the attacks. Proposed scheme provided additional degree of freedom in security, robustness, payload capacity and visual transparence. Proposed scheme could also be used to communicate or store the watermarked image as erasure code, to reduce communication errors over a network, due to the use of FRAT.

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