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AFW extraction based on MCA

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Abstract. This paper improves the learning dictionary construction method for morphological component analysis (MCA) to separate the atrial and ventricular signals. The incoherence is added into the objective function to reduce the sparsity ratio between the atrial and ventricular dictionaries. By using the dictionaries, atrial and ventricular activities are separated from the location of the coefficients. We test the methods on both the synthetic and real atrial data. While extracting AFW from synthetic data, we use the Poisson relation as the measure. The result shows that we can obtain greater relation value using the method this paper presents than using the methods of ABS and PCA. We also conduct spectral analysis on AFW extracted from real atrial data.

Keywords: Atrial fibrillation wave, morphological component analysis, sparse, learned dictionary

1. Introduction

Clinically, atrial fibrillation is the most common arrhythmia, with a morbidity of 0.4 to 1.0 percent. The morbidity gets higher as people get older and over 80 percent of people aged over 80 are suffered from atrial fibrillation [1]. High incidence of atrial fibrillation will lead to high risks of coronary heart disease, high blood pressure, cardiac failure and other diseases. On ECG atrial fibrillation's main feature is disorder of RR intervals, disappearance of P wave and occurrence of jagged AFW. In ECG, AFW shows atrial activity while QRST shows ventricular activities. AFW can be used to distinguish atrial fibrillation types and to predict whether atrial fibrillation will recur or not. Therefore it is vital to extract AFW accurately.

So far the most-use algorithm to extract AFW is ABS [2]. It is simple but has its drawbacks: when the QRS wave distorts greatly, QRST wave remnant will persist in extracted AFW wave; and when ECG signals have ectopic beats, ectopic beats will be mutilated as a whole. The other method is time and space QRST elimination [3]. It uses space changes to compensate for different leaded ECG signals' offsets on timeline and the effect on human physical characteristics to increase accuracy. However it only applies to processing multi-lead ECG signals.

The other type of methods hypothesizes that ECG signals are the linear combination of atrial and ventricular activities. These methods are principal component analysis and independent component analysis. The principal component analysis [4, 5] maps the signals into three subspaces, which

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correspond to atrial activity, ventricular activity and noise signals respectively. Its advantage is that it allows QRST waveform changes, but it does not excel in identifying ventricular space. Independent component analyses [1] are usually used to process multi-lead ECG signals. The key and difficult point is how to identify the part that corresponds to ventricular activity and analyze ECG signals for long period of time. Spectral analysis is good for identifying ventricular activity, but needs prior knowledge of the frequency of ventricular activity's signal.

Recently some scholars put forward atrial fibrillation signal extraction based on wavelet transform [6] and neural network algorithm [7, 8]. The first method expands from seeing AFW as interference. Its disadvantage is much residue of QRST wave in AFW. The neural network algorithm starts at multi-lead signals. It works great at extracting signals but needs large amount of samples for training and its results depend highly on the quality of the samples.

MCA is a signal separation method based on sparse representation. Its main idea is to separate the signal using their morphological components' differences. For human heart, the bioelectricity generated by atrium and ventricle are separated by surrounding tissues when they are conducted, making the atrial cell activity and the ventricular cell activity independent from each other biologically. Therefore ECG signals can be seen as a mixed signal linearly composed by atrial and ventricular activities and other interference signals. Both the atrial and ventricular signals are sparse and it means the value of each source signal is non-zero at very few moments. At this time, the two kinds of signals' irrelevance mean they are highly difficult to be expressed by using same atoms. Therefore this paper uses MCA to separate the atrial and ventricular signals. The second section explains sparse representation and MCA theories, and constructed a proper way to separate signals. The third section shows the results of related experiments.

2. Method

2.1. Sparse representation

The sparse representation of signals is part of the compressive sensing theory. In [9], the mathematical expression of signal *f(t)*'s sparse decomposition is shown as:

$$f(t) = \sum_{i=1}^{n} x_i \psi_i(t) \tag{1}$$

In which Ψ is an orthogonal base and the so-called sparse signal is that the signal can be expressed by linear combination of limited orthogonal bases and would not distort when loses small coefficients. We can use Eq. (2) to solve the sparse representation of signals:

$$\hat{x} = \arg\min_{\alpha} \|x\|_0 \quad \text{s.t.} \quad A\hat{x} = y \tag{2}$$

Based on different starting points, there are two types of solving algorithms. The first type is greedy pursuit, which approaches the signals through an iterative process and chooses a best locally optimal solution at each iterative process, to gradually eliminate the differences between the approximation signals and source signals. The typical algorithm includes MP. The second typical type is convex relaxation method, whose main idea is to put the problem of NP in Eq. (2) into a linear programming problem in norm 1. [10] proves that the two solutions share same result and raises the BP solution. As greedy pursuit is relatively simple, we adopt the MP solution to solve the sparse representation.

2.2. Ventricular activity cancellation using MCA

Suppose the electrical signals generated by atrial and ventricular activities have their corresponding dictionaries that can conduct sparse representation and the atrial activity dictionary can only be expressed as atrial signals sparsely, then

$$\alpha = \arg\min_{\alpha} \|\alpha\|_0 \quad \text{s.t.} \quad X_{AA} = D_{AA}\alpha \tag{3}$$

$$w = \arg\min_{w} \|w\|_0 \quad \text{s.t.} \quad X_{VA} = D_{VA}w \tag{4}$$

X_{AA} and X_{VA} represent the signals of atrial and ventricular activities respectively. D_{AA} and D_{VA} are their corresponding dictionaries. α and w are the sparse coefficients of those signals in corresponding dictionaries. ECG signals X can be expressed as

$$X = X_{VA} + X_{AA} + X_N \tag{5}$$

X_N represents the noise in signals, and it is non-sparse in both sparse domains. Merge Eqs. (3)-(5) and signal X 's sparse representation can be seen as solution of Eq. (6):

$$\{\alpha, w\} = \arg\min\left\{\|\alpha\|_0 + \|w\|_0\right\} \quad \text{s.t.} \quad \|X - D_{VA}w - D_{AA}\alpha\|_2^2 \le \varepsilon \tag{6}$$

[11] proves that in Eq. (4), when ECG signal $\mathbf{x}$, which is composed of atrial and ventricular activities, can be expressed as

$$\|v\|_0 = \|\alpha\|_0 + \|w\|_0 \le \frac{1}{2}\left(1 + \frac{1}{M(D)}\right) \tag{7}$$

Atrial and ventricular activities can be estimated through MP and BP, in which $D = [\mathbf{D}_{VA}; \mathbf{D}_{AA}]$; $M(D)$ is used to describe the richness of the dictionary. It is defined as:

$$M(D) = \max_{\substack{j,k \\ j \ne k}} (d_j^T d_k) \tag{8}$$

d_k is an atom of the normalized dictionary D. Eq. (7) is a multi-objective no-convex function. Inspired from the methods of MP, Eq. (7) can also be written as:

$$\{\alpha, w\} = \arg\min \left\{ \|\alpha\|_1 + \|w\|_1 + \lambda \|X - D_{VA}w - D_{AA}\alpha\|_2^2 \right\} \tag{9}$$

For Eq. (10), Starck [11] introduced residual into it and present a numerical solution. In each circle the coefficients would be processed by a soft-threshold method. The drawback of this method is that it is very difficult to decide the value of the thresholds and the number of iterations could also impact the final results. In this paper we see Eq. (10) as:

$$\{\alpha, w\} = \arg\min \left\{ \|w\|_1 + \|\alpha\|_1 + \lambda \left\| X - \begin{Bmatrix} D_{VA} \\ D_{AA} \end{Bmatrix} \{w, \alpha\} \right\|_2^2 \right\} \tag{10}$$

Here, w and α are the sparse coefficients obtained under the dictionaries D_{VA} and D_{AA} respectively. Eq. (11) contains a 1-norm part and a square part, therefore can be solved by MP or BP. And suppose that atrial and ventricular activities can be uniquely sparsely represented under the dictionaries D_{VA} and D_{AA} respectively, we can easily reconstructed atrial signals and ventricular signals separately. Therefore the core problem is how to design dictionaries D_{VA} and D_{AA}.

2.3. Dictionary design

2.3.1. Design of dictionaries

The selection of dictionaries affects the extraction results and even, the wrong dictionary may make it impossible to extract AFW. Therefore this paper's priority is to construct sparse dictionaries that suit AFW. For Eq. (11), the most ideal dictionary should be able to express the atrial signals as sparse as possible through dictionary D_{AA}, but does not work through dictionary D_{VA}. The dictionary should work the contrary for ventricular signals. Therefore the principles for selecting the dictionaries are:

$$\Gamma(D_{VA}) = \max \left\{ \frac{\|w\|_0}{\|\alpha\|_0} \right\} \tag{11}$$

In which, $w = \arg\min_w \|w\|_0 \quad \text{s.t.} \quad X_{VA} = D_{VA}w$ and $\alpha = \arg\min_\alpha \|\alpha\|_0 \quad \text{s.t.} \quad X_{AA} = D_{AA}\alpha$.

However, it is extremely difficult to get both dictionaries through optimization method. A simple way is to design a dictionary though the signal's prior knowledge and select the most effective dictionary through tests. [12] points out that the Fourier coefficient and wavelet coefficient can be used as smooth signals' sparse transformation bases and oscillator signals can be represented sparsely by Gabor dictionary. Also, when separate the texture part and smooth part of an image, we can choose bi-orthogonal wavelet dictionary and local ridgelet transform for the smooth part, and Gabor dictionary and cosine transformation for the texture parts. The mentioned methods share a common point. Several dictionaries are chosen by experience and then be evaluated by Eq. (12). Best fitted dictionaries are then set as the used dictionaries. The drawback of such methods is they rely too much on experience. Therefore we consider learned dictionaries. One direct way to design a learned dictionary is adding Eq. (12) to the objective function. However this will add the difficulties of solving the problem. Actually, under the constraints of Eqs. (4) and (5), to improve the fitness of dictionaries means to reduce the relevance between dictionaries. Therefore we define:

$$R_{(VA,\,AA)} = \left\| D_{VA}^{T} D_{AA} \right\|_2^2 \tag{12}$$

Supposed that the dictionaries are normalized, therefore $R_{(i,\,j)}$ could get the max value when $D_{VA} = D_{AA}$. $R_{(i,\,j)} = 0$ means each atom of one dictionary are incoherent to other atoms in another dictionary. Therefore we can list the objective functions：

$$\{\alpha, w, D_{VA}, D_{AA}\} = \arg\min\left\{\|\alpha\|_1 + \|w\|_1 + \lambda_1\|X_{VA} - D_{VA}w\|_2^2 + \lambda_2\|X_{AA} - D_{AA}\alpha\|_2^2 + \lambda_3\|D_{VA}^T D_{AA}\|_2^2\right\} \tag{13}$$

Since the Eq. (14) is separately convex in each variable, we proposed a block coordinate descent method, iteratively minimizing firstly with respect to encoding variables α, w, and then to the dictionaries D_{VA} and D_{AA}.

- fix D_{VA} and D_{AA} , update α, w

Supposing D_{VA} and D_{AA} are fixed, we can update encoding variables α, w iteratively, in the way of:

$$\{\hat{\boldsymbol{\alpha}}\} = \arg\min\left\{\|\boldsymbol{\alpha}\|_1 + \lambda_2\|\mathbf{x}_{AA} - \mathbf{D}_{AA}\hat{\boldsymbol{\alpha}}\|_2^2\right\} \tag{14}$$

$$\{\hat{\mathbf{w}}\} = \arg\min\left\{\|\mathbf{w}\|_1 + \lambda_1\|\mathbf{x}_{VA} - \mathbf{D}_{VA}\hat{\mathbf{w}}\|_2^2\right\} \tag{15}$$

Eqs. (15) and (16) are the problems of sparse encoding. We can use basic pursuit or orthogonal matching pursuit method to solve them.

- fix α, w, update D_{VA} and D_{AA}

When α, w are fixed, updating D_{VA} and D_{AA} means solving Eq. (17):

$$\{D_{VA}, D_{AA}\} = \arg\min\left\{\lambda_1\|X_{VA} - D_{VA}w\|_2^2 + \lambda_2\|X_{AA} - D_{AA}\alpha\|_2^2 + \lambda_3\|D_{VA}^T D_{AA}\|_2^2\right\} \tag{16}$$

Eq. (17) is not convex with joint of D_{VA} and D_{AA}. So we use gradient descent method to update D_{VA} and D_{AA} iteratively:

$$\begin{aligned} D_{VA}^{k+1} &= g\left(D_{VA}^{k} - \frac{1}{\delta}\left(D_{VA}^{k} w w^T + 2\frac{\lambda_3}{\lambda_1} D_{AA}^{k}\left(D_{AA}^{k}\right)^T - 2x_{VA}w^T\right)\right) \\ D_{AA}^{k+1} &= g\left(D_{AA}^{k} - \frac{1}{\delta}\left(D_{AA}^{k}\alpha\alpha^T + 2\frac{\lambda_3}{\lambda_2} D_{VA}^{k+1}\left(D_{VA}^{k+1}\right)^T - 2x_{AA}\alpha^T\right)\right) \end{aligned} \tag{17}$$

Table 1

Fitnesses in Different Dictionaries on Synthesized Data

	$\Gamma(D_{VA})$
proposed	26.3
Db4/sine	2.06
Db6 /sine	2.01
Haar/sine	5.13
Boir3.3 /sine	2.07

where $g(d) = d / \max\{1, \|d\|\}$ denotes the projection on the unit ball.

2.3.2. Analysis on dictionary fitness

As the proposed way of constructing dictionaries is based on a dictionary training method in statistics, there is a possibility that the trained dictionaries are effective for fixed samples but are invalid to process new ECG data. We select the data in and outside the samples and evaluate the dictionary constructed based on Eq. (11). The ventricular activities come from MIT–BIH arrhythmia database. There are over 109,000 labeled ventricular beats from 15 different heartbeat types. These beats are contained in 48 recordings each of which have duration of 30 minutes and include two leads. The modified limb lead II and one of the modified leads V1, V2, V4 or V5. The sampling frequency is 360 Hz. the data have already filtered by a band-pass filter at 0.1–100 Hz and the resolution is 200 samples per mV. As the amplitude of P wave in normal heartbeats are small, we neglect the influence of P wave when constructing the dictionary and testifying the results. AFW data are synthesized and generated by the method in paper [3]. Take modified limb lead II for example, we choose 100 normal heartbeat data from 36 recording as training samples to construct sparse dictionary for ventricular activities. The samples to evaluate the sparse dictionary are selected from the other 12 recordings. The frequency of the constructed F wave f_0 ranges from 6-8Hz, frequency variation frequency f_f ranges from 0.1 Hz-0.2 Hz. Frequency variation $\Delta f = 0.2Hz$, harmonics $M = 5$, $a_l = [150, 75, 45]\mu V$, amplitude variation is set as $\Delta a_l = [50, 25, 15]\mu V$ amplitude variation frequency is set as $\omega_a = 0.08Hz$. A total of 20 groups of data are constructed. Table 1 lists the sparseness of the two data in several common dictionaries.

The dictionary constructed by proposed method can obtain the largest fitness when used as the signals for ventricular activities and the lowest fitness when used as signals for AFW. It demonstrates the proposed method may better fit for constructing dictionaries when using MCA-based method for AFW extraction.

For real AF signals, a single beat is segmented into two parts for training atrial and ventricular dictionary respectively. Since the QRS of adults and children usually last for 0.06-0.1s and 0.04-0.08s, we take the sample of 0.1s before R peak as the start and the sample of 0.1s after R peak as the end for the sample to train ventricular dictionary. The other parts are used to train atrial dictionary. The data are come from PTB diagnose database which will be introduced later. Fitness of Different Dictionary couples on Real Atrial Fibrillation Data are listed in Table 2.

Table 2

Fitness of Different Dictionary couples on Real Atrial Fibrillation Data

	$\Gamma(D_{VA})$
proposed	33.2
Db4/sine	3.17
Db6 /sine	1.55
Haar/sine	4.72
Boir3.3 /sine	3.29

3. Experiments and result

3.1. Synthesized data

In this paper, both simulated AF signals and recorded ECG data are employed to test the performance of present method. The synthesis signals are generated by adding simulated atrial activity signals to sinus rhythm recordings following the method mentioned in [3]. The results are given by comparing to ABS and PCA. We first processed the interference in ECG signals before extracting AFW and eradicated the baseline drift under 0.5 Hz and high-frequency noises above 100 Hz. Then we use the method based on wavelet to test the R wave and align each heartbeat data using R peak point as standard. For ABS, the heartbeat samples are averaged to obtain QRST template. The difference between each heartbeat data and the template is the extraction result of AFW. For PCA, the method described in literature [5] is used to extract the aligned ECG data. Figure 1 shows the extraction result of synthesized AF by using the method proposed in this paper. The Possion correlation (ρ) and the mean square error (MSE) are used to evaluate the three methods.

$$\rho = \frac{E\left[\hat{S}_{AA} S_{AA}\right]}{\hat{\sigma}_{AA}\sigma_{AA}} \qquad MSE = \frac{1}{N}\sum_{i=1}^{N}(S_{AA}^{(i)} - \hat{S}_{AA}^{(i)})^2$$

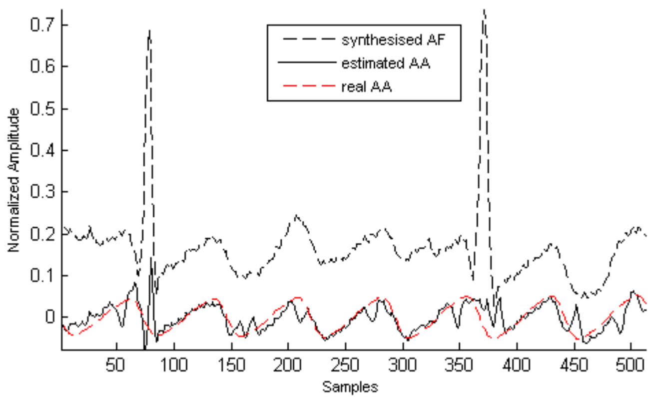

Fig. 1. The extraction result of synthesized atrial fibrillation data (black dotted line shows the synthesized atrial fibrillation data, black full line shows the AFW extracted and the red part shows the synthesized F wave).

Table 3

Pearson correlation indexes for each method

Method	ρ	MSE
Present	426.66	5.46e-04
PCA	389.31	1.28e-04
ABS	345.97	5.26e-03

In which $E[\bullet]$ is an expected operator $\hat{\sigma}_{AA}$ and σ_{AA} are the standard deviation of the extracted AFW and real AFW.

3.2. Real atrial fibrillation data

Apply the same method in actual AFW extraction. The data come from the PTB Diagnostic ECG Database [13], which consists of 594 records from 290 individuals. Each record contains fifteen leaded data. The sampling frequency is 1000 HZ. Digitizing the sample with 16-bit precision and the voltage value remain between minus 16 to 16 millivolts. We select the atrial fibrillation data from five patients suffering from paroxysmal atrial fibrillation and the serial numbers for the data are patient 153/s391lre, patient 201/s0420_re, patient 254/s489_re, patient 257/s493_re, patient 286/s0546_re.

We use the wavelet-based method for denoising these ECG signals each of which lasts 10s, with an aim of eliminating the high frequency noise and base drift. The mother wavelet we choose is DB6 for it is similar to QRS wave in the profile. Then we used the mentioned method for extracting F waves. The result can be found in Figure 2. The dominant frequency of extracted F waves is shown in Table 4.

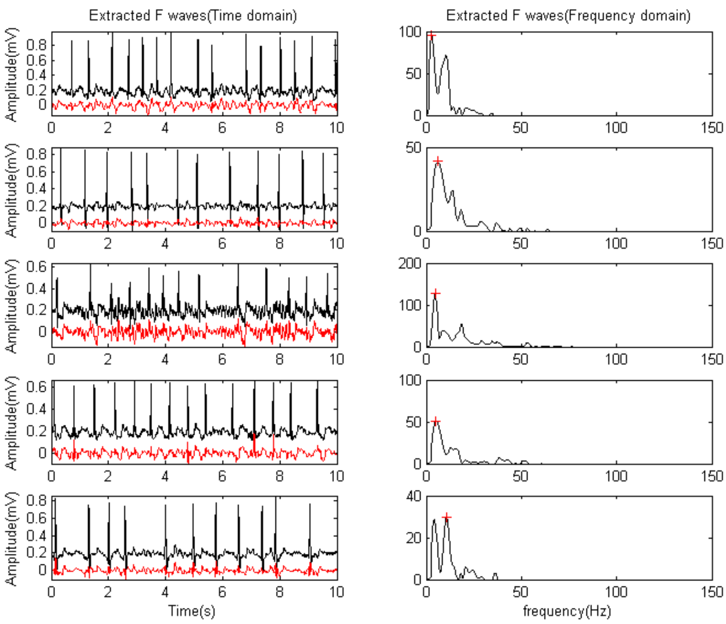

Fig. 2. The result of extracted F wave and their frequency spectrum (The left part shows the extracted F waves. In these parts, the dark line represents the ECG signals after being pre-processing and the red line are extracted F waves. The right part shows frequency spectrum of extracted F waves, where the red crosses denote peaks).

Table 4

Main F wave frequency for each patient

	p1	p2	P3	P4	P5
f_{peak}(Hz)	3.42	5.03	4.63	5.43	10.06

Figure 1 shows result of extracting F waves by mentioned method on simulated AFW. Comparison between extracted F waves and simulated F waves demonstrates the success of presented method. In Table 3 we use Possion correlation and mean square error as the standard to evaluate the extracting effect.Figure 2 shows the extracted F waves from real AFW and their frequency spectrums.

3.3. Analysis of experiment results

Figure 1 shows result of extracting F waves by mentioned method on simulated AFW. Comparison between extracted F waves and simulated F waves demonstrates the success of presented method. In Table 3 we use Possion correlation and MSE as the standard to evaluate the extracting effect. It shows the method proposed in this paper gets bigger Possion correlation value, which means the extracted F waves by proposed method are more similar to the simulated F waves in morphology. Besides, the proposed method also gets a minor MSE value which means it get a minor error between the extracted features and simulated features. Therefore the method proposed out-performs ABS and PCA. Figure 2 shows the extracted F waves from real AFW and their frequency spectrums. We find that the peaks are concentrating around 5 Hz (see in Table 4).

4. Conclusion

This paper uses morphological component analysis to separate the signals of atrial and ventricular activities in atrial fibrillation data. This method is built based on the hypothesis that the two kinds of activities are sparse under different domains. The paper's main contribution lies in that it first proves the feasibility of MCA can extract from atrial fibrillation and then presents a novel method of constructing dictionaries for atrial and ventricular activities. By using the mixed dictionary consisted of these two dictionaries, the atrial and ventricular activities are separated by reconstructing signals using different parts of coefficients and the corresponded dictionary. We extracted both simulated and real atrial fibrillation data. In the experiment of extracting F wave in simulated atrial fibrillation data, we found that the method proposed in this paper can get bigger Possion correlation value and smaller MSE value than using that of ABS and PCA. The peak value of the frequency spectrum of F wave extracted from real atrial fibrillation data center around 5 Hz and the concentration is high in each individual F wave frequency spectrum.

However, there is room for improvement with regard to particular conditions. In some conditions, when the ventricular signals only contains the R or the T wave looks like spike, the ventricular signals are similar to the target AFW in shape. That may cause proposed method losing the robust quality, although the results turn out good. According to our knowledge, clinically, this kind of ECG signal only takes a very small portion of the total ECG recordings. So, in most conditions, our method can separate the atrial signals from ventricular signals with significant robust quality, which means it's useful in AFW Extraction.

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