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Towards estimating fiducial localization error of point-based registration in image-guided neurosurgery

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Abstract. Fiducial Localization Error (FLE) is one of the major reasons of inaccuracy in point-based spatial registration of Image-Guided Neurosurgery System (IGNS), and minimizing FLE is the fundamental way to improve spatial registration accuracy. A reliable estimation of FLE is needed, as it cannot be measured directly in real application of IGNS. In this paper, we propose a method to estimate the FLE in a point-based registration of IGNS. Test fiducial point sets were generated in one coordinate system around the given fiducial point set by utilizing simple random sampling. Further, these points were registered to the fiducial point set in the other coordinate system. The average position of the test fiducial point sets with small FRE are calculated and its displacement from the given fiducial point set is the parameter used to estimate the FLE of each fiducial point. The correlation between the displacement and the FLE of each fiducial point is greater than 0.75 when nine or more fiducial points were utilized. This correlation gradually increases up to 0.9 with the increase of the number of fiducial points.

Keywords: Image-guided neurosurgery, fiducial localization error, point-based registration

1. Introduction

Point-based rigid body registration, which calculates a transformation between the two coordinate systems by optimally matching two fiducial point sets with know point-to-point correspondence, is widely used in many research areas. For example, in image-guided surgery, this technique can be used to track the navigation tools, register images with different modalities, register the patient space to the image space and serve as an initialization step in surface based patient-to-image registration [1-5]. The point-based registration problem can be solved analytically [6], but the two coordinate systems cannot be registered perfectly because of the imperfectness of the correspondence.

There have been a number of studies on the accuracy of point-based registration, and three types of errors are usually used in these analyses: fiducial localization error (FLE), fiducial registration error (FRE) and target registration error (TRE) [7, 8]. FLE can be roughly regarded as the error in localizing the fiducial point, and it is the fundamental cause of the inaccuracy in point-based registration. FRE is the distance between fiducial points after registration, and it is the only error that can be calculated in a

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single instance of registration. The TRE for a target point is the distance between its registered point and its true corresponding point in the other space. TRE is usually the most clinically relevant error, but it cannot be calculated or measured directly in application. There have been a number of studies demonstrating the estimation of TRE at a specific point [9-12].

Since FLE is the basic cause of TRE, estimation of FLE in a single instance of registration is very important in the optimization of point-based registration. For example, a reliable estimation of FLE can be directly used to find optimal weights in weighted point-based registration. Determining the fiducial point with the largest FLE according to estimation can help the user to adjust the fiducial point with the largest FLE literately to optimize registration [13]. In addition, most of the TRE estimation methods are based on certain assumptions of the properties of the FLE, such as independent or dependent, isotropic or anisotropic, homogeneous or heterogeneous, biased or unbiased. Also some of the TRE estimation methods are directly based on the value of FLE. Therefore, a reliable estimation of FLE would be beneficial for TRE estimation.

In this paper, we have identified a parameter that is calculated from a single instance of point-based registration and this parameter is highly correlated with the FLE. Hence, this parameter can be utilized as the basis for FLE estimation and for the optimization of point-based registration.

The rest of this paper is organized as follows: in section II, the definition of the point-based registration problem is reviewed and the proposed algorithm to calculate the parameter for FLE estimation is described in detail. Experiments and results are given in section III. Finally, section IV concludes this paper.

2. Method

2.1. Point-based registration

For the convenience of introducing the proposed algorithm and experiments, here the point-based rigid body registration problem is briefly introduced. Given two three dimensional spaces, called floating space and reference space, two fiducial point sets $\mathbf{P}=\{\mathbf{p}_1,\mathbf{p}_2,\cdots\mathbf{p}_N\}$ and $\mathbf{Q}=\{\mathbf{q}_1,\mathbf{q}_2,\cdots\mathbf{q}_N\}$ are selected from them, respectively. The number of fiducial points in each space is N, and $\mathbf{p}_i$ and $\mathbf{q}_i$ are a pair of corresponding fiducial points. Because of the imperfectness in selecting the fiducial points, usually $\mathbf{p}_i$ and $\mathbf{q}_i$ do not correspond to each other exactly. Suppose that $\mathbf{P}^*=\{\mathbf{p}^*_1,\mathbf{p}^*_2,\cdots\mathbf{p}^*_N\}$ is the true corresponding point sets of $\mathbf{P}$ in the reference space, the FLE of fiducial point i can be defined as:

$$FLE_i=\left|\mathbf{p}^*_i-\mathbf{q}_i\right| \tag{1}$$

If the value of $\mathbf{P}^*$ is known, then a unique transformation from floating space to the reference space can be calculated, and the two point sets can be aligned perfectly. However, it is impossible to determine $\mathbf{P}^*$ in real application. Therefore a point-based registration algorithm is used to calculate an optimal pair of rotation $\mathbf{R}^*$ and translation $\mathbf{t}^*$ in the mean square sense:

$$\arg\min_{\mathbf{R}^*,\mathbf{t}^*}\sum_{i=1}^{N}\left|\mathbf{R}\mathbf{p}_i+\mathbf{t}-\mathbf{q}_i\right|^2 \tag{2}$$

After registration, the FRE of the fiducial point i is defined as:

$$FRE_i = \left|\mathbf{R}^*\mathbf{p}_i + \mathbf{t}^* - \mathbf{q}_i\right| \tag{3}$$

The objective of the proposed algorithm, which is described in detail in the following subsection, is to calculate the value that is highly correlated with FLE.

2.2. The algorithm to calculate the value highly correlated to FLE

For corresponding fiducial point sets $\mathbf{P} = \{\mathbf{p}_1, \mathbf{p}_2, \cdots \mathbf{p}_N\}$ and $\mathbf{Q} = \{\mathbf{q}_1, \mathbf{q}_2, \cdots \mathbf{q}_N\}$, the general idea of this algorithm is to register randomly generated fiducial point sets around $\mathbf{Q}$ and move $\mathbf{Q}$ to the mean position with small FREs. After some number of iteration of updating $\mathbf{Q}$, the displacement of the fiducial points in $\mathbf{Q}$ from its original position is determined and this is the parameter that is highly correlated to FLE. The algorithm is stated in detail as follows:

- It starts with a pair of fiducial point sets $\mathbf{P} = \{\mathbf{p}_1, \mathbf{p}_2, \cdots \mathbf{p}_N\}$ and $\mathbf{Q} = \{\mathbf{q}_1, \mathbf{q}_2, \cdots \mathbf{q}_N\}$ in the floating and the reference space respectively.
- For each fiducial point $\mathbf{q}_i$ in the reference space, generate a test fiducial point $\mathbf{q}_{test-i}$ by random sampling in a cube around $\mathbf{q}_i$, and form a test fiducial point set $\mathbf{Q}_{test}$.
- Register $\mathbf{P}$ and $\mathbf{Q}_{test}$ with SVD algorithm [6], calculate the mean FRE for all the fiducial points, which is designated as $\overline{FRE}_{test}$.
- Steps from 2 to 3 are repeated for 2,000 times and then sort the $\overline{FRE}_{test}$. Choose the 250 $\mathbf{Q}_{test}$ that result in smaller $\overline{FRE}_{test}$. Then calculate the mean position for each fiducial point. The point sets of the mean position are annonated as $\overline{\mathbf{Q}}_{test}$, and $\overline{\mathbf{q}}_{test-i}$ that represent i^{th} point.
- Set $\mathbf{Q}$ equal $\overline{\mathbf{Q}}_{test}$, and repeat steps from 1 to 5. If the decrease in $\overline{FRE}_{test}$ is small or it begins to increase, then go to step 6.
- Calculate the displacement from $\mathbf{q}_i$ to $\overline{\mathbf{q}}_{test-i}$ $Disp_i = \left|\overline{\mathbf{q}}_{test-i} - \mathbf{q}_i\right|$ and $Disp_i$ is the value that is needed.

3. Experiments and results

3.1. Experiments with random fiducial points

First, the algorithm was tested with different number of randomly distributed fiducial points. The test is conducted as follows:

- Randomly generate N fiducial points $\mathbf{P} = \{\mathbf{p}_1, \mathbf{p}_2, \cdots \mathbf{p}_N\}$ in a cube of size 200*200*200 in the floating space.
- Transform all the points in $\mathbf{P}$ by a random rigid body transformation to get another set of points $\mathbf{P}^* = \{\mathbf{p}^*_1, \mathbf{p}^*_2, \cdots \mathbf{p}^*_N\}$, so that $\mathbf{p}^*_i = \mathbf{R}_{real}\mathbf{p}_i + \mathbf{t}_{real} (i = 1, 2, \cdots N)$. Therefore, $\mathbf{P}^*$ represents the true corresponding point set of $\mathbf{P}$ in the reference space, and $\mathbf{p}^*_i$ is the true

corresponding point of $\mathbf{p}_i$. $\mathbf{R}_{real}$ is a rotation matrix, representing a uniformly distributed random rotation between 0 and 2π about each axis. t_{real} is a translate vector, representing a uniformly distributed translate between -1000 and 1000 along each axis.

- Adding perturbation to each fiducial point in $\mathbf{P}^*$ according to four different FLE models to obtain $\mathbf{Q} = \{\mathbf{q}_1, \mathbf{q}_2, \cdots \mathbf{q}_N\}$, which represents the fiducial point actually selected in the reference space. Therefore, $\mathbf{q}_i = \mathbf{p}^*_i + Perturb \quad (i = 1, 2, \cdots N)$. Where $Perturb$ is used to represent the displacement vector from the true corresponding point when selecting $\mathbf{q}_i$ in the reference space, and it is a three-dimensional random variable generated according to the FLE model.
- Calculate $FLE_i = |\mathbf{p}_i - \mathbf{q}_i|$. Calculate $Disp_i$ by the proposed algorithm, where $\mathbf{Q}_{test}$ was generated by simple random sampling around each fiducial point in a cube size 2*2*2. The threshold of $\overline{FRE_{test}}$ for stopping iteration of updating $\mathbf{Q}_{test}$ was set at 0.01.
- Repeat steps 3 and 4 for 200 times. Calculate the correlation coefficient between $200N$ FLE_i and $Disp_i$, and obtain the slope of FLE_i against $Disp_i$ by linear regression.

Four types of FLE models were used in these experiments, and the perturbations were generated as follows. Here, r represents a random value belonging to normal distribution $N(0,1)$, and the FLE of different fiducials are regarded independent.

Type 1. Isotropic homogeneous unbiased FLE: For all the fiducial points, every element of the $Perturb$ is r.

Type 2. Anisotropic homogeneous unbiased FLE: For all the fiducial points, the first and the third element of the $Perturb$ is r, and the second element of the $Perturb$ is $2r$.

Type 3. Anisotropic heterogeneous unbiased FLE: On the basis of the second type of FLE, the $Perturb$ for the first and the third fiducial points were doubled.

Type 4. Anisotropic heterogeneous biased FLE: On the basis of the third type of FLE, a bias is added to $Perturb$. The bias is a random three dimensional variable with each element uniformly distributed between 0 and 1.

The number of fiducial points used includes: 5, 7, 9, 11, 13, 15, and 17. Experiments were not done with more than 17 fiducial points, because in most point-based registration, there are usually no more fiducial points. For each FLE type and each fiducial point number, the test experiment was repeated for 20 times, and the statistical distribution, correlation coefficients and the slopes of each combination of the FLE type are determined. These fiducial point numbers are displayed in Figures 1 and 2, respectively.

In Figure 1, it can be seen that the *Disp* is highly correlated to *FLE*. As the increase of the number of fiducial points, the correlation becomes larger and more stable. In the experiment with nine

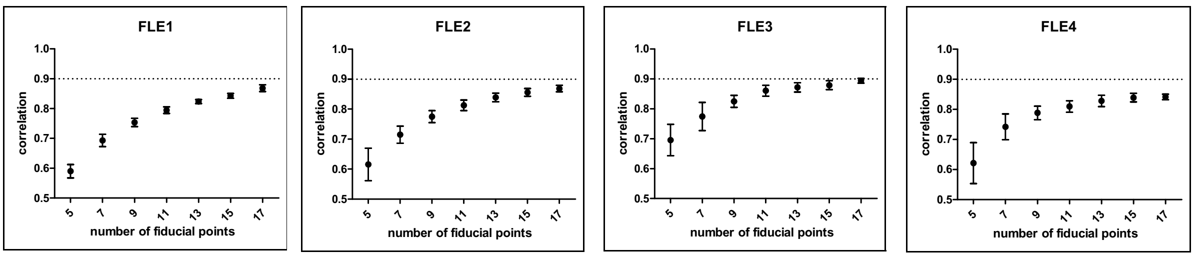

Fig. 1. Correlation between the *FLE* and *Disp* against the number of fiducial points under different *FLE* models.

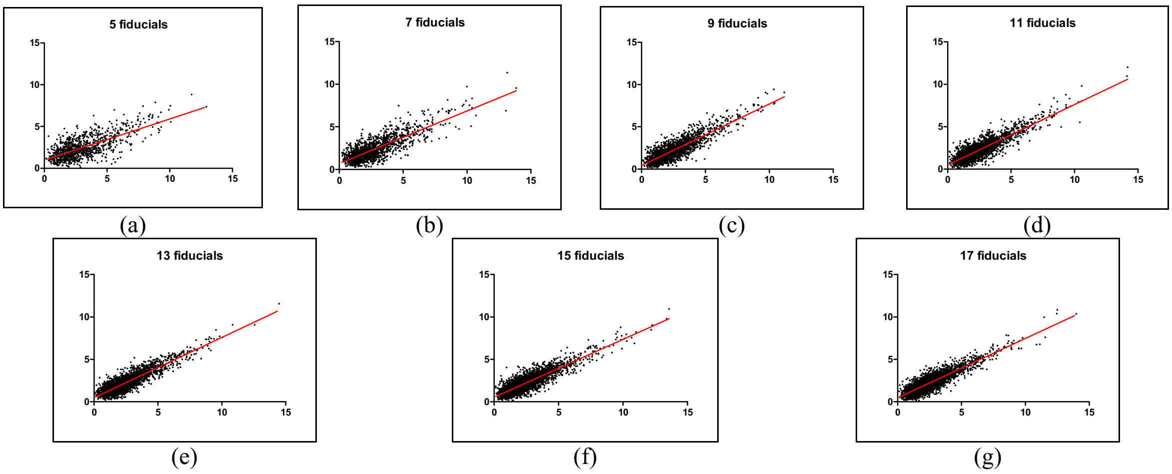

Fig. 2. Plots of *Disp_FLE* for the first test experiment when *FLE* 2 was used. (a)~(g) are the plots for the fiducial point number 5 to 17, respectively. Horizontal axis is the *FLE*, vertical axis is the *Disp*, and a black point is a *Disp_FLE* pair.

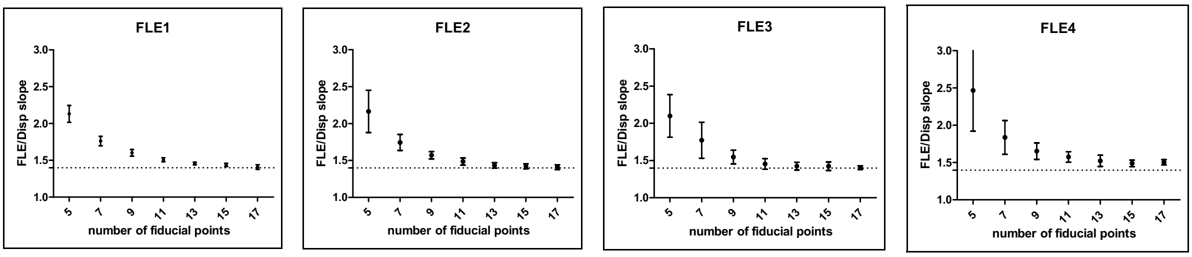

Fig. 3. Slope of *FLE/Disp* obtained from linear regression against the number of fiducial points under different FLE models.

fiducial points, which is practical in many applications, the correlations were 0.753±0.013, 0.775±0.020, 0.825±0.020, and 0.789±0.023, for FLE 1, FLE 2, FLE 3, and FLE 4, respectively.

Figure 2 plots the $Disp - FLE$ pairs of the first test experiment for different number of fiducial points when FLE 2 was used. The red line is the regression line.

Figure 3 indicates that the statistical distribution of the slopes of FLE/Disp is different with different types of FLE and different number of fiducial points, and the standard deviation of the slopes is fairly large when the number of fiducial points is small, both of which make it impossible to find a single linear model to estimate FLE from Disp. However, with the increase of fiducial points, the slope approaches 1.4 and becomes more stable. For the case of 13 fiducial points, the slopes were 1.457±0.018, 1.434±0.036, 1.426±0.051, and 1.523±0.076, for FLE 1, FLE 2, FLE 3, and FLE 4, respectively. Therefore, when the number of fiducial points is large, for example larger than 13, it is possible to establish a linear model to estimate FLE according to Disp. When the number of the fiducial points is small, establishing a linear model will need more prior knowledge, such as the FLE model and the number of fiducial points, and even with this knowledge, the model will still be less accurate.

3.2. Experiment with a fiducial point set used in neuronavigation

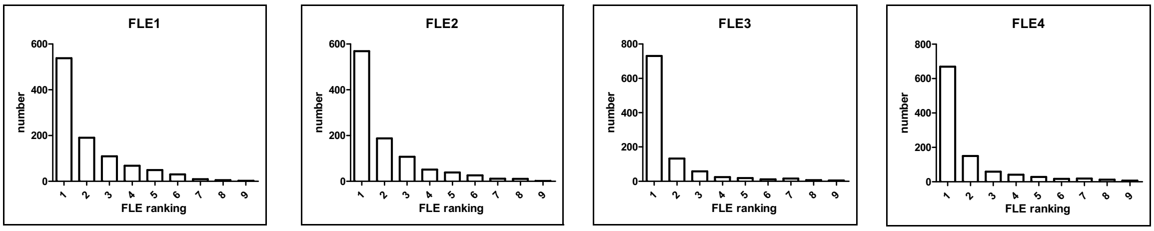

Fig. 4. Histogram of the *FLE* ranking of the fiducial point with the largest *Disp* under different FLE models.

Spatial registration in neuronavigation is usually done by point-based registration. In this experiment, fiducial points from a real neuronavigation case were used. Similar steps were followed as detailed in the experimental section 3.1 to calculate the correlation coefficient between *Disp* and *FLE* for the four different FLE models. The difference is that a set of nine fiducial points from a real neuronavigation case instead of randomly generated fiducial points was used, and step 2 and 3 of the experiment described at the beginning of section 3.1 was repeated for 1,000 times instead of 200 times to calculate the correlation coefficient. The correlation coefficient between *Disp* and *FLE* were 0.759, 0.759, 0.808, and 0.784 for FLE 1, FLE 2, FLE 3, and FLE 4, respectively.

Then, we try to find how accurate it is to estimate the fiducial point with the largest *FLE* with *Disp*. For the four types of FLE model, step 2 and 3 of the experiment described at the beginning of section 3.1 was repeated for 1000 times, in each of which the *FLE* and the *Disp* of each fiducial point were calculated and sorted. In the 1000 times of repetition, we picked up the fiducial point with the largest *Disp* and counted the number of ranking of its *FLE*. The histogram of the *FLE* ranking of the fiducial point with the largest *Disp* is shown in Figure 4. For example, the bin height of FLE1 in Figure 4 is 538, 190, 109, 68, 49, 30, 9, 5 and 2 for FLE ranking from 1 to 9. This means that for the total 1000 times of estimation, in 538 times, the fiducial point with the largest *Disp* was the point with the largest *FLE*; in 190 times, the fiducial point with the largest *Disp* was the point with the 2nd largest *FLE*, and so on.

In Figure 4, we see that by selecting the fiducial point with the largest *Disp*, we have a big chance to select the fiducial point with the largest *FLE*. When considering the more realistic error model in neuronavigation, the FLE 3 and the FLE 4, the probability of selecting the fiducial point with the largest (and the second largest) *FLE* is 73.0% (and 13.3%), and 66.9% (and 15.0%), respectively.

4. Discussion and conclusion

In this paper, we have determined the value *Disp* for each fiducial and this value is strongly correlated to the fiducial's FLE in point based registration. With the help of this value, it is possible to implement a weighted point-based registration or pick out the fiducial point with the largest FLE and optimize its position, so that the accuracy of spatial registration in IGNS can be improved.

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