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Whatever happened to autostress design?

T.V. Galambos*
University of Minnesota, Minneapolis, MN, USA

Abstract. This paper is a review of the history of inelastic design criteria for steel girder bridges. The AASHTO Specification's evolution from Working Stress Design (WSD) to Load Factor Design (LFD) to Load and Resistance Factor Design (LRFD) will be the connecting thread of the paper. The author has been involved in some way in much of the research that is behind the inelastic design criteria in the AISC and the AASHTO steel specifications. Brief accounts will be given of the theoretical and experimental background for the inelastic provisions in the LFD and LRFD methods for girder bridge design. The background and the implementation of the "Auto-Stress" method will be discussed.

Keywords: Steel bridges, plastic design, shakedown theory, design standards

1. Introduction

The subject of this paper is the history of the application of the theory of inelastic behavior to the design of steel I-girder bridges. The fundamental premise in contemporary design practice is that a beam-type member possesses sufficient ductility to attain full plasticization of the cross section under predominantly flexural action. Much theoretical and experimental research was performed on this subject for most of the previous Century, and well established rules have evolved to ascertain ductile behavior by limiting cross section geometries and bracing spacing requirements. "Compact and properly braced" girders are designed so that the maximum moment obtained from a linearly elastic structural analysis is equal to the "plastic moment M_p" [36]. This approach to design is universally employed in most of the world's design standards for steel structures, including Chapter 6 of the *Standard Specifications for Highway Bridges* of the American Association of State Highway and Transportation Officials (AASHTO).

It has long been recognized, however, that for a structure with ductile members and connections, there is an unused reserve of strength beyond the attainment of the first "plastic hinge", and that this reserve can be utilized in the design. In the limit, a "plastic mechanism" forms, and the structure is said to have reached its "ultimate strength". The method that evolved around the mechanism theory was extensively researched world- wide in the Post-WWII decades, and the resulting procedure of "plastic design" has been part of the "*Specification for Structural Steel Buildings*" of the American Institute of Steel Construction since 1963. The aim of this paper is to tell the history of the application of the plastic method to the design of steel and composite concrete/steel girder bridges.

2. Early beginnings

Plastic theory was already employed in the design of continuous beams in 1914 in Budapest, Hungary. Research in the ensuing decades developed the theorems and the methods of analysis, and the theory was extensively verified by thousands of experiments all over the world. As far as application to beam-type bridge structures, there was one uncomfortable problem: the plastic design theory assumed "proportional" loading, that is, the static load set on the structure had to increase proportionally until the set reached the collapse load. Of course, no such assumption holds for a bridge, where moving vehicles of different weights travel along the bridge. In the 1920-s Gruning, in Germany, found

*Corresponding author. T.V. Galambos, University of Minnesota, Minneapolis, MN, USA. E-mail: galam001@umn.edu.

that if a structure is initially loaded beyond its elastic limit, but below its plastic collapse load, a set of self-equilibrating residual forces will remain in the structure after the load is removed. Subsequent variable repeated load applications that exceed the elastic limit load can then be accommodated without further yielding.

A rigorous theory was provided in the 1930-s by Bleich and Melan, who formulated what became subsequently known as the "Shakedown Theory", the theory of "Variable Repeated Loading" or "Incremental Collapse Mechanism" theory. In subsequent years the terms "Autostress Design Method", "Alternate Load Factor Design Method" and "Unified Autostress Method" were used in the late 1980-and early 1990-s editions of the AASHTO Specifications, to be replaced finally by the name "Moment Redistribution Method" in the latest (2010) edition.

After WWII the original shakedown theory was further expanded and a number of relatively small-scale experiments were performed to firm up the principles of the effect of variable repeated loading on ductile beam and frame structures. This work took place at Brown, Lehigh, California and Darmstadt (Germany) Universities, and it was summarized in ASCE Manual No. 41 (2nd edition 1971) "*Plastic Design in Steel*". The consensus of the authors of Manual 41 was that the theoretical shakedown limit was generally exceeded by the experimental strength, which was mostly near to the plastic mechanism collapse load. Their conclusion was that for building structures the probability of reaching the plastic collapse load once was much greater than attaining a shakedown load just a little smaller by many repeated applications.

3. Classical shakedown theory

It is possible to support loads beyond the formation of the first plastic hinge in a continuous multi-span steel bridge girder of "compact" cross section with appropriately spaced lateral-torsional bracing. The purpose of this paper is to document the research that was performed to implement the utilization of this load reserve for bridge design and bridge rating.

Two possibilities of inelastic strength can be exploited in the design of such girders:

1) *The ultimate limit state*: A one-time factored load set, such as the AASHTO *STRENGTH I* load combination: *[1.2 D + 1.75 (L + I)] ≤ plastic mechanism load.*
2) *The shakedown limit state:* A repeatedly applied overload set, such as the AASHTO *SERVICE II* load combination *[1.2D + 1.3(L + I)] ≤ shakedown limit load.*

The following discussion will consider the application of the shakedown limit state for design. It is assumed that the overload is larger than the elastic limit load and less than the plastic collapse load. The first such overload event will produce a plastic hinge at a location on the girder where the moment from an elastic analysis exceeds the plastic moment. At this point there will be an inelastic rotation, or a "kink" in the girder. When the overloaded vehicle passes from the bridge, there remains a set of self-equilibrating "residual" moments. The next and subsequent passes of the overload will be resisted purely elastically, because the sum of the residual and the applied elastic moment will be less than or equal to the plastic moment.

This beneficial process thus produces, so-to-speak, more "elbow room" for the elastic space available to the variable applied live load, and the girder is then said to have "shaken down". The resulting inelastic permanent deflections and rotations are quite small and do not impair the riding quality of the bridge. This fact has been demonstrated by many researchers, both experimentally and computationally, as will be shown subsequently. This process, however, cannot continue indefinitely. A shakedown load limit is reached when in one or more of the spans of the continuous girder the sum of the residual moments and the elastic moments exactly equals the plastic moment at enough locations so that a virtual kinematic mechanism forms. This mechanism is akin to the plastic collapse mechanism. Any load larger than the shakedown limit, will produce additional inelastic rotations at the hinge locations with each passage, until excessive deflection makes the bridge useless.

Mathematically shakedown can be defined in the following manner: At the locations i of the extreme elastic moments M_e in each span and at each interior support

$$-M_{p_i} \leq m_i + M_{e_i}^{min}$$
$$m_i + M_{e_i}^{max} \leq +M_{p_i} \tag{1}$$

m_i is the residual moment, and M_p is the plastic moment capacity of the cross section.

The conditions defined by Eq. 1 can be interpreted or exploited in various ways:

1) The development of a residual moment field in a continuous bridge girder occurs naturally from inadvertent occasional overloads. As a result an excessive concern over exceeding code-specified stress limits is not warranted.
2) A residual moment field can be intentionally introduced during construction by fabricating an artificial "kink" at interior supports, thus allowing the use of an enlarged elastic range, and thus a larger live load.
3) Traditionally the AASHTO Specifications have a 10% redistribution of stresses at supports. This is an empirical recognition of the ductility of steel beams.
4) In the ASHTO 2010 Specification the magnitude of the maximum residual moments is limited so that the "kink" at any interior support does not impair smooth drivability.
5) By considering Eqs.1 at locations of extreme elastic moment in each span and each internal support as a set of constraints, the shakedown limit load can be calculated by a mathematical optimization process (a "linear program").

While the writers of the Plastic Design Manual [5] considered deflection stability to be of no interest in the design of building structures, a number of researchers have for the past 40 or so years been very interested in using the principles of the shakedown theory to find benefits in bridge design and in bridge load rating. This effort will be the subject of the following parts of this paper.

4. Research at Washington University 1965–1970

The American Iron and Steel Institute (AISI), with additional assistance by the National Science Foundation (NSF) and the McDonnell-Douglas Corporation, sponsored research on the shakedown of continuous bridges under moving loads at Washington University in St. Louis under the author's direction. The aim of the research was to discover if such multi-span bridges could actually shake down, and to develop criteria that could be used for strength design. The research was performed by the then graduate student Dr. Dale Eyre. The results of this research are reported in Eyre [19], Eyre and Galambos [14–18].

The initial phase of the research was to conduct a number of small-scale tests of two-span beams. The specimens were 1.5" × 0.75" (38 mm × 19 mm) steel bars with a span-to-depth ratio of 20. Variable repeated loads were applied at 1 or 2 fixed locations in each span. A constant value representing dead load was present at each load point while the additional load was incremented after the cycles of repeated loading indicated either that the deflections stabilized or that they became excessive. While some valuable lessons were learned from these tests, it was desired to test larger beams with simulated moving loads. A two-span W8X20 (A36 steel) beam of 160 inch (4.1 m) length in each span was tested in as shown at the left in Fig. 1. The beam was laterally braced according to the plastic design requirements of the then applicable AISC Specification.

Two beams were tested: one was loaded by a single load at four locations in each span, and the other one was loaded by a double load scheme, as shown on the left drawing in Fig. 1. The essential results of the second test are illustrated in the drawing in the right of Fig. 1. The ordinate on the graph denotes the number of repeated cycles. The abscissa represents the vertical deflection of the second load point from each exterior support, Load points 2 and 7. Also marked are the magnitudes of the varying applied simulated moving live loads. The values shown are the total of the two adjacent applied jack forces. The constant dead load was 3 kips (13 kN) at each load point. The two graphs show the maximum deflection at each of the load points for each cycle of load application. At each load increment the deflection jumped during the first application, but it then remained essentially constant as the cycles were repeated as many times as were judged to show that the deflections were definitely "shaken" down. The circled marks on the left side of the graph represent the residual deflection after the removal of the live load. The test was terminated when the deflection became excessive at a total live load of 24 kips (107 kN). This occurred because of lateral buckling between the lateral braces. This failure, however, was at live loads in excess of the theoretical shakedown load of 21.6 kips (96 kN) and it was even larger than the theoretical plastic mechanism load. These theoretical values were determined for the measured dimensions and yield points. The magnitude of the final shaken down deflection was about 3.5 times the elastic limit deflection. However, this deflection at the theoretical shakedown load was a little less than the twice the elastic limit load.

After the completion of the laboratory experiments Dr. Eyre made extensive theoretical and numerical studies that explored the role of stain hardening and plastic zone spreading on the behavior of two-span beams subject to moving variable repeated loads. From these

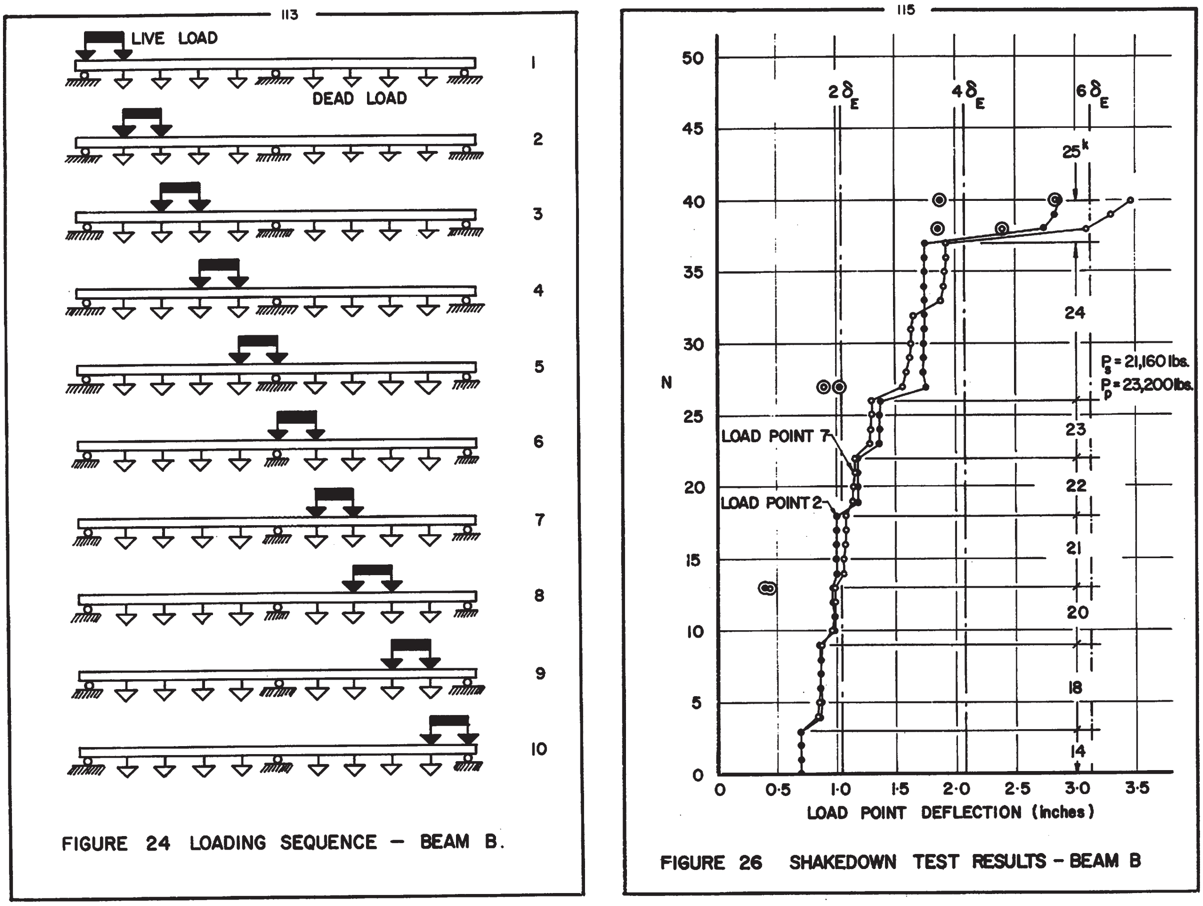

Fig. 1. Testing scheme and test results from [19].

works he was remarkably successful in predicting the experimental results. He also expanded the studies to the shakedown behavior of grid-type structures that simulate bridges with multiple girders.

The basic lesson from this work is that the *shakedown* load computed with the ideal elastic-plastic theory is conservative. The repeated load cycles can reach the strength predicted by the plastic mechanism theory. The tests showed that a design method that permits live loads above the elastic limit load up to the "simple" shakedown load may be safely used in design, since there is only a relatively small residual inelastic hinge rotation, and there is a reserve of strength up to and even beyond the plastic mechanism load.

5. Research at US Steel company laboratory

Research to develop inelastic design criteria for adoption in the AASHTO specification was performed at the US Steel research laboratory in Monroeville, PA during the 1970-s and 1980-s. This work was inspired and led by the late Dr. Geerhard Haajer and his collaborators Charles Shilling, Philip Carskaddan, and Michael Grubb [24, 25]. The dominating idea of this research was that under strict limits of cross section geometry and lateral bracing, the strength limit load (*AASHTO STRENGTH I*) could be defined by a plastic mechanism, and the overload limit (*AASHTO SERVICE II*) could be calculated by the shakedown theory. The strength criterion can be readily met by compact prismatic non-composite I-girder bridges. However, such bridges are only a small proportion of the total bridge girder population. The limiting criterion with the shakedown condition is that the residual hinge rotation must not impair rideability. These objections were successfully addressed by the original U.S. Steel research team, and by new improvements made by later researchers.

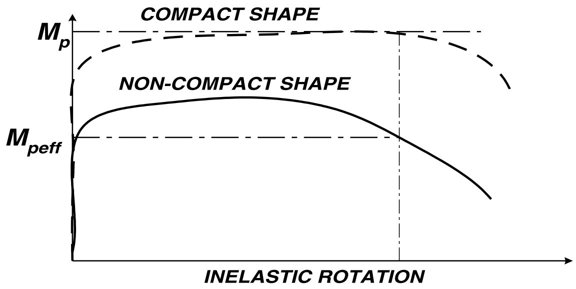

Fig. 2. Definition of effective plastic moment (Schilling, 1990).

In order to expand the population of I-girder geometries for use of the shakedown method, the concept of the "effective plastic moment" was developed [12]. The effective plastic moment is defined as the moment on the downward slope of the non-compact girder's moment-versus-hinge rotation curve that is equal to the inelastic rotation capacity of a fully compact girder. This is illustrated in Fig. 2. A series of six tests were performed on welded girders by Schilling [33, 34] to experimentally verify the concept. Test specimen lengths varied from about 7 to 19 ft, (2.1 to 5.8 m) and the depths 2 to 3 ft. (0.6 to 0.9 m). The beams were simply supported at their ends, and the load was applied in the center. This set-up simulated the condition at the interior supports of continuous girders. A bearing stiffener was inserted at the load point, and the remaining parts of the member had equally spaced transverse stiffeners. Three tests were made to simulate the effect of the re-bars at the compositely designed support cross section, and three tests were made to find the moment- rotation curves of slender-web beams. The availability of such M-θ curves, or the conservative empirical formula derived from them, were then employed to construct the *Unified Autostress Method* [35]. This vastly expanded the population of available I-girder geometries. However, restrictions were still required, and these remain to this day: The girders must be prismatic, the plastic hinges may only exist at the interior supports, and no plastic rotation is permitted at locations of maximum positive moment in composite members.

Based on the Schilling experiments and on the extensive theoretical and computational work of the whole team, a design standard for autostress design was proposed to AASHTO (Haaijer et al., 1983). AASHTO accepted the recommendations and issued a Guide Specification [3] and the criteria became fully incorporated into the AASHTO LRFD Specification in [4] in *Sec. 6.10.11 Inelastic Analysis Procedures*. Two inelastic limit states are included: 1) *plastic mechanism* formation under *AASHTO STRENGTH I* (1.25D + 1.75 L) loading, provided that plastic hinge rotations only occur at the interior supports, and 2) the restriction of residual "kinks" at the interior supports under *AASHTO SERVICE II* loads. These "kinks" were to be determined by the Unified Autostress Method. The bridge engineer was then referred to the publication that explained the method [35]. Unfortunately this involved considerable additional programming for utilization in the design office.

6. Inelastic design in the 2010 AASHTO specifications

The utilization of inelastic strength reserve for continuous plate girder bridges was further investigated by three independent activities during the: 1) The construction and in-service load testing of a bridge designed by the Unified Autostress Method on a Forest Service Road in the State of Washington [32], 2) The testing of a 40% scale two-span bridge model at the Turner-Fairbank Highway Research Center in McLean, VA (Grubb, & Moore, 1990), and 3) by additional beam-testing and extensive analytical studies [7–10, 26, 29–31, 37, 38].

The bridge in Washington State is probably still in service. During the period of its load test and the period of the study it showed no noticeable distress. The laboratory bridge at the FHWA research center was part of a major multi-year study of a many issues of interest to bridge engineers. The two spans were 56 ft. (17 m) each in length. Three 27 in. (0.7 m) deep welded girders were spaced at 7 ft. (2.1 m). The total width of the bridge was 19 ft. (5.8 m). Concentrated loads were applied by 18 hydraulic jacks to simulate various combinations of live loads. The steel girders were designed by the autostress provisions of the AASHTO Guide Specification [3]. After the application of the overload (roughly corresponding to the *SERVICE II* loading) there was no noticeable buckling or yielding in the steel girders. At the AASHTO strength limit, there was hardly any detectable damage. Loading continued to about 2.5 times the AASHTO strength limit load when the jack capacity was reached. By this time there was considerable local buckling at the interior supports but the bridge showed no sign of collapsing.

As a result of the additional component testing and analytical work, new inelastic design criteria were proposed and subsequently adopted by AASHTO. These recommendations are incorporated as Appendix B6 *MOMENT REDISTRIBUTION FROM INTERIOR-*

PIER I-SECTIONS IN STRAIGHT CONTINUOUS-SPAN BRIDGES of the 2010 AASHTO Specifications. In the adopted inelastic criteria all the parametric computer work was done beforehand, and the following simple rule for the inelastic design were adopted:

$$M_r = |M_e| - M_{pe}$$
$$M_r \le 0.2\,|M_e| \tag{2}$$

The operations required by Eq. (2) must be performed for each interior continuous support location. In this equation M_r equals the residual moment at the support. $|M_e|$ is the absolute value of the elastic moment envelope at that location. M_{pe} is the effective plastic moment that is dependent on the geometry of the cross section, on the spacing of the lateral bracing and on the presence of transverse stiffeners on each side of the plastic hinge. The residual moment must not exceed 20% of the maximum elastic moment. Two checks are to be made at each location: for the elastic moment envelope due to 1) the *SERVICE II* loading, and 2) the *STRENGTH I* loading. The underlying maximum "kink" built into the background calculations is 0.009 radians ($\approx 0.5°$) for the *SERVICE II* loading and 0.03 radians ($\approx 2°$) for the *STRENGTH I* loading. This method is very simple to apply since the designer already has the elastic moment envelope available as a part of the design process. The computed residual moments are next distributed to the interior of the span of the beam by linear superposition. The total moment at each location in the interior of each span has to be checked by the applicable criteria of the AASHTO Specification. For the designers who wish to use a more generous inelastic reserve, Appendix B6 includes criteria for a "Refined Procedure".

7. Research at the University of Minnesota

During the late 1980-s NCHRP sponsored a research project at the University of Minnesota to study the application of the shakedown method to the rating of existing continuous girder bridges [21]. The final report of this research contained a sample draft of a guide specification for inelastic bridge rating. The theoretical development and the numerical applications in the form of Fortran computer programs were based on the doctoral theses of Michael Barker [6] and Burl Dishongh [13]. This work expanded both the depth and the scope of the previous research.

The research of Dishongh proposed a method whereby the bridge rating engineer could specify maximum acceptable inelastic "kinks" at interior supports, and then check if the vehicle that is to be rated, met this limit. A major improvement of the proposed method was the use of bi-linear moment-curvature relations instead of the "effective plastic moment" scheme of the autostress method. The new M-θ curves applied a "softening" relationship for the non-compact cross sections at the interior supports, and a "hardening" curve for the composite section inside the spans [28]. This "*Residual Deformation Analysis*" was implemented by a "hands-on" computer program. The method is, however, restricted to a single linear girder.

The research of Barker expanded the application of the shakedown analysis to the whole bridge. The bridge was idealized as a grid system that incorporated the main girders, the diaphragms and the slab. The outcome of the analysis is the shakedown limit load, as well as the plastic collapse load, of the whole bridge. The analysis assumes a classical ideal elastic plastic moment-curvature relation. Again a computer program was also provided in the final report. The use of the checking methods proposed by Dishong and Barker awaits the availability of modern commercial software for its practical implementation in a bridge design office.

Toward the end of the NCHRP project the Minnesota researchers became aware that a 1/3 scale, 3-span, composite 4-girder bridge was tested at the structures laboratory of Iowa State University at Ames IA to study the effectiveness of various strengthening schemes [27]. The model was not previously subject to any intentional inelastic loading. This essentially undamaged bridge was made available to the Minnesota team. A regime of repeated loadings was applied to the model to study shakedown and finally to subject the bridge to collapse [11]. The scheme of load application is shown in Fig. 3.

The test results furnished proof of the ability of the analytical methods developed by Dishongh and Barker to predict the shakedown and ultimate strength of the two-dimensional specimen. A further verification was obtained by a Finite Element program *BOVAS*. Thel work at the University of Minnesota thus provided yet more confidence to the AASHTO inelastic method. The test was the first shake-down test of amulti-girder bridge in a structural laboratory.

The Iowa test, however, introduced an unexpected surprise: When the repeated load cycle produced moments only slightly above the calculated yield moment, the inelastic deflections were predicted very accurately according to the assumption of composite

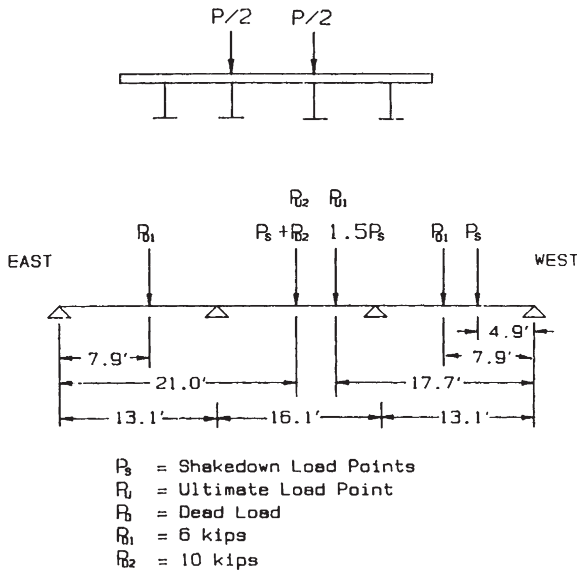

Fig. 3. Scheme of load application for the Iowa State bridge [11].

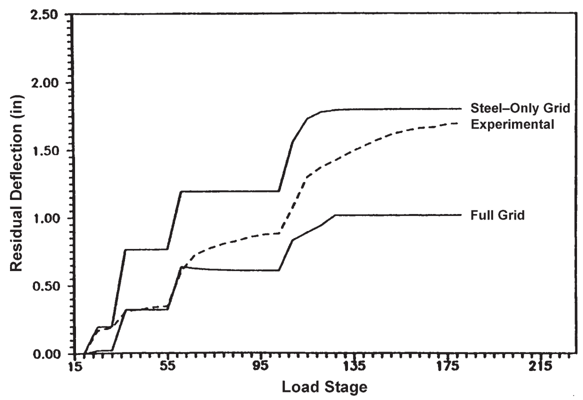

Fig. 4. Shakedown deflections of Iowa State bridge [11].

behavior. However, as the load magnitudes increased, the deflections at shakedown became larger, eventually approaching the values predicted by assuming non-composite response. The slab showed considerable deterioration. The cycle number-versus residual-deflection graph is shown in Fig. 4.

The solid lines in Fig. 4 define the predicted values of the residual deflections under the assumption of composite behavior (lower curve) and non-composite behavior (upper curve). The experimental curve follows the composite path at first (at $P = 20$ kips (90 kN), see definition of P on Fig. 3), but then it deviates more and more towards the non-composite prediction. The final load cycle was $P = 28$ kips (125 kN). No definite cause for this unexpected behavior was stated in the final report to the NCHRP.

The following possible combinations of effects could be responsible:

1) Some shear connectors could have been damaged or even failed during the previous research project.
2) Shear connectors could have gradually failed by low-cycle fatigue during the 182 repetitions of the shakedown test-cycles, until finally only the steel beams were active.
3) The slab deteriorated to an extent that the bridge no longer acted as a unit.

It turned out that this type of behavior under repeated load events near the static plastic collapse load was also observed by other experiments, notably in an experiment at Monash University in Australia [23].

Stimulated by the results of the Iowa bridge test a 1/2-scale bridge was tested in the structures laboratory of the University of Minnesota to study the behavior of the shear connectors in a composite bridge under the effect of a real moving live load. The model was a two-span bridge of two 32 ft. (9.8 m) spans. Two W14X22 rolled beams supported a 4 in. (0.1 m) deep and 8 ft. (2.4 m) wide concrete slab. The beams were spaced 6 ft.(1.8 m) apart. The shear connectors were 3/8th inch (9.5 mm) welded stud connectors. One span had 50% of the AASHTO required shear connectors for static strength, and the other span had 80%. The moving load was a four-wheeled bogie that was weighted by lead ingots and pulled back-and-forth by a cable on pulleys connected to an overhead crane. The movement was effected by slowly moving the crane carriage along the crane rails. The bogie had a transverse wheel spacing of 6 ft (1.8 m). and a longitudinal wheel spacing of 4 ft. (1.2 m). The key element of the instrumentation was the measurement of the slip between the slab and the beam at various locations along the beam.

The following table compares the theoretical and the experimental incremental collapse loads:

SPAN	WEST 80% AASHTO connectors	EAST 50% AASHTO connectors
Theoretical Composite	65.1 kip	72.7 kip
Theoretical Non-Composite	49.1 kip	54.0 kip
Experimental	64.1 kip	54.4 kip

The differences in the theoretical shakedown load limits between the two spans are due to differences in the yield points of the steels. From the table it is evident that the span with 80% of the required shear connectors almost reached the theoretical incremental collapse load for the composite model, while the span with the 50% required connectors barely made it above the prediction for the non-composite model.

The author of the report on this work makes the following observations [20]:

1) "*Based on these findings and observations made on the model after failure of the spans it appears as though the shear studs failed thereby causing loss of the composite action at the critical section in positive bending and resulting in sudden collapse of the span. Based on the shape of the horizontal shear vs slip graphs it appears quite possible that the shear studs failed as a result of some type of low cycle fatigue under alternating plasticity. Also due to the deflections in the structure at failure there may have been a sizable uplift component which helped to pry the weakened studs off the beams*". (p. 169 in Flemming [20]).
2) "*The bottom line of this test is that the interfacial degradation (slip) did not stabilize in the case of extreme overloads. This being the case the structure itself can never truly shakedown or stabilize in the sense of vertical deflections. In a real bridge with adequate shear connection based on current AASHTO design one would expect to see the same trends that were found in this experiment to hold true, albeit to a smaller degree.*" (p.192).

8. Summary and conclusion

The theoretical, numerical and experimental work on the possible utilization of inelastic reserve strength over the past 40 or so years that was briefly reviewed in this paper has resulted in the simple and safe criteria permitted in Appendix B of the latest edition of the AASHTO Specifications. Using these criteria would yield modest economic gains in design, and in modest increases in the permissible weight of permit vehicles.

It is difficult to make generalized conclusions from the above described three tests on laboratory composite steel multi-girder bridges. The bridge tested at the FHWA research laboratory proved that the "autostress" design method works well, and is conservative, as long as the live load is applied statically but not repetitively. The Iowa test was not designed for loads beyond the elastic limit. The slab system and possibly also the shear connectors failed under extreme repetitive load cycles. The bridge was never meant for the abuse that the shakedown tests administered to it. Was the testing then of no practical use? The data taken during the extreme loading demonstrated that the theoretical models and the computational tools developed by Dishongh and Barker were able to predict behavior remarkably well. Thus future research can use these tools for research or in design. Beyond that, the test did not provide much toward the advancement in inelastic bridge design practice.

The Flemming bridge test at Minnesota is to be taken very seriously. Unfortunately, while the report was issued in 1994, no part of the findings were published, and the vast amount of data on deflections, reactions, moments and especially shear connector slip have not been further analyzed. In hindsight, it would probably have been more useful if at least one of the spans would have had the full complement of required shear connectors.

The conclusion of Flemming that shear connector yielding under illegal or unintentional overloaded vehicles may eventually lead to the fracture of shear connectors in low cycle fatigue, even if the design obeys the AASHTO Specification, is plausible, but as yet unproven on real bridges on the road system. One approach to test the prediction would be to install slip gages on a bridge in the field, and make long-term readings. Another suggestion would be to subject an out-of-service continuous multi-span composite bridge to increasingly heavy trucks or military tanks. This would be an extremely expensive exercise, and it could be also dangerous.

In the meantime, the fruits of this expensive research and development effort, while available for use in design and rating, are ignored by the bridge design community. Maybe some group will rediscover and use them!

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