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Strength in coalitions: Community detection through argument similarity

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Abstract. We present a novel argumentation-based method for finding and analyzing communities in social media on the Web, where a community is regarded as a set of supported opinions that might be in conflict. Based on their stance, we identify argumentative coalitions to define them; then, we apply a similarity-based evaluation method over the set of arguments in the coalition to determine the level of cohesion inherent to each community, classifying them appropriately. Introducing conflict points and attacks between coalitions based on argumentative (dis)similarities to model the interaction between communities leads to considering a meta-argumentation framework where the set of coalitions plays the role of the set of arguments and where the attack relation between the coalitions is assigned a particular strength which is inherited from the arguments belonging to the coalition. Various semantics are introduced to consider attacks' strength to particularize the effect of the new perspective. Finally, we analyze a case study where all the elements of the formal construction of the formalism are exercised.

Keywords: Argument Similarity, Communities, Coalitions, Strength of Attacks between communities

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1. Introduction

The identification of *communities* in social media and the *detection of stances* in Tweets has become increasingly important in recent times [15,18,19,26,28] as a result of the tangible effect that these platforms have on the public opinion. In this domain, identifying communities implies analyzing the position of contributing agents concerning a particular topic or their respective *argumentative stance*; several tools can be used for this purpose, for instance [19] describes an approximation solution based on a supervised classifier that finds stances, and classifies them, over a graph representation. Most of the explored work on identifying stances focuses on the classification of tweets as "in favor" (support), "against" (dispute), or "neutral" (comments or questions) regarding a previous tweet in a conversation [35,37]. Most of these methods focus on analyzing tweets to characterize the relationship between messages in a set.

In this work, we propose an *argumentation-based* method to analyze stances in a debate exchange and formally characterize the relationships between these stances using similarity, understanding the similarity as an attribute of the relationship, not just of the message. Thus, a similarity measure associated with arguments will allow us to find defined positions or communities in social media; to do that, we use argumentation to formalize the exchange of views. In a general sense, *argumentation* can be defined as the study of the interaction of arguments in favor and against a position or claim to determine which are acceptable. *Formal Argumentation Theory* provides several formalisms to model emerging behavior, creating different platforms to perform defeasible reasoning and solve several problematic situations [5–7,20,44]. In the context of our research, an argument and an argumentative stance will be considered synonyms.

In [16], Dung introduced *Abstract Argumentation Frameworks* (AFs), intending to create a tool for modeling situations where an agent considers arguments supporting a claim. The framework allows the representation of attack relations between abstract entities called arguments and provides different acceptability semantics that, in essence, characterize different criteria under which sets of arguments can be accepted together. Cayrol and Lagasquie-Schiex [12] extended Dung's framework by considering two independent types of interactions between arguments: the relationships of attack and support. This formalism, called *Bipolar Argumentation Frameworks* (BAFs), models situations where arguments may give support for other arguments, supplying additional reasons to believe in it. Furthermore, they extend Dung's acceptability semantics, considering the support relationship between arguments.

Based on the bipolar formalism, in [11] the authors presented an approach to use a similarity degree measure between arguments to characterize the attack and support relations in a BAF [2,12]. They consider gradual relations between arguments to define more flexible sets of acceptable arguments. An attractive application of this idea is that similar arguments linked by a support relation could represent a community characterized by a similar stance. The detection of communities in social media will be based on the support and similarity relations between arguments, while the classification of the communities will be done according to the similarity degree between the stances that conform to them. To do this, we will take advantage of the notion of coalition presented in [13] and the framework proposed in [11].

Briefly speaking, to represent a community's strength, first, we analyze the arguments proposed by each community. Then, we consider the context where the argumentation discussion is put into play since the opinions of a particular community may vary according to the argumentative context. Finally, we perform a comparison procedure where arguments are analyzed considering the set of descriptors (a tag or a label describing an aspect to which an argument is connected) that have in common in

A	**Con1:** More than one thousand scientists disagree that human activity is primarily responsible for global climate change.
B	**Con2:** The Cook review of 11,944 peer-reviewed studies found that 66.4% of the studies had no stated position on anthropogenic global warming, and while 32.6% of the studies implied or stated that humans are contributing to climate change, only 65 papers (0.5%) explicitly stated: "that humans are the primary cause of global warming."
C	**Pro1:** The rise in atmospheric CO2 over the last century was caused by human activity, as it occurred at a rate much faster than natural climate changes could produce.
D	**Con3:** Rising levels of atmospheric CO2 do not necessarily cause global warming, which contradicts the core thesis of human-caused climate change.
E	**Pro2:** A National Climate Assessment report said human-caused climate changes, such as increased heat waves and drought, "are visible in every state".
F	**Con4:** Human-produced CO2 is re-absorbed by oceans, forests, and other "carbon sinks," negating any climate changes.
G	**Undef1:** A 2012 Purdue University survey found that 47% of climatologists challenge the idea that humans are primarily responsible for climate change and instead believe that climate change is caused by an equal combination of humans and the environment (37%), mostly by the environment (5%), or that there's not enough information to say (5%).
H	**Undef2:** According to a report from the Tropical Meteorology Project at Colorado State University, specific hurricanes, were not a direct consequence of human-caused global warming.

Fig. 1. Arguments in favor and against the possible causes of climate change.

combination with the defined context. Thus, the closer or more similar the arguments of a particular community are, the greater the strength of the position proposed by the community is. Note that we mainly refer to discursive communities. This clarification is necessary because it will allow us to regard communities as subgroups with cohesive thinking, inspiring us to find a cohesive measure to characterize their behavior.

In this regard, it is pertinent to find a way to determine how close, or *cohesive*, these communities are. Precisely, the cohesion associated with a community expresses how united its members are, how integrated, how fraternal and supportive they are to each other, how much they strive to think together, and how willing they are to work together to achieve collective goals and support a specific outcome.

The following example will illustrate the ideas involved in this research.

Example 1. Figure 1 shows a set of opinions extracted from the *ProCon* website[1] in favor of (pro) or against (con) the following proposition *"Is Human Activity Primarily Responsible for Global Climate Change?"*. We can roughly distinguish three communities that give opinions regarding the responsibility of humans for climate change: one of them supports the idea that human activity is responsible for

[1] See https://climatechange.procon.org. The ProCon website states that its goal is "To promote civility, critical thinking, education, and informed citizenship by presenting the pro and con arguments to debatable issues in a straightforward, nonpartisan, freely accessible way."

climate change (arguments E and C), another confronts the previous one with the opposite position (arguments A, B, D and F), and lastly, there is one representing an intermediate posture between the other two (arguments G and H). Each of these communities aggregates different points of view, providing various aspects supporting the community's overall stance.

By analyzing these well-defined general postures, we will obtain the details of the beliefs each community backs; but, by closely examining the opinions in each community, we can determine each community's inherent *strength*.

We structured this presentation as follows: In Section 2, we briefly explore the notion of *coalition* in a BAF, and we summarize the main ideas behind the *Similarity Valued Argumentation Framework* (S-BAF) [11]; then, in Section 3, we examine the notion of *community*. In Section 4, we detail a mechanism to determine *coalitions based on a similarity measure* and, subsequently, *communities from coalitions*. Furthermore, we detail a mechanism to understand the attacks between them, while our introductory example is developed in Section 5. Related research is discussed in Section 6, while Section 7 is devoted to conclusions and future lines of further our investigations. Finally, in Appendix A, we include the necessary proofs of the theorems and propositions introduced in the main text; in Appendix B, we propose a basic algorithm to compute coalitions.

2. Background

In [12], Cayrol and Lagasquie-Schiex proposed an approach to model two different forms of interaction between arguments, one of these positive, when an argument provides *support* to another, and the other negative, as an argument performs an *attack* to another argument, thus extending Dung's abstract argumentation frameworks by adding the support relation. This approach, known as *bipolar argumentation*, is characterized as an *Bipolar Argumentation Framework* or BAF, and allows to represent an intelligent agentś "bipolar" behavior since reasons in favor of a claim can be regarded as supportive while reasons against it can be viewed as contrary. A BAF is defined as a 3-tuple constituted of a set of atomic arguments and two binary relations representing the attack and support relationships between arguments. We begin by recalling the definition of BAF as presented in [12].

Definition 1 (Bipolar Argumentation Framework (BAF))**.** A Bipolar Argumentation Framework is a 3-tuple $\Theta = \langle \mathtt{Args}, \mathrm{R}_a, \mathrm{R}_s \rangle$, where $\mathtt{Args}$ is a set of arguments, and R_a and R_s are two disjoint binary relations defined on $\mathtt{Args}$ called attack and support, respectively.

The graph description introduced in Dung [16] is extended in BAF, adding the representation of the support relation; thus, G_Θ will denote a bipolar argumentation graph. As mentioned, Cayrol and Lagasquie-Schiex [12] presented the extensions of the attack and support notions introducing the *supported* and *secondary* defeats, combining a sequence of supports with a direct defeat; this move allows to explore the interaction between supporting and defeating arguments.

Definition 2 (Defeat in BAF)**.** Let $\Theta = \langle \mathtt{Args}, \mathrm{R}_a, \mathrm{R}_s \rangle$ be a BAF, and A, B two arguments in $\mathtt{Args}$. Then, it is said that:

- A is a *supported defeat* for B iff there exists a sequence $\mathtt{A}_1\ \mathrm{R}_1\ \ldots\ \mathrm{R}_n\ \mathtt{A}_{n+1}$, with $n \geqslant 1$, where $\mathtt{A}_1 = \mathtt{A}$ and $\mathtt{A}_{n+1} = \mathtt{B}$, such that $\mathrm{R}_i = \mathrm{R}_s$, $1 \leqslant i \leqslant n-1$, and $\mathrm{R}_n = \mathrm{R}_a$, $\mathtt{A}_i \in \mathtt{Args}$, $1 \leqslant i \leqslant n+1$.
- A is a *secondary defeat* for B iff there exists a sequence $\mathtt{A}_1\ \mathrm{R}_1\ \ldots\ \mathrm{R}_n\ \mathtt{A}_{n+1}$, with $n \geqslant 2$, where $\mathtt{A}_1 = \mathtt{A}$ and $\mathtt{A}_{n+1} = \mathtt{B}$, such that $\mathrm{R}_1 = \mathrm{R}_a$, and $\mathrm{R}_i = \mathrm{R}_s$, $2 \leqslant i \leqslant n$, $\mathtt{A}_i \in \mathtt{Args}$, $1 \leqslant i \leqslant n+1$.

Considering the simplest case of defeat in any BAF, a sequence of two arguments $\mathtt{A}\ \mathtt{R}_a\ \mathtt{B}$ is also regarded as a supported defeat from A to B, *i.e.*, a direct defeat is also a supported defeat.

There exist other forms of attack that are considered in other interpretations of BAF, some of them summarized in [55] as *indirect attacks* [39] that considers *mediated attacks*, *extended attacks*, and the *tiered indirect attacks*. However, in the present work, we consider only the simple forms of attack (defeat) presented in Definition 2. However, extending this proposal to consider a more refined BAF version is possible. We will consider this extension in future work.

Following the Cayrol and Lagasquie-Schiex approach [12], in some sense, a set of arguments must keep a minimum of coherence to model one side of any reasonable dispute adequately. They propose that the coherence of an acceptable set of arguments can be kept *internally* by requiring the set not to contain an argument that attacks another one in the same set. Meanwhile, *external* coherence can be maintained by requiring that the set does not include both a supporter and an attacker of the same argument. Internal coherence can be obtained by extending the definition of *conflict-free set* proposed in [16], and external coherence can be captured by the notion of a *safe* set.

Definition 3 (Conflict-freeness and Safety Properties in BAF)**.** Let $\Theta = \langle \mathtt{Args}, \mathtt{R}_a, \mathtt{R}_s \rangle$ be a BAF, and $\mathtt{S} \subseteq \mathtt{Args}$ be a set of arguments. We say that S is *conflict-free* iff $\nexists\, \mathtt{A}, \mathtt{B} \in \mathtt{S}$ s.t. there is an attack (direct, or supported, or secondary) from A to B. We say that S is *safe* iff $\nexists\, \mathtt{A} \in \mathtt{Args}$ and $\nexists\, \mathtt{B}, \mathtt{C} \in \mathtt{S}$ s.t. there is an attack (direct, or supported, or secondary) from B to A, and either there is a sequence of support from C to A, or $\mathtt{A} \in \mathtt{S}$.

Conflict-freeness requires considering the direct, supported, and secondary attacks. Additionally, Cayrol and Lagasquie-Schiex show that the notion of a safe set is powerful enough to encompass the concept of conflict-freeness, *i.e.*, if a set is safe, it is also conflict-free. The closure under $\mathtt{R}_s$ was introduced in [12] is a requirement that only concerns the support relation.

Definition 4 (Closure Property in BAF)**.** Let $\Theta = \langle \mathtt{Args}, \mathtt{R}_a, \mathtt{R}_s \rangle$ be an BAF. $\mathtt{S} \subseteq \mathtt{Args}$ be a set of arguments. S is closed under $\mathtt{R}_s$ iff $\forall\, \mathtt{A} \in \mathtt{S}, \forall\, \mathtt{B} \in \mathtt{Args}$ if $\mathtt{A}\ \mathtt{R}_s\ \mathtt{B}$ then $\mathtt{B} \in \mathtt{S}$.

Succinctly, these are some salient features of this formalism:

- It allows to represent relationships between arguments through a *bipolar interaction graph* that has two kinds of edges: one of them to represent the *attack relation* and the other for the *support relation*.
- It introduces the identification of special types of attack. The notions of *supported* and *secondary* attack combine sequences of *supports* with a *direct attack* considering the interaction between supporting and attacking arguments.
- It describes the notions of internal and external coherence. The set of arguments must keep a minimum of coherence to be able to model adequately one side of any reasonable dispute. This form of consistency is both internal and external when a subset of arguments is considered, and can be obtained by extending the definition of *conflict free set* proposed in [16] in the internal case, while the external coherence can be captured by the notion of *safe* set presented in [12].
- It redefines conflict-free sets. As we mentioned in the previous item, the notion of conflict-freeness requires considering both supported and secondary attacks. Additionally, the notion of a safe set is powerful enough to encompass the concept of conflict-freeness, *i.e.*, if a set is safe, it is also conflict-free. Another critical requirement to be a safe set is that it should be closed under the support relationship [12].

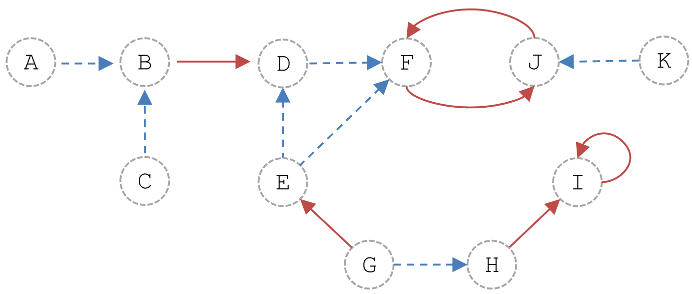

Fig. 2. Representation of the attack and support relations in BAF, the G_Θ graph.

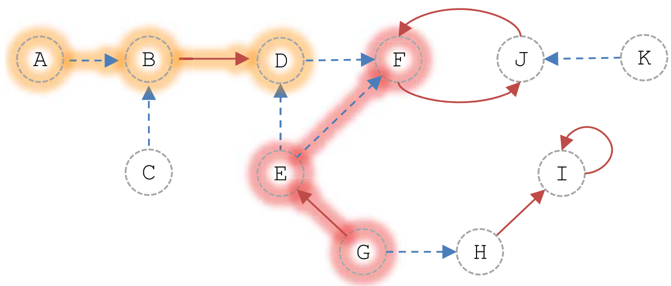

Fig. 3. Representation of the secondary and supported attacks in BAF.

In this work, we will introduce the concept of a coalition as it is revealed through considering similarity to help identify argumentative stances in a conversation and characterizing the relationships between these stances.

Example 2. Next, we present a BAF example described as $\Theta = \langle \mathtt{Args}, \mathtt{R}_a, \mathtt{R}_s \rangle$, where:

$$\mathtt{Args} = \{\mathtt{A}, \mathtt{B}, \mathtt{C}, \mathtt{D}, \mathtt{E}, \mathtt{F}, \mathtt{G}, \mathtt{H}, \mathtt{I}, \mathtt{J}, \mathtt{K}\}$$

$$\mathtt{R}_a = \big\{(\mathtt{B}, \mathtt{D}), (\mathtt{G}, \mathtt{E}), (\mathtt{H}, \mathtt{I}), (\mathtt{I}, \mathtt{I}), (\mathtt{F}, \mathtt{J}), (\mathtt{J}, \mathtt{F})\big\}$$

$$\mathtt{R}_s = \big\{(\mathtt{A}, \mathtt{B}), (\mathtt{C}, \mathtt{B}), (\mathtt{E}, \mathtt{D}), (\mathtt{E}, \mathtt{F}), (\mathtt{D}, \mathtt{F}), (\mathtt{G}, \mathtt{H}), (\mathtt{K}, \mathtt{J})\big\}$$

Conflict-freeness requires considering the direct, supported, and secondary attacks. Figure 2 shows the bipolar argumentation graph of this particular BAF where we can identify the followings defeat relations: the argument G is a secondary defeater for F, while C and A are supported defeaters for argument D (See Fig. 3, where a secondary attack is highlighted in red, while a supported attack is emphasized in orange). Additionally, Cayrol and Lagasquie-Schiex show that the notion of a safe set is powerful enough to encompass the concept of conflict-freeness, *i.e.*, if a set is safe, it is also conflict-free. The closure under $\mathtt{R}_s$ was introduced in [12] is a requirement that only concerns the support relation. Also, the argument K is a supported defeater for the argument F. Furthermore, G is a supported defeater for I, while I is a direct defeater for itself.

This formalism can approach a representation of how human beings think by recognizing the bipolar nature of a debate when discussing a particular topic; however, it is not feasible to clearly distinguish stances on a specific subject. For this reason, in [13], the authors proposed a notion of coalitions between supporting arguments that will be discussed next.

2.1. Coalitions in bipolar argumentation framework

The capability of representing support among arguments available in Bipolar argumentation frameworks becomes relevant when it is necessary to reason with maximal and coherent sets of arguments that are collectively related through that relationship. Cayrol and Lagasquie-Schiex in [13] perform an in-depth analysis of the bipolar framework abstraction introducing the notion of *coalition* that aims to obtain the maximal set of coherent arguments which collaborate to justify a conclusion, *i.e.*, arguments that do not attack each other directly or indirectly. The following definition formally introduces the notion of a coalition.

Definition 5 (Coalitions in a BAF)**.** Let $\Theta = \langle \texttt{Args}, \mathtt{R}_a, \mathtt{R}_s \rangle$ be a BAF, and G_Θ be a bipolar argumentation graph. A subset $\mathcal{C}_\Theta \subseteq \texttt{Args}$ is a *coalition* in Θ iff $\mathcal{C}_\Theta$ is a maximal conflict free set in Θ such that the subgraph G'_Θ induced by $\mathcal{C}_\Theta$ is connected only by support relations.

Note that if (i) $\texttt{A}, \texttt{B} \in \texttt{Args}$, (ii) $\texttt{A}\ \mathtt{R}_s\ \texttt{B}$ and (iii) there is no attack (direct, supported, or secondary attack) from A to B, then there exists a coalition over Θ which contains A and B.

A coalition represents a relationship on the set of arguments; therefore, the notion of attack between them introduces a *meta-argumentation framework* that provides a higher locus where to interpret and analyze the set of supported arguments and the attacks between those sets:

Definition 6 (Attack between coalitions in BAF)**.** Let $\Theta = \langle \texttt{Args}, \mathtt{R}_a, \mathtt{R}_s \rangle$ be a BAF, and let $\mathcal{C}_1$ and $\mathcal{C}_2$ be two coalitions over Θ. If there exist $\texttt{A} \in \mathcal{C}_1$ and $\texttt{B} \in \mathcal{C}_2$ such that $\texttt{A}\ \mathtt{R}_a\ \texttt{B}$, then the coalition $\mathcal{C}_1$ c-attacks (or just attacks) $\mathcal{C}_2$.

Example 3. Continuing with Example 2, Fig. 4 shows the following four coalitions: $\mathcal{C}_1 = \{\texttt{A}, \texttt{B}, \texttt{C}\}$, highlighted green; $\mathcal{C}_2 = \{\texttt{E}, \texttt{D}, \texttt{F}\}$, highlighted purple; $\mathcal{C}_3 = \{\texttt{G}, \texttt{H}\}$, highlighted blue; and $\mathcal{C}_4 = \{\texttt{J}, \texttt{K}\}$, highlighted orange.

These are maximal conflict-free sets, and $\mathcal{C}_1$, $\mathcal{C}_2$, $\mathcal{C}_3$, and $\mathcal{C}_4$ are maximal sets closed under $\mathtt{R}_s$. Additionally, we have that: $\mathcal{C}_1$ attacks $\mathcal{C}_2$, because B attacks D; the coalitions $\mathcal{C}_3$ and $\mathcal{C}_4$ attack the coalition $\mathcal{C}_2$, since J attacks F, and G attacks E; and finally, $\mathcal{C}_2$ attacks $\mathcal{C}_4$, as F attacks J.

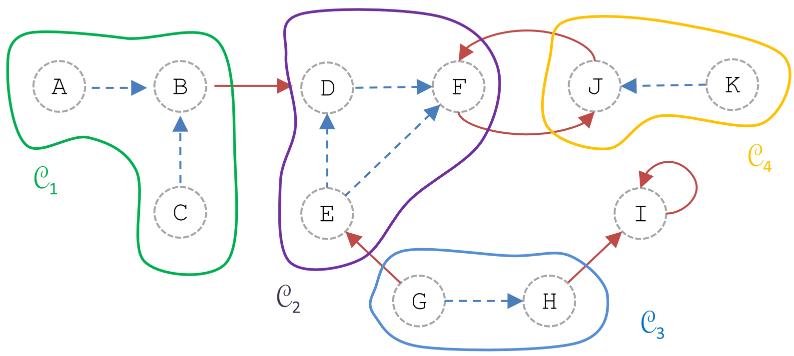

Fig. 4. Coalitions in BAF.

The notions presented in [13] initially do not provide the tools to analyze how strong the coalitions, and the attacks between them, are. Also, note that argument $\mathtt{I}$ does not belong to any coalition, missing information about the discursive domain that would be important to consider. Furthermore, it would be possible that under certain conditions, a coalition *assimilates* another. We will address these observations in the following sections.

2.2. A similarity valued argumentation framework

Despite the representation capacity that bipolar frameworks offer, some argument's characteristics should be taken into account to improve the performance of their semantics. For example, a very natural tool for argument-based reasoning is the notion of similarity among arguments: during an argumentation process, we sometimes tend to group arguments according to their shared characteristics or to the topics to which they refer. It can be argued that any comparison process requires defining a context in which such comparison can be meaningful [23,48]. These intuitions can be applied to arguments as follows: two arguments may be similar in a given context but may be entirely unrelated (or even incomparable) under different circumstances. Reasoning that considers similarities between arguments represents a natural form that is used in everyday human reasoning [11].

Budán et al. introduced a *Similarity-based Bipolar Argumentation Framework* (or S-BAF) in [11] where the interested reader may find relevant examples and discussions illustrating the central ideas in the framework; in what follows, we will recall the definitions and notions concerning S-BAF we need here. The authors described a mechanism for considering the context of the comparison between arguments, and that context is based on a set of descriptors the arguments being analyzed have in common, where a descriptor is a tag or a label describing an aspect to which an argument is connected. With the purpose of the comparison, they introduced *enriched arguments*, that is, arguments decorated with additional information.

We will make some notational conventions to facilitate the following definitions. We assume a set $\mathcal{D}$ of available descriptors corresponding to the domain where the argumentation is carried out. Each descriptor has a set of values associated; thus, for a descriptor $d \in \mathcal{D}$, $\mathcal{V}_d$ will be the set of semantic values corresponding to descriptor d.

Definition 7 (Enriched Argument)**.** Let $\Theta = \langle \mathtt{Args}, \mathrm{R}_a, \mathrm{R}_s \rangle$ be a BAF, $\mathtt{A}$ be an abstract argument in Θ, and $\mathcal{D}$ be a set of descriptors. An *enriched argument* is a pair $\mathrm{A} = \langle \mathtt{A}, \delta_{\mathtt{A}} \rangle$, where $\delta_{\mathtt{A}}$ is a finite non-empty set of pairs $(d, \mathcal{V}_d^{\mathtt{A}})$, where $d \in \mathcal{D}$ and $\mathcal{V}_d^{\mathtt{A}} \subseteq \mathcal{V}_d$. The set of all enriched arguments will be denoted as $\mathbb{A}\mathrm{rgs}$.

Next, we introduce the notion of context of the argumentation.

Definition 8 (Context)**.** Let $\mathcal{D}$ be a set of descriptors, a *context* $\mathbb{C}$ will be represented as $\mathbb{C} = \{(d, w_d) | d \in \mathcal{D}, w_d \in [0, 1]\}$, *i.e.*, a context is a set of ordered pairs where $d \in \mathcal{D}$ is a descriptor and $w_d \in [0, 1]$ is the weight associated with d. We denote with $\mathcal{D}^{\mathbb{C}}$ the set of descriptors mentioned in the context $\mathbb{C}$, *i.e.*, $\mathcal{D}^{\mathbb{C}} = \{d | (d, w_d) \in \mathbb{C}\}$.

Using the additional information provided by the context, it is possible to represent and determine similarities between arguments by introducing means to enrich the analysis of the relationships between them and distinguish between arguments that are weakly related to those with stronger relationships. In this direction, it is possible to compute an argument's similarity degree between two arguments. To do that, we consider the descriptors that arguments have in common and the weight those descriptors have in the process comparison in a specific context comparison $\mathbb{C}$. This context is defined as a subset of the description that specifies a point of analysis. Given a context $\mathbb{C}$, for any argument $\mathrm{X} \in \mathbb{A}\mathrm{rgs}$, we denote the descriptors in X that appear on the context $\mathbb{C}$ as $\mathcal{D}_{\mathrm{X}}^{\mathbb{C}}$, *i.e.*, $\mathcal{D}_{\mathrm{X}}^{\mathbb{C}} = \mathcal{D}_{\mathrm{X}} \cap \mathcal{D}^{\mathbb{C}}$.

Definition 9 (Similarity coefficient for a descriptor)**.** Let $\mathbb{A}\mathrm{rgs}$ be a set of enriched arguments, $\mathrm{A} = \langle \mathtt{A}, \delta_{\mathtt{A}} \rangle$ and $\mathrm{B} = \langle \mathtt{B}, \delta_{\mathtt{B}} \rangle$ two enriched arguments in $\mathbb{A}\mathrm{rgs}$, and $\mathbb{C}$ a context. We define the similarity coefficient for each descriptor $d \in \mathcal{D}_{\mathrm{A}}^{\mathbb{C}} \cap \mathcal{D}_{\mathrm{B}}^{\mathbb{C}}$ with weight w_d, denoted $\mathsf{Coef}_d(\mathrm{A}, \mathrm{B})$, as follows:

$$\mathsf{Coef}_d(\mathrm{A}, \mathrm{B}) = \begin{cases} \frac{|\mathcal{V}_d^{\mathtt{A}} \cap \mathcal{V}_d^{\mathtt{B}}|}{|\overline{\mathcal{V}_d^{\mathtt{A}} \cap \mathcal{V}_d^{\mathtt{B}}}|} \cdot w_d & \text{if } |\overline{\mathcal{V}_d^{\mathtt{A}} \cap \mathcal{V}_d^{\mathtt{B}}}| \neq 0 \\ w_d & \text{otherwise} \end{cases}$$

Intuitively, finding the similarity coefficient between two arguments for a particular descriptor requires finding the number of semantic values common to that descriptor in both arguments and dividing it by the number of semantic values for the descriptor that the arguments do not have in common, and then weighing the resulting value according to the relevance associated with the descriptor in the definition of the context considered [24,29,32].

Definition 10 (Similarity degree between arguments)**.** Let $\mathbb{A}\mathrm{rgs}$ be a set of enriched arguments, $\mathrm{A} = \langle \mathtt{A}, \delta_{\mathtt{A}} \rangle$ and $\mathrm{B} = \langle \mathtt{B}, \delta_{\mathtt{B}} \rangle$ be two enriched arguments in $\mathbb{A}\mathrm{rgs}$, and $\mathbb{C}$ be a context. The similarity degree between arguments in a context $\mathbb{C}$, denoted $\mathsf{Sim}_{\mathbb{C}}$, is defined as a function $\mathsf{Sim}_{\mathbb{C}} : \mathbb{A}\mathrm{rgs} \times \mathbb{A}\mathrm{rgs} \to [0, 1]$, such that:

$$\mathsf{Sim}_{\mathbb{C}}(\mathrm{A}, \mathrm{B}) = \begin{cases} \alpha_n & \text{if } \mathcal{D}_{\mathrm{A}}^{\mathbb{C}} \cap \mathcal{D}_{\mathrm{B}}^{\mathbb{C}} = \{d_1, \ldots, d_n\} \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

where $\alpha_1 = \mathsf{Coef}_{d_1}(\mathrm{A}, \mathrm{B})$ and $\alpha_i = \odot(\alpha_{i-1}, \mathsf{Coef}_{d_i}(\mathrm{A}, \mathrm{B}))$ with $2 \leqslant i \leqslant n$, and, the operator $\odot$ should be either a *T-norm* satisfying the following properties: commutative, associative, monotonically increasing, and with 1 as its neutral element; or $\odot$ should be a *T-conorm*, satisfying commutative, associative, monotonically decreasing with 0 as its neutral element.

Note that the order in which the descriptors are considered in computing $\mathsf{Sim}_{\mathbb{C}}(\cdot, \cdot)$ is irrelevant since the operator $\odot$ satisfies commutativity and associativity. Furthermore, considering the monotony property and the fact that *T-norms* and *T-conorms* are defined in the interval [0, 1], we can ensure that the resulting value is non-negative.

The abstract concepts presented earlier will be illustrated in the following example.

Example 4. Suppose that the arguments `A` and `B` of our abstract example represent the following opinions regarding students and their homework:

`A`: Research published in the High School Journal indicated that "students who spent between 31 and 90 minutes each day on homework scored about 40 points higher on the SAT-Mathematics subtest than their peers, who reported spending no time on homework each day, on average."

`B`: Research by the Institute for the Study of Labor (IZA) concluded that "increased homework led to better GPAs and higher probability of college attendance for high school boys. In fact, students who attended college did more than three hours of additional homework per week in high school."

Analyzing the arguments above, we observe that they have the following descriptors and values:

$$\delta_{\mathtt{A}} = \big\{\big(\textit{mentions\_student}, \{\textit{yes}\}\big); \big(\textit{refers\_good\_practice}, \{\textit{yes}\}\big); \big(\textit{time\_mention}, \{\textit{yes}\}\big);$$
$$\big(\textit{presents\_results}, \{\textit{scored\_better\_Mathematics\_subtest}\}\big); \big(\textit{based\_on\_evidence}, \{\textit{yes}\}\big)\big\}.$$
$$\delta_{\mathtt{B}} = \big\{\big(\textit{mentions\_student}, \{\textit{yes}\}\big); \big(\textit{refers\_good\_practice}, \{\textit{yes}\}\big); \big(\textit{time\_mention}, \{\textit{yes}\}\big);$$
$$\big(\textit{presents\_results}, \{\textit{better\_GPAs}; \textit{higher\_probability\_college\_atendance}\}\big);$$
$$\big(\textit{based\_on\_evidence}, \{\textit{yes}\}\big)\big\}.$$

Now, suppose that the context for the arguments comparison is the following:

$$\mathbb{C} = \big\{(\textit{mentions\_student}, 0.4); (\textit{refers\_good\_practice}, 0.4); (\textit{presents\_results}, 0.2)\big\},$$

Per each descriptor, we have:

- For the descriptor *mentions_student*: the two arguments have a single value in common and no different ones. So, the $\mathsf{Coef}_d(\mathrm{A}, \mathrm{B}) = 0.4$.
- For the descriptor *refers_good_practice*: the two arguments have a single value in common and no different ones. So, the $\mathsf{Coef}_d(\mathrm{A}, \mathrm{B}) = 0.4$
- For the descriptor *presents_results*: arguments have different values for this descriptor, and no common value. So that, according to the similarity coefficient definition, the $\mathsf{Coef}_d(\mathrm{A}, \mathrm{B}) = 0$.

Now, considering the *bounded sum T-conorm*, we have that the $\mathsf{Sim}_{\mathbb{C}}(\mathrm{A}, \mathrm{B}) = 0.8$, given that:

$$\alpha_1 : \min(0.4 + 0.4, 1) = \min(0.8, 1) = 0.8$$
$$\alpha_2 : \min(0.8 + 0, 1) = \min(0.8, 1) = 0.8$$

The similarity value obtained reflects that the both arguments refer to a good practice for the students, but each argument gives different reasons (results) for this assertion.

The next step is to define a cohesion degree between supporting arguments and a controversy degree between conflicting arguments. In the following definition, we introduce the enriched BAF framework based on the original BAF. Formally,

Definition 11 ((Induced) Enriched BAF)**.** Let $\Theta = \langle \mathtt{Args}, \mathtt{R}_a, \mathtt{R}_s \rangle$ be a BAF, the enriched BAF induced is defined as $\overline{\Theta} = \langle \mathbb{A}\mathrm{rgs}, \mathbb{R}_s, \mathbb{R}_a \rangle$, where $\mathbb{A}$rgs is the set of enriched arguments corresponding to arguments in $\mathtt{Args}$, and $\mathbb{R}_a$ and $\mathbb{R}_s$ are the attack and support relationships in $\mathbb{A}$rgs that are induced by $\mathtt{R}_a$ and $\mathtt{R}_s$, respectively.

Given that the Enriched BAF is based on the classic BAF, the former contains the same arguments and induced relations that the classic BAF, except that in the Enriched BAF, the arguments are decorated with supplementary information. The additional information added to the arguments will be helpful later in extending the formalism.

Now, the *cohesion degree* of a set of supporting enriched arguments and the *controversy degree* associated with a set of attacking enriched arguments can be formally introduced.

Definition 12 (Cohesion & Controversy degrees)**.** Let $\overline{\Theta} = \langle \mathbb{A}\text{rgs}, \mathbb{R}_a, \mathbb{R}_s \rangle$ be the enriched BAF induced from the BAF $\Theta = \langle \texttt{Args}, \mathtt{R}_a, \mathtt{R}_s \rangle$. Given a set of enriched arguments $\mathbb{S} \subseteq \mathbb{A}\text{rgs}$ and a context $\mathbb{C}$, let $\mathsf{Sim}_{\mathbb{C}}$ be a similarity degree function for $\mathbb{C}$, and $\mathbb{R}_a^{\mathbb{S}} = \{(\mathrm{X}, \mathrm{Y}) \in \mathbb{R}_a \mid \mathrm{X}, \mathrm{Y} \in \mathbb{S}\}$ be the subset of $\mathbb{R}_a$ restricted to the arguments of $\mathbb{S}$ and $\mathbb{R}_s^{\mathbb{S}} = \{(\mathrm{X}, \mathrm{Y}) \in \mathbb{R}_s \mid \mathrm{X}, \mathrm{Y} \in \mathbb{S}\}$ be the subset of $\mathbb{R}_s$ restricted to the arguments of $\mathbb{S}$ then we have:

- The *cohesion degree* for $\mathbb{S}$, denoted as $\mathsf{Coh}_{\mathbb{C}}(\mathbb{S})$, is defined as:

$$\mathsf{Coh}_{\mathbb{C}}(\mathbb{S}) = \begin{cases} \beta_n & \text{if } \mathbb{R}_s^{\mathbb{S}} = \{(\mathrm{A}_1, \mathrm{B}_1), \ldots, (\mathrm{A}_n, \mathrm{B}_n)\} \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

  where $\beta_1 = \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_1, \mathrm{B}_1)$ and $\beta_i = \oplus(\beta_{i-1}, \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_i, \mathrm{B}_i))$ with $2 \leqslant i \leqslant n$.
- The *controversy degree* for $\mathbb{S}$, denoted as $\mathsf{Cont}_{\mathbb{C}}(\mathbb{S})$, is defined as:

$$\mathsf{Cont}_{\mathbb{C}}(\mathbb{S}) = \begin{cases} \gamma_n & \text{if } \mathbb{R}_a^{\mathbb{S}} = \{(\mathrm{A}_1, \mathrm{B}_1), \ldots, (\mathrm{A}_n, \mathrm{B}_n)\} \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

  where $\gamma_1 = \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_1, \mathrm{B}_1)$ and $\gamma_i = \otimes(\gamma_{i-1}, \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_i, \mathrm{B}_i))$ with $2 \leqslant i \leqslant n$.

Both $\mathsf{Coh}_{\mathbb{C}}(\cdot)$ and $\mathsf{Cont}_{\mathbb{C}}(\cdot)$ can be obtained independently using a recursive function instantiated with T-norms or T-conorms, essentially in the same manner as with the similarity function $\mathsf{Sim}_{\mathbb{C}}$, depending on the user modeling intentions. It is required that the functions chosen satisfy commutativity, associativity, and monotonicity and have an identity element. These properties ensure that the order of the calculations does not affect the result [27]. Note that both degrees deliver a non-negative real number; more precisely, both are defined as $2^{\mathbb{A}\text{rgs}} \longrightarrow [0, 1]$.

Next, for simplicity, we present our abstract example where the similarity degree associated with each relationship (attack or support) was previously established (for more details, see [11]). Note that, in this formalism, both attacks and support are treated likewise. If an argument X attacks or supports another argument Y, the similarity measure is assigned to the relationship without differentiating the kind of relationship. Mainly it is because we want to be balanced in dealing with the positive and negative actions over a discussion.

Example 5. We continue with our abstract example, the graph in Fig. 5 shows the similarity degree associated with the arguments in each relation. Intuitively, we can observe that the attack between the arguments B and D is weaker than the attack between G and E. Note that the attack between F and J have the same similarity degree that the attack from J to F since the similarity relation is symmetric. Additionally, we can also differentiate the weakest support relationship existing in the whole model: the one between G and H. Based on the similarity degree obtained in each relation, we compute the cohesion coefficient associated with the set of supporting arguments (considering a product T-norm) and the controversy coefficient associated with attacking arguments (considering a max T-conorm). Thus, we have that:

$$\mathsf{Coh}_{\mathbb{C}}\big(\{(\mathrm{E}, \mathrm{D}), (\mathrm{D}, \mathrm{F})\}\big) = 0.42 \qquad \mathsf{Coh}_{\mathbb{C}}\big(\{(\mathrm{A}, \mathrm{B})\}\big) = 0.8$$

$$\mathsf{Coh}_{\mathbb{C}}\big(\{(\mathrm{C}, \mathrm{B})\}\big) = 0.6 \qquad \mathsf{Coh}_{\mathbb{C}}\big(\{(\mathrm{E}, \mathrm{F})\}\big) = 0.8$$

$$\mathsf{Coh}_{\mathbb{C}}\big(\{(\mathrm{G}, \mathrm{H})\}\big) = 0.5 \qquad \mathsf{Coh}_{\mathbb{C}}\big(\{(\mathrm{K}, \mathrm{J})\}\big) = 0.6$$

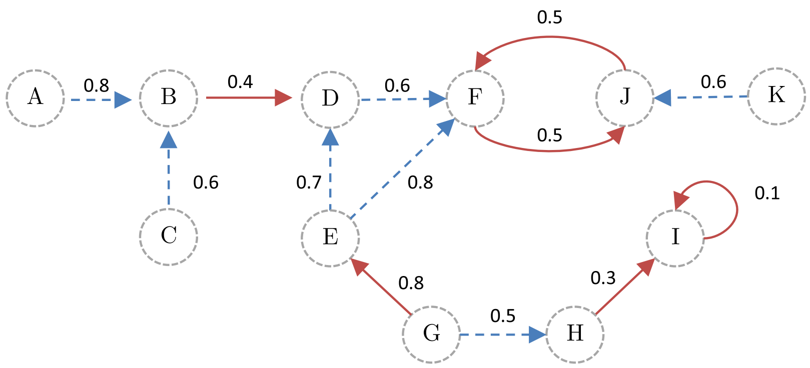

Fig. 5. Similarity in the bipolar argumentation framework (Figs 2 and 4).

$$\mathsf{Cont}_{\mathbb{C}}(\{(\mathrm{B},\mathrm{D})\}) = 0.4 \qquad \mathsf{Cont}_{\mathbb{C}}(\{(\mathrm{G},\mathrm{E})\}) = 0.8$$
$$\mathsf{Cont}_{\mathbb{C}}(\{(\mathrm{H},\mathrm{I})\}) = 0.3 \qquad \mathsf{Cont}_{\mathbb{C}}(\{(\mathrm{I},\mathrm{I})\}) = 0.1$$
$$\mathsf{Cont}_{\mathbb{C}}(\{(\mathrm{H},\mathrm{I}),(\mathrm{I},\mathrm{I})\}) = 0.03 \qquad \mathsf{Cont}_{\mathbb{C}}(\{(\mathrm{F},\mathrm{J})\}) = 0.5$$
$$\mathsf{Cont}_{\mathbb{C}}(\{(\mathrm{J},\mathrm{F})\}) = 0.5 \qquad \mathsf{Cont}_{\mathbb{C}}(\{(\mathrm{F},\mathrm{J}),(\mathrm{J},\mathrm{F})\}) = 0.25$$

Observe that, in this particular case, the cohesion associated with the support relation is analyzed considering the support chain presented in the argumentation model (see Figure 5). At the same time, the controversy measure is obtained by analyzing the pairs of attacking arguments.

The enriched BAF $\overline{\Theta}$ will be extended to include the degrees just defined.

Definition 13 (Similarity-based BAF)**.** Let $\overline{\Theta} = \langle \mathbb{A}\mathrm{rgs}, \mathbb{R}_s, \mathbb{R}_a \rangle$ be an enriched BAF and $\mathbb{C}$ a context, a *Similarity-Based Bipolar Argumentation Framework* (or S-BAF) is defined as a tuple $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}} \rangle$, where $\mathsf{Sim}_{\mathbb{C}}$ is a similarity degree function for enriched arguments in $\mathbb{A}\mathrm{rgs}$, and $\mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}$ and $\mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}}$ are, respectively, the cohesion and controversy degree functions defined over $\overline{\Theta}$ in the context $\mathbb{C}$.

When no confusion may arise, we will avoid mentioning the $\overline{\Theta}$ enriched BAF as a superscript of the cohesion and controversy degree operators, writing instead $\langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}, \mathsf{Cont}_{\mathbb{C}} \rangle$, making the notation more straightforward.

Additionally, in S-BAF, the support and attack relations will have a particular interpretation since a threshold $\tau \in [0, 1]$ will be considered in the specification of the type of attack being analyzed. The strong-support relation in S-BAF occurs when the cohesion associated with the relationship is greater than the threshold τ and when this value is less than the τ, we will say we have a weak-support relation. Consequently, the attacks in an S-BAF will be of two types: (i) *strong*, when the cohesion and controversy values associated with the attack are greater than the threshold τ; in this situation, we have *strong-direct attack*, *strong-supported attack*, and *strong-secondary attack*, or (ii) *weak*, if at least one of the values is less than τ; then, in this case, we have *weakly-direct attack*, *weakly-supported attack*, and *weakly-secondary attack*.

Now, and considering the elements introduced above, the authors in [11] redefine the classical notions of *conflict-free* and *safe* sets in a BAF that are the basis for a new family of semantics (see *op. cit.* for more details).

Definition 14 (Conflict-freeness and Safety properties in a S-BAF)**.** Let $\Phi = \langle\overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}, \mathsf{Cont}_{\mathbb{C}}\rangle$ be a S-BAF, where $\overline{\Theta} = \langle\mathbb{A}\text{rgs}, \mathbb{R}_s, \mathbb{R}_a\rangle$ is the enriched BAF, and $\tau \in [0, 1]$ be a given threshold. Then:

- $\mathbb{S}$ is a *strongly-conflict-free* set iff there is no A, B $\in \mathbb{S}$ such that there exists a strong or weak attack from A to B.
- $\mathbb{S}$ is a *τ-conflict-free* set iff there is no A, B $\in \mathbb{S}$ such that there exists a strong attack from A to B and $\mathsf{Cont}_{\mathbb{C}}(\mathbb{S}) > \tau$.
- $\mathbb{S}$ is a *weakly-conflict-free* set iff there is no A, B $\in \mathbb{S}$ such that there exists a strong attack from A to B.
- $\mathbb{S}$ is a *strongly-safe* set iff there is no A $\in \mathbb{A}$rgs and no pair B, C $\in \mathbb{S}$ such that there exists a strong or weak attack from B to A, and either there is a sequence of support from C to A, or A $\in \mathbb{S}$.
- $\mathbb{S}$ is *τ-safe* set iff there is no A $\in \mathbb{A}$rgs and no pair B, C $\in \mathbb{S}$ such that there exists a strong attack from B to A, $\mathsf{Cont}_{\mathbb{C}}(\mathbb{S} \cup \{A\}) > \tau$, and either there is a sequence of support from C to A such that $\mathsf{Coh}_{\mathbb{C}}(\{C, \dots, A\}) > \tau$, or A $\in \mathbb{S}$.
- $\mathbb{S}$ is *weakly-safe* set iff there is no A $\in \mathbb{A}$rgs and no pair B, C $\in \mathbb{S}$ such that there is a strong attack from B to A and either there is a sequence of support from C to A such that $\mathsf{Coh}_{\mathbb{C}}(\{C, \dots, A\}) > \tau$, or A $\in \mathbb{S}$.

In the following step, in [11] the authors extended the notions of defense for an argument with respect to a set of arguments. Furthermore, the paper introduces different definitions of admissibility, from the most general and strong to the most specific and weak. The most general is based on the classical notion of admissibility, where only the attack relations are considered, both the strong and the weak ones.

Definition 15. Let $\mathbb{S} \subseteq \mathbb{A}$rgs be a set of arguments, and A $\in \mathbb{A}$rgs an argument. Then:

- $\mathbb{S}$ is a *strong defense* for A iff for all B $\in \mathbb{A}$rgs such that if B is a strong or weak (direct, supported, or secondary) attacker of A then there exists C $\in \mathbb{S}$ where C is a strong (direct, supported, or secondary) attacker of B.
- $\mathbb{S}$ is a *weak defense* for A iff for all B $\in \mathbb{A}$rgs such that if B is a strong or weak attacker (direct, supported, or secondary) attacker of A then there exists C $\in \mathbb{S}$ where C is a weak attacker (direct, supported, or secondary) attacker of B.

Then, this notion is extended by considering external coherence and under different attack and support degrees among arguments. Finally, external coherence is strengthened by requiring the closure under the support relation ($\mathbb{R}_s$).

Definition 16. Let $\Phi = \langle\overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}}\rangle$ be a S-BAF with the underlying enriched bipolar argumentation framework $\overline{\Theta} = \langle\mathbb{A}\text{rgs}, \mathbb{R}_s, \mathbb{R}_a\rangle$, and $\mathbb{S} \subseteq \mathbb{A}$rgs be a set of enriched arguments. Then:

- $\mathbb{S}$ is *d-strongly-admissible* if $\mathbb{S}$ is strongly-conflict-free and strongly-defends all its elements.
- $\mathbb{S}$ is *d-τ-admissible* if $\mathbb{S}$ is τ-conflict-free and there exists a strong or weak defense for all its elements.
- $\mathbb{S}$ is *d-weakly-admissible* if $\mathbb{S}$ is weakly-conflict-free and there exists a strong or weak defense for all its elements, or $\mathbb{S}$ is strongly-conflict-free and weakly-defends all its elements.
- $\mathbb{S}$ is *s-strongly-admissible* if $\mathbb{S}$ is strongly-safe and strongly-defends all its elements.
- $\mathbb{S}$ is *s-τ-admissible* if $\mathbb{S}$ is τ-safe and there exists a strong or weak defense for all its elements.

- $\mathbb{S}$ is *s-weakly-admissible* if $\mathbb{S}$ is weakly-safe and there exists a strong or weak defense for all its elements, or $\mathbb{S}$ is strongly-safe and weakly-defends all its elements.
- $\mathbb{S}$ is *c-strongly-admissible* if $\mathbb{S}$ strongly-conflict-free, closed under $\mathbb{R}_s$ and strongly-defends all its elements.
- $\mathbb{S}$ is *c-τ-admissible* if $\mathbb{S}$ τ-conflict-free, closed under $\mathbb{R}_s$ and there exists a strong or weak defense for all its elements.
- $\mathbb{S}$ is *c-weakly-admissible* if $\mathbb{S}$ weakly-conflict-free, closed under $\mathbb{R}_s$ and there exists a strong or weak defense for all its elements, or $\mathbb{S}$ strongly-conflict-free, closed under $\mathbb{R}_s$ and weakly-defends all its elements.

In this manner, in [11] it is argued that admissibility becomes a characteristic of a set of arguments that can be perceived from different perspectives. The most restrictive admissible sets do not admit conflicts and defend all their elements with values of controversy greater than the given threshold. A more flexible admissibility property is when a certain level of controversy associated with the set, which is limited by the threshold, is acceptable. In this case, the arguments' defense can oscillate between strong and weak. Finally, the most flexible set allows the existence of conflicts where the controversy associated with them is strictly less than the threshold; *i.e.*, the controversy is analyzed individually for each pair of conflicting arguments. In the last two cases, the arguments' defense can fluctuate between strong and weak.

From the notions of coherence (internal and external) and admissibility, it is possible to introduce different acceptability semantics. In [11], Budan et al. introduced a more fine-grained definition of preferred extensions as follows:

Definition 17. Let $\Phi = \langle\overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}^{\overline{\Theta}}, \mathsf{Cont}_{\mathbb{C}}^{\overline{\Theta}}\rangle$ be a S-BAF with the underlying enriched bipolar argumentation framework $\overline{\Theta} = \langle\mathbb{A}\text{rgs}, \mathbb{R}_s, \mathbb{R}_a\rangle$, and $\mathbb{S} \subseteq \mathbb{A}\text{rgs}$ be a set of enriched arguments. Then:

- $\mathbb{S}$ is a *d-strongly-preferred (resp. s-strongly-preferred, c-strongly-preferred) extension* of Φ if $\mathbb{S}$ is $\subseteq$-maximal among the d-strongly-admissible (resp. s-strongly-admissible, c-strongly-admissible) subsets of $\mathbb{A}$rgs.
- $\mathbb{S}$ is a *d-τ-preferred (resp. s-τ-preferred, c-τ-preferred) extension* of Φ if $\mathbb{S}$ is $\subseteq$-maximal among the d-τ-admissible (resp. s-τ-admissible, c-τ-admissible) subsets of $\mathbb{A}$rgs.
- $\mathbb{S}$ is a *d-weakly-preferred (resp. s-weakly-preferred, c-weakly-preferred) extension* of Φ if $\mathbb{S}$ is $\subseteq$-maximal among the d-weakly-admissible (resp. s-weakly-admissible, c-weakly-admissible) subsets of $\mathbb{A}$rgs.

Next, we analyze our running example to obtain the different types of acceptable argument sets, where the properties of conflict-freeness and safety are considered.

Example 6. We continue analyzing the Example 5 presented in Fig. 5, introducing a threshold $\tau = 0.48$. With that addition we obtain:

- Weakly-direct attacks, with controversy coefficient lower than τ, are from B to D, H to I, and from I to I.
- Strongly-direct attacks, with a controversy coefficient greater than τ, are from F to J, J to F, and from G to E.
- Weakly-supported attacks are from C to D (since $\mathsf{Coh}_{\mathbb{C}}(\{(\mathrm{C}, \mathrm{B})\}) \geqslant \tau$ and $\mathsf{Cont}_{\mathbb{C}}(\{(\mathrm{B}, \mathrm{D})\}) < \tau$), from A to D (because $\mathsf{Coh}_{\mathbb{C}}(\{(\mathrm{A}, \mathrm{B})\}) \geqslant \tau$ and $\mathsf{Cont}_{\mathbb{C}}(\{(\mathrm{B}, \mathrm{D})\}) < \tau$), from E to J (given that $\mathsf{Coh}_{\mathbb{C}}(\{(\mathrm{E}, \mathrm{D}), (\mathrm{D}, \mathrm{F})\}) < \tau$ and $\mathsf{Cont}_{\mathbb{C}}(\{(\mathrm{F}, \mathrm{J})\}) \geqslant \tau$), and from G to I (due to $\mathsf{Coh}_{\mathbb{C}}(\{(\mathrm{G}, \mathrm{H})\}) \geqslant \tau$ and $\mathsf{Cont}_{\mathbb{C}}(\{(\mathrm{H}, \mathrm{I})\}) < \tau$).

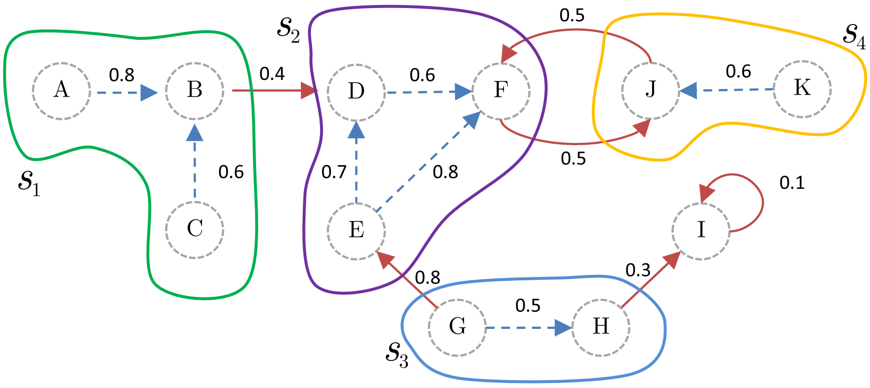

Fig. 6. Analysis of admissibility in S-BAF.

- Strongly-supported attacks, with the controversy and cohesion coefficients greater than τ, are from E to J, and from K to F.
- A *strongly-secondary attack*, with the controversy and cohesion coefficients greater than τ, is from G to F.
- A *weakly-secondary attack* is from B to F because $\mathsf{Cont}_{\mathbb{C}}(\{(\mathrm{B},\mathrm{D})\}) < \tau$) and $\mathsf{Coh}_{\mathbb{C}}(\{(\mathrm{D},\mathrm{F})\}) \geqslant \tau$.

Additionally, we have:

- $S_1 = \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$, is *strongly-conflict-free*, *strongly-safe*, because there are no elements in the set that simultaneously support and attack external arguments. This set does not receive attacks, therefore is d-strongly-admissible, s-strongly-admissible and c-strongly-admissible sets (because is closed under support relation), however it is not a maximal set (see Fig. 6).
- $S_2 = \{\mathrm{D}, \mathrm{E}, \mathrm{F}\}$ is *strongly-conflict-free* and *strongly-safe*. However, the set does not defend D from the attack from B. Furthermore, there is a conflict cycle between the arguments F and J (see Fig. 6).
- $S_3 = \{\mathrm{H}, \mathrm{G}\}$ is *strongly-conflict-free* and *strongly-safe*. This set does not receive attacks, therefore is d-strongly-admissible, s-strongly-admissible, and c-strongly-admissible set. However, it is not a maximal set (see Fig. 6).
- $S_4 = \{\mathrm{K}, \mathrm{J}\}$ is *strongly-conflict-free* and *strongly-safe*. However, there is a conflict cycle between the arguments F and J (see Fig. 6).
- $S_5 = \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{H}, \mathrm{K}, \mathrm{J}, \mathrm{G}\}$ is a *strongly-conflict-free*, *strongly-safe* set and it is closed under support relation. The set S_5 strongly-defends J from F attacks; therefore, it is a d-strongly-admissible, a s-strongly-admissible, and a c-strongly-admissible set. This is a maximal set, therefore is a d-strongly-preferred, a s-strongly-preferred, and a c-strongly-preferred extension (see Fig. 7).
- $S_6 = \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{H}, \mathrm{K}, \mathrm{J}, \mathrm{G}, \mathrm{I}\}$ is a τ*-conflict-free*, *strongly-safe* set and it is closed under support relation. The set S_6 strongly-defends J from F attacks; therefore, it is d-τ-admissible, a s-strongly-admissible, and a c-τ-admissible set. This is a maximal set, therefore it is a d-τ-preferred extension and a c-τ-preferred extension (see Fig. 7).

As we can see, the analysis of a BAF enriched with the added notion of threshold becomes complex, but it provides more information to obtain a set of arguments with the property of conflict-freeness or safety. Moreover, different levels can be defined, where the notion of conflict-free and safety can lead to weakening allowing the acceptance of arguments that otherwise would not have been accepted. Next,

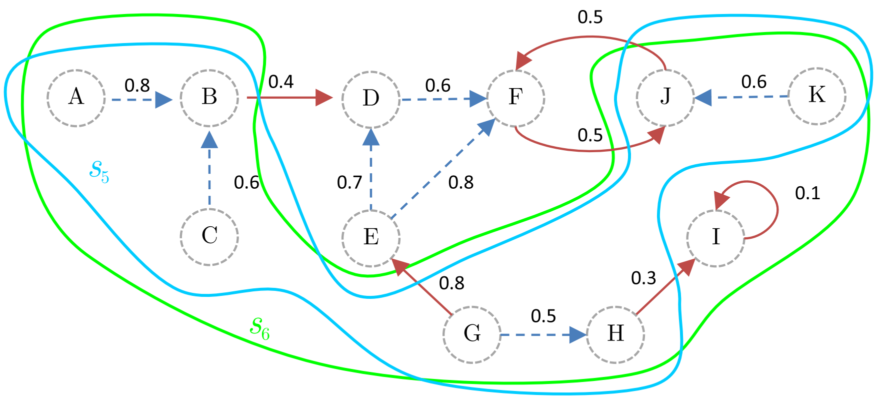

Fig. 7. Preferred extensions in S-BAF.

we will analyze how these arguments can be grouped in *communities* using the notion of a coalition and how they are related in an argumentation domain.

3. Conceptualizing communities in argumentation

A *human community* is a social unit considered a discrete constituent of society with shared norms, values, customs, or identity. Communities may share a sense of being placed in a given geographical area (*e.g.*, country, village, town, or neighborhood) or a virtual space through digital platforms, including associations expanding outside direct genealogical relations, which also define a sense of community, becoming essential to the identity, practice, and roles in the various usual social institutions such as family, workplace environment, governmental organizations, or any other social construct to which individuals consider themselves as participants [25]. As we can see, formulating a definition of the *community* term is a complex task that is being approached from different perspectives [9]. It is possible to find several notions about this kind of organization, like the ones that follow:

- A community can be defined as a group of people that interact and have common interests. They can share or not geographical localities [9].
- A community is a way to decompose a social network by clustering nodes with strong links [14]. This meaning implies a structural representation of the community as a *graph*.
- A community is a gathering of people assembled around a topic of common interest [22]. Its members participate in the community by exchanging information, obtaining answers to personal questions or problems, improving their understanding of a subject, sharing common passions, or playing interests.
- A community can be considered a network of people (possibly distributed in different locations) that share specific beliefs such as solidarity, identity, or a set of rules that govern their behavior [9].
- Communities, or clusters or modules, are a group of vertices in a graph that probably share common properties or play similar roles [17].
- According to a structural perspective, a community is a set of nodes strongly linked to each other and loosely linked to other nodes. However, it is also a set of nodes that share the same interests, based on a semantic position [14].

From the perspective of Social Psychology, a person is attracted to a group in which they can serve as inspiration or give an opinion according to the culture of the social organization [34]. This characteristic has a significant effect on the cohesion of a community. However, another critical concept exists when we refer to these groups: the *consensual validation* that represents the uniformity and conformity in the community. It is important to note that the members of the community share feelings, opinions, beliefs, priorities, or goals. In other words, the community has structural and semantic aspects [14]. From a different standpoint, Sarason in [45] argues that a psychological sense of community is the perception of being *similar* to others. There is an acknowledged interdependence with others, a willingness to maintain it by giving to or doing what others expect from them, including the feeling that one is part of a larger, dependable, stable structure.

Adopting a practical stance, McMillan and Chavis in [34] identify four elements of the "sense of community" involving the four aspects of membership, sense of influence, the fulfillment of needs, and a shared emotional connection: (i) the feeling of belonging to a group or, in other words, a sense of *membership*, (ii) the feeling of being essential to the group or having *influence* in this group, (iii) the members of the community are integrated into it, and they can fulfill their necessities leading to the *reinforcement* of that feeling, and (iv) the group members share an *emotional connection* represented by the belief that they have and will continue having a shared history and places, spending time together, and partaking of comparable experiences. Concerning this, a "sense of community index (SCI)" was developed by Chavis and colleagues[2] to assess the sense of community in neighborhoods, and the index is used to characterize schools, the workplace, and a variety of types of communities [38].

All the observations above outline an essential characteristic of a community: its members have a shared context unique to them. The context informs all the activities inside the community and provides the background information necessary for reasoning and acting by its members.

Now, considering our specific application domain, Porter in [40] defines a *virtual community* as an aggregate of individuals or business partners (in connection with one or more organic communities) that interact on a shared (or complementary) interest and in which a common language implements the interaction and eventually a possible common paralanguage, led by some protocols or shared norms. Taking as a basis this definition, Prodnik in [41] establishes a virtual community as a social construction where the language is the basis of its organization, and the technology in general and the internet, in particular, have a predominant role. Given the importance of the language in virtual communities, arguments can be helpful in detecting them, reflecting the thoughts of the organization's members in a given discussion. Thus, arguments may have an opinion for or against a specific statement, representing communities interacting in a particular argumentative discourse.

In this work, when referring to a *community*, it will be understood as a group of agents presenting different postures through a set of arguments expressing supporting and conflicting positions in a setting akin to a debate (see Fig. 8). Support can be interpreted as a relationship among the group members through common opinions; consequently, coalitions in BAF will represent a community. Furthermore, the support relation will have associated a measure of internal cohesion of the community following [11]; thus, this measure can be considered an SCI in the argumentation domain, reflecting to a certain degree the four characteristics mentioned above. Finally, the conflict relationship between different communities represents how different positions on a specific statement are brought into play. Measuring the degree of conflict between different positions is essential in determining how strong the relationship is, considering the degree of controversy between communities.

[2]See https://www.senseofcommunity.com/.

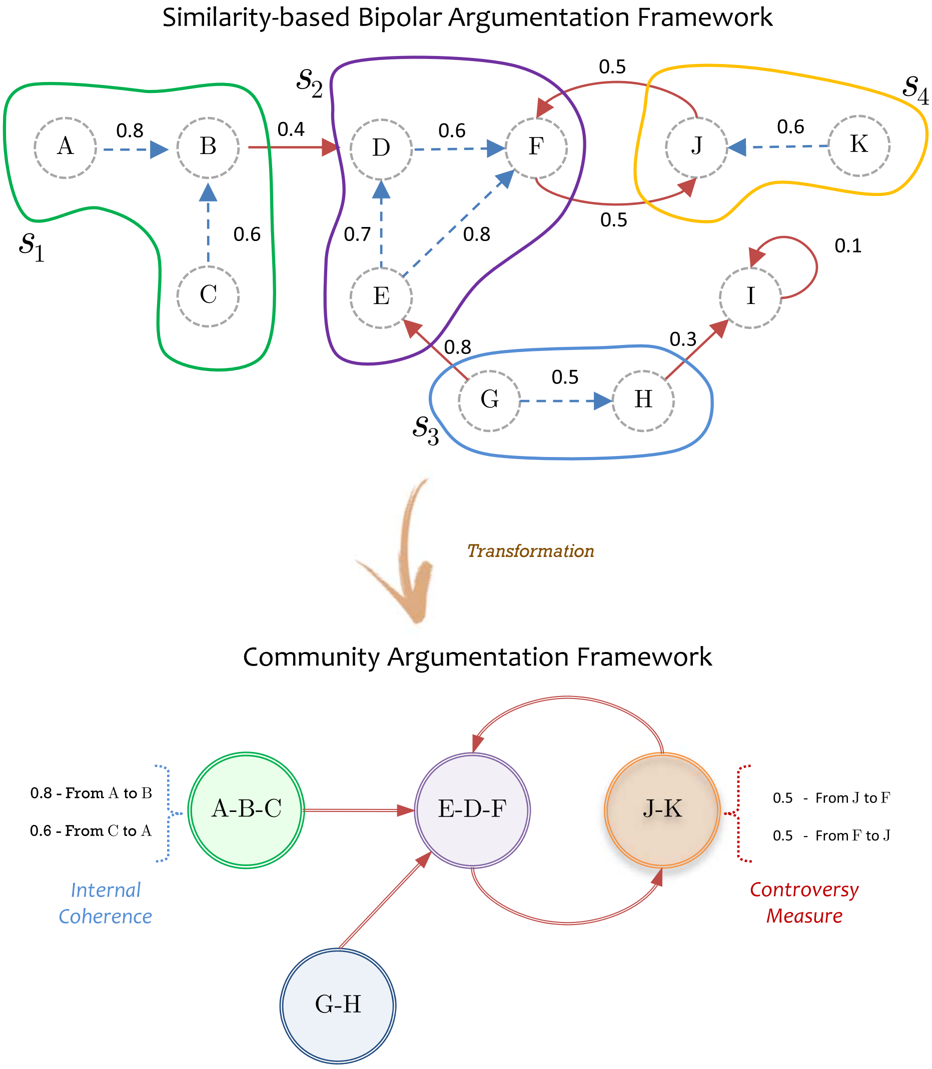

Fig. 8. Communities in bipolar argumentation frameworks.

4. Communities from valuated-similarity coalitions

Given a system that represents knowledge as arguments and considers the existing conflicts and supports between these arguments, a primary goal is to find sets of arguments that can be kept together by handling the conflicts appropriately while fulfilling relevant properties. It is feasible to create maximal cohesive sets of arguments by taking advantage of the mechanism proposed in [11] to collect in a set as many conflict-free and related-by-support arguments as possible, ensuring coherence of the whole set. Nevertheless, it is also interesting to include some degree of controversy by considering the addition of attacks and maintaining a coherence threshold in a dialogue or debate. Thus, it is possible in the proposed framework to find *sets of arguments* or *stances* that conform to a *community*, where for the present work, a *community* is a set of consistent stances in favor (or against) a specific topic. The threshold has two different meanings in the valuation proposed here. On the one hand, the threshold is the maximal degree of controversy that a community can admit without losing the essence that identifies it as a discursive and coherent community. On the other hand, the threshold represents the minimal level of coherence required by a set of opinions to be considered at least as a community with a moderately solid and consolidated position among its members. There might be many possible threshold settings; in each case, it is essential to determine the most appropriate one to use. This threshold setting is a methodological issue involving the semantics of the domain. The question could be tackled by devising experiments using examples where the desired conclusion is well known or by performing tests using the cognitive evaluation of human subjects to approximate their assessment of the valuations obtained after their interactions. Furthermore, according to [46], it could be challenging to find the correct value for a heuristic threshold; a complete discussion of the generality of this choice for representing uncertain information can be found in [50]. This issue exceeds the scope of our work. However, given the practical usefulness of this parameter, we plan to return to this question in future works.

When analyzing social media conversations as an exchange of arguments, it is natural to find many arguments in favor of a conclusion; generally, these arguments are similar but have some nuances in their meanings. To recognize *communities*, we propose considering an argumentation graph where the arguments are decorated with labels that will allow us to determine how similar the supported and attacked arguments are. Aiming at that, we introduce a bipolar argumentation graph whose arcs are labeled with a similarity degree between the related arguments, as follows:

Definition 18 (S-valued bipolar argumentation graph)**.** Given $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}, \mathsf{Cont}_{\mathbb{C}} \rangle$, an S-BAF where $\overline{\Theta} = \langle \mathbb{A}\mathrm{rgs}, \mathbb{R}_s, \mathbb{R}_a \rangle$ is the underlying bipolar argumentation framework. An *s-valued Bipolar Argumentation Graph*, denoted G_Φ, is the argumentation graph where the nodes are the elements of $\mathbb{A}\mathrm{rgs}$ and the arcs between nodes depict the $\mathbb{R}_s$ (dashed arcs) and $\mathbb{R}_a$ (full arcs) relationships, where the arcs are decorated with the similarity degree $\mathsf{Sim}_{\mathbb{C}}$ between the related arguments.

Now, it is necessary to revisit the concept of coalitions given by Cayrol and Lagasquie-Schiex in [12] to extend it by formalizing how a similarity degree can influence the support relations. Formally:

Definition 19 (S-coalitions)**.** Given an S-BAF $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}, \mathsf{Cont}_{\mathbb{C}} \rangle$, where $\overline{\Theta} = \langle \mathbb{A}\mathrm{rgs}, \mathbb{R}_s, \mathbb{R}_a \rangle$ is the underlying bipolar argumentation framework, let G_Φ be the s-valued bipolar interaction graph over Φ, and $\mathcal{C} \subseteq \mathbb{A}\mathrm{rgs}$ be a set of enriched arguments. Then, we say that $\mathcal{C}$ is an s-coalition iff it is a maximally strongly-conflict-free set such that the sub-graph G'_Φ induced by $\mathcal{C}$ is connected only by support relations. We will denote as $\mathcal{C}_\Phi$ the set of coalitions obtained from Φ.

Note that self-attacking arguments are disregarded in this approach according to the classic definition of a coalition where no attacks are permitted (Definition 5). In other words, an opinion that contradicts itself cannot be part of a discourse community. However, in future research, the attack might spur different coalition classes by weakening the strong-conflict-free condition by admitting certain conflicting opinions within a community.

The following result follows naturally from the definition of S-coalitions.

Proposition 1. *Let* $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}, \mathsf{Cont}_{\mathbb{C}} \rangle$ *be a* S-BAF*, where* $\overline{\Theta}$ *is an enriched bipolar argumentation framework* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rgs}, \mathbb{R}_s, \mathbb{R}_a \rangle$*. Each enriched argument, which is not self-attacking, belongs to an s-coalition.*

Once the set of coalitions is obtained, we can use the internal coherence of each element in this set to characterize an s-coalition. Note that, as an s-coalition is a set of enriched arguments, we can use the cohesion function established in Definition 12 to determine a cohesion measure associated with that s-coalition.

Definition 20 (Types of s-coalitions)**.** Let $\Phi = \langle\overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}, \mathsf{Cont}_{\mathbb{C}}\rangle$ be an S-BAF, where $\overline{\Theta} = \langle\mathbb{A}\text{rgs}, \mathbb{R}_s, \mathbb{R}_a\rangle$ is the underlying bipolar argumentation framework, $\mathcal{C} \in \mathcal{C}_\Phi$ be a coalition obtained from Φ, and $\tau \in [0, 1]$ be a threshold. Then:

- $\mathcal{C}$ is a *strong-coalition* iff $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}) = 1$.
- $\mathcal{C}$ is a τ-coalition iff $\tau \leqslant \mathsf{Coh}_{\mathbb{C}}(\mathcal{C}) < 1$.
- $\mathcal{C}$ is a *weak-coalition* iff $0 \leqslant \mathsf{Coh}_{\mathbb{C}}(\mathcal{C}) < \tau$.

Intuitively, a *strong-coalition* does not admit conflict between its arguments, assuring that these pieces of knowledge refer to the same aspects of the argumentation process, *i.e.*, the opinions allude to precisely the same values for each considered descriptor. In a τ*-coalition*, even though the arguments do not contradict each other, they essentially refer to the same aspects, *i.e.*, the opinions allude to the same values for each descriptor considered but contain some descriptors whose values differ. Lastly, in a *weak-coalition*, although the arguments do not contradict each other, they can refer to the same aspects differently, or some of them might refer to different aspects of the issue, *i.e.*, either the opinions mainly allude to each considered descriptor differently, or each opinion refers to different descriptors. The threshold is a sensitive value to determining what type the s-coalition is a group of supported opinions. A higher threshold value allows more flexibility in the descriptors' values for each considered descriptor.

A coalition can be regarded as an argument at a meta-level built from argumentation stances that deal with contextual features with different degrees of strength. In an everyday discussion, even when stances do not contradict each other, the different members of the coalition do not have the same strength. So, we can associate a coalition to *discourse communities*. In this direction, a possible way to detect and classify these discourse communities is to find s-coalitions. Furthermore, the following criteria for distinguishing discourse communities from coalitions can be considered:

Definition 21 (s-discourse-communities)**.** Let $\Phi = \langle\overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}, \mathsf{Cont}_{\mathbb{C}}\rangle$ be an S-BAF, with $\overline{\Theta} = \langle\mathbb{A}\text{rgs}, \mathbb{R}_s, \mathbb{R}_a\rangle$ the underlying bipolar argumentation framework. Let $\mathcal{C}_\Phi$ be the set of s-coalitions obtained from Φ, and $\mathcal{C} \in \mathcal{C}_\Phi$ be an s-coalition. Then:

- $\mathcal{C}$ represents an *sd-robust-community* when $\mathcal{C}$ is a *strong-coalition*.
- $\mathcal{C}$ represents an *sd-moderate-community* when $\mathcal{C}$ is a τ*-coalition*.
- $\mathcal{C}$ represents an *sd-fragile-community* when $\mathcal{C}$ is a *weak-coalition*.

Those criteria are defined taking into account: (i) the collective or consensual validation as a distinctive feature in an argumentative process, (ii) the necessity of modeling the community as a computable concept, and (iii) the requirement to distinguish the connection level (emotional or any other kind of support), as well as any other distinctive features in the discourse communities.

According to the characteristics of a community, an *sd-robust-community* represents a maximum level of consensual validation and strong support (possibly emotional) connection. In contrast, an *sd-fragile-community* shows a minimum level of consensus and support connection inside the organization. Next, we introduce an example to clarify the preceding ideas.

Example 7. Continuing our Example 6, using a product T-norm to obtain the cohesion value, considering a $\tau = 0.48$, and analyzing the abstract argumentation framework represented in Figure 9, we have that the coalitions $\mathcal{C}_1 = \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$, $\mathcal{C}_2 = \{\mathrm{E}, \mathrm{D}, \mathrm{F}\}$ are *weak* because the $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_1) = 0.2 < \tau$,

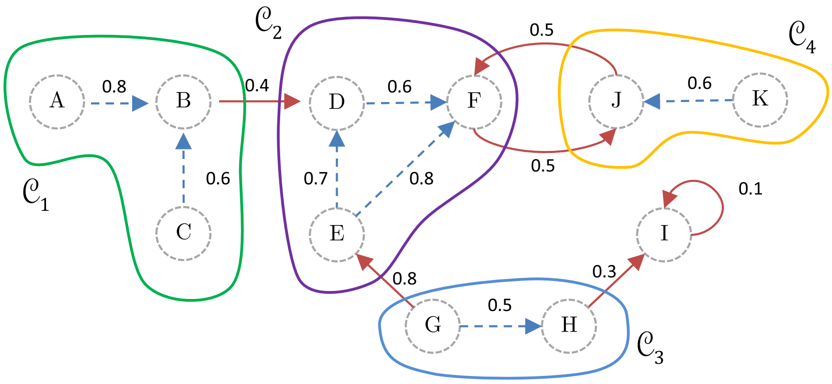

Fig. 9. Communities in bipolar argumentation frameworks.

$\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_2) = 0.34 < \tau$. On the other hand, $\mathcal{C}_3 = \{\mathrm{G}, \mathrm{H}\}$ and $\mathcal{C}_4 = \{\mathrm{K}, \mathrm{J}\}$ are τ*-coalitions* since the $\tau \leqslant \mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_3) = 0.5 < 1$ and $\tau \leqslant \mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_4) = 0.6 < 1$. However, if we choose a different function to obtain the cohesion of the sets, the max T-conorm for instance, we have that: $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_1) = 0.8 > \tau$; the same occurs with $\mathcal{C}_2$. Under this perspective, all the communities are τ*-coalitions*. At the level of semantic analysis, by using a product T-norm to obtain the cohesion value, we find that the $\mathcal{C}_1$ and $\mathcal{C}_2$ are *sd-fragile-communities*, while in the second interpretation that relies on a max T-conorm, we conclude that $\mathcal{C}_1$ and $\mathcal{C}_2$ are *sd-moderate-communities*. Finally, $\mathcal{C}_3$ and $\mathcal{C}_4$ are *sd-moderate-communities* in either analysis.

The difference between an *sd-robust-community* and an *sd-moderate-community* is a design choice. Still, we believe the intuition behind an *s-coalition* is that the stances (arguments) in it must be fully supported, taking into account the aspects they refer to. It is possible to follow a simple procedure to find and classify s-coalitions computationally from an s-valued bipolar argumentation graph: first, consider the paths following the support relation; then, calculate the coherence value for each path found considering the threshold τ given; and finally, determine the corresponding s-coalition type based on these coherence values.

The nature of the support relation allows us to establish some properties. For example, when we find a strong support relation inside a coalition, it is natural to think that the cohesion associated with the supported arguments would be high.

Proposition 2. *Let* $\Phi = \langle\overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}, \mathsf{Cont}_{\mathbb{C}}\rangle$ *be an* S-BAF, *where* $\overline{\Theta}$ *is the underlying bipolar argumentation framework* $\overline{\Theta} = \langle\mathbb{A}\mathrm{rgs}, \mathbb{R}_s, \mathbb{R}_a\rangle$, *and let* $\mathrm{A}, \mathrm{B} \in \mathbb{A}\mathrm{rgs}$ *two enriched arguments such that* $(\mathrm{A}, \mathrm{B}) \in \mathbb{R}_s$, *and* $\mathbb{R}_a$ *does not contain* (A, B) *or* (B, A). *If* $\mathrm{A}\ \mathbb{R}_s\ \mathrm{B}$ *is either a strong-support or weak-support relation, then there exists at least a weak-coalition containing both* A *and* B.

Now, it is necessary to introduce an attack relationship between conflicting coalitions. The characterization of these new attacks considers the attacks between the arguments that are part of these coalitions as formalized in the following definition.

Definition 22 (Internal attacks between s-coalitions)**.** Given the S-BAF $\Phi = \langle\overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}, \mathsf{Cont}_{\mathbb{C}}\rangle$, where $\overline{\Theta} = \langle\mathbb{A}\mathrm{rgs}, \mathbb{R}_s, \mathbb{R}_a\rangle$ is the underlying bipolar argumentation framework, let $\mathcal{C}_\Phi$ be the set of s-coalitions obtained from Φ, and $\mathcal{C}, \mathcal{C}' \in \mathcal{C}_\Phi$ be two s-coalitions. We will say that there exists an attack point from $\mathcal{C}$ to $\mathcal{C}'$ iff there are two enriched arguments $\mathrm{A} \in \mathcal{C}$ and $\mathrm{B} \in \mathcal{C}'$ such that $(\mathrm{A}, \mathrm{B}) \in \mathbb{R}_a$. We will denote as $\mathbb{R}_a^{[\mathcal{C},\mathcal{C}']}$ the set of all attacks points between two coalitions $\mathcal{C}$ and $\mathcal{C}'$.

Intuitively, it is possible to say that if there is an attack between two arguments that *belong* to two different coalitions, then it is natural to raise this conflict to the coalition level and define now an attack *between* these coalitions.

Definition 23 (S-coalitions Attacks)**.** Let $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}, \mathsf{Cont}_{\mathbb{C}} \rangle$ be an S-BAF, where $\overline{\Theta} = \langle \mathbb{A}\mathrm{rgs}, \mathbb{R}_s, \mathbb{R}_a \rangle$ is the underlying bipolar argumentation framework, let $\mathcal{C}_\Phi$ be the set of s-coalitions obtained from Φ. We will define the attack relation between s-coalitions derived from Φ, denoted $\mathbb{R}_a^{\mathcal{C}_\Phi}$, as

$$\mathbb{R}_a^{\mathcal{C}_\Phi} = \{(\mathcal{C}, \mathcal{C}') | \mathcal{C}, \mathcal{C}' \in \mathcal{C}_\Phi \text{ and } \mathbb{R}_a^{[\mathcal{C},\mathcal{C}']} \neq \emptyset\}$$

Furthermore, it is interesting to study the strength of the attack from one coalition to another by considering the strength of the attacks that define the existing points of conflict. Formally:

Definition 24 (Strength of attack between s-coalitions)**.** Given an S-BAF $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}, \mathsf{Cont}_{\mathbb{C}} \rangle$, where $\overline{\Theta} = \langle \mathbb{A}\mathrm{rgs}, \mathbb{R}_s, \mathbb{R}_a \rangle$ is the underlying bipolar argumentation framework, let $\mathcal{C}_\Phi$ be the set of s-coalitions obtained from Φ, $\mathcal{C}, \mathcal{C}' \in \mathcal{C}_\Phi$ be two s-coalitions, and $\mathbb{R}_a^{[\mathcal{C},\mathcal{C}']} = \{(\mathrm{A}_1, \mathrm{B}_1), \ldots, (\mathrm{A}_n, \mathrm{B}_n)\} \subseteq \mathbb{R}_a$ be the set of all attack points between $\mathcal{C}$ and $\mathcal{C}'$ with $\mathbb{R}_a^{[\mathcal{C},\mathcal{C}']} \neq \emptyset$. The attack strength, or *attack degree* between $\mathcal{C}$ and $\mathcal{C}'$, denoted $\mathsf{Str}_{\mathcal{C}}^{\Phi}(\mathcal{C}, \mathcal{C}')$, is defined as:

$$\mathsf{Str}_{\mathcal{C}}^{\Phi}(\mathcal{C}, \mathcal{C}') = \delta_n,$$

where δ_n is defined as $\delta_1 = \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_1, \mathrm{B}_1)$ and $\delta_i = \otimes(\delta_{i-1}, \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}_i, \mathrm{B}_i))$ with $2 \leqslant i \leqslant n$.

The attack degree can be obtained by instantiating the $\mathsf{Sim}_{\mathbb{C}}(\cdot, \cdot)$ similarity function with T-norms or T-conorms, considering the user modeling preferences. This measure returns a non-negative real number in $[0, 1]$, *i.e.*, it is defined as $\mathsf{Str}_{\mathcal{C}}^{\Phi} : 2^{\mathbb{A}\mathrm{rgs}} \to [0, 1]$.

Once the attacks between s-coalitions are identified, and their strength is computed, we begin by using the attack degree to distinguish between strong and weak attacks. This classification can be employed to define different semantics by using different forms of acceptability; for instance, conflicting s-coalitions could be part of a set of acceptable s-coalitions when the attack degree is not strong enough to be considered a defeat.

Definition 25 (Classification of attacks between s-coalitions)**.** Given an S-BAF $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}, \mathsf{Cont}_{\mathbb{C}} \rangle$, with $\overline{\Theta} = \langle \mathbb{A}\mathrm{rgs}, \mathbb{R}_s, \mathbb{R}_a \rangle$ as the underlying bipolar argumentation framework, let $\mathcal{C}_\Phi$ be the set of s-coalitions obtained from Φ, $\mathcal{C}, \mathcal{C}' \in \mathcal{C}_\Phi$ be two s-coalitions such that $(\mathcal{C}, \mathcal{C}') \in \mathbb{R}_a^{\mathcal{C}_\Phi}$, and $\tau \in [0, 1]$ be a threshold. We say that:

- $\mathcal{C}$ strongly-attacks $\mathcal{C}'$ iff $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}) \geqslant \tau$ and $\mathsf{Str}_{\mathcal{C}}^{\Phi}(\mathcal{C}, \mathcal{C}') \geqslant \tau$,
- $\mathcal{C}$ weakly-attacks $\mathcal{C}'$ iff $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}) < \tau$ or $\mathsf{Str}_{\mathcal{C}}^{\Phi}(\mathcal{C}, \mathcal{C}') < \tau$.

The previous definition formalizes the intuition that a strong attack considers two necessary elements: the strength of attack and the s-coalition internal cohesion measure applied to the set of the enriched arguments in the s-coalition. Once the s-coalitions and associated attacks are obtained from the S-BAF, we can formalize a new meta-argumentation framework to analyze a new kind of semantics concerning the set of communities.

Definition 26 (Meta-argumentation framework)**.** Given $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}, \mathsf{Cont}_{\mathbb{C}} \rangle$, an S-BAF with $\overline{\Theta} = \langle \mathbb{A}\mathrm{rgs}, \mathbb{R}_s, \mathbb{R}_a \rangle$ as the underlying bipolar argumentation framework, we define the *meta-argumentation framework associated with* Φ, as a 3-tuple $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$, where $\mathcal{C}_\Phi$ is the set of s-coalitions obtained from Φ, $\mathbb{R}_a^{\mathcal{C}_\Phi}$ is an attack relation between s-coalitions derived from Φ, $\mathsf{Str}_{\mathcal{C}}^{\Phi}$ is the attack strength function defined over Φ.

Note that in the new meta-argumentation framework, the set $\mathcal{C}_\Phi$ of coalitions plays the role of the argument set, and the relation $\mathbb{R}_a^{\mathcal{C}_\Phi}$ represents the set of attacks. Henceforth, we will describe this meta-argumentation framework $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$ through a weighted directed graph $\mathsf{G}_{\mathcal{C}_\Phi}$, called meta-argumentation graph, with a unique kind of edge representing attacks between coalitions. Furthermore, each edge is assigned a weight representing the strength behind the attack it represents under the interpretation of attack strength.

Next, we will introduce the measure of controversy associated with a set of s-coalitions, where the different types of attacks are analyzed to specify how contradictory they are.

Definition 27 (Controversy degree for a s-coalition set)**.** Given a meta-argumentation framework $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$, where $\mathcal{C}_\Phi$ is a set of s-coalitions, $\mathbb{R}_a^{\mathcal{C}_\Phi}$ is an attack relation, $\mathsf{Str}_{\mathcal{C}}^{\Phi}$ is the attack strength function, $\mathcal{S} \subseteq \mathcal{C}_\Phi$ be a set of s-coalitions, and $\mathbb{R}_a^{\mathcal{S}} = \{(\mathcal{C}_1, \mathcal{C}_2), \dots, (\mathcal{C}_{n-1}, \mathcal{C}_n)\} \subseteq \mathbb{R}_a^{\mathcal{C}_\Phi}$. The controversial measure for $\mathcal{S}$, denoted $\mathsf{Cont}_{\mathcal{C}}^{\Phi}(\mathcal{S})$, is defined as:

$$\mathsf{Cont}_{\mathcal{C}}^{\Phi}(\mathcal{S}) = \begin{cases} \lambda_n & \text{if } \mathbb{R}_a^{\mathcal{S}} \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

where $\lambda_1 = \mathsf{Str}_{\mathcal{C}}^{\Phi}(\mathcal{C}_1, \mathcal{C}_2)$ and $\lambda_i = \otimes(\lambda_{i-1}, \mathsf{Str}_{\mathcal{C}}^{\Phi}(\mathcal{C}_{i-1}, \mathcal{C}_i))$ with $2 \leqslant i \leqslant n$.

The instantiation of the controversy degree function is a design decision, *i.e.*, users can choose what they consider more appropriate for the problem at hand. Two possible choices are the T-norms and T-conorms. This measure returns a non-negative real number in [0, 1], *i.e.*, it is defined as $\mathsf{Cont}_{\mathcal{C}}^{\Phi} : 2^{\mathbb{A}\mathrm{rgs}} \to [0, 1]$.

Proposition 3. *Given the* S-BAF $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}, \mathsf{Cont}_{\mathbb{C}} \rangle$ *where* $\overline{\Theta}$ *is the underlying bipolar argumentation framework described as* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rgs}, \mathbb{R}_s, \mathbb{R}_a \rangle$*, the meta-argumentation framework* $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$ *associated with* Φ*, and* $\mathsf{Cont}_{\mathcal{C}}^{\Phi}$ *a controversy degree function defined over* $\Phi^{\mathcal{C}}$*. Let* $\mathcal{S} \subseteq \mathcal{C}_\Phi$ *be a set of coalitions, and* $\mathbb{S} \subseteq \mathbb{A}\mathrm{rgs}$ *be the enriched arguments involved in* $\mathcal{S}$*, then* $\mathsf{Cont}_{\mathbb{C}}(\mathbb{S}) = \mathsf{Cont}_{\mathcal{C}}^{\Phi}(\mathcal{S})$.

Given that the controversy associated with a set of coalitions is the same as the controversy associated with the set of enriched arguments involved, the previous result establishes a common point between the S-BAF and the meta-argumentation framework.

Now, based on the semantic analysis done in [11], we introduce the notions of *conflict-free* s-coalition sets in our meta-argumentation framework $\Phi^{\mathcal{C}}$. Thus, it is possible to determine the set of communities that can coexist within an argumentative model.

Definition 28 (Conflict-freeness in $\Phi^{\mathcal{C}}$)**.** Given a meta-argumentation framework $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$, where $\mathcal{C}_\Phi$ is the set of s-coalitions, $\mathbb{R}_a^{\mathcal{C}_\Phi}$ is an attack relation between s-coalitions, and $\mathsf{Str}_{\mathcal{C}}^{\Phi}$ the strength of attack function. Given a controversy degree function $\mathsf{Cont}_{\mathcal{C}}^{\Phi}$ defined over $\Phi^{\mathcal{C}}$. Let $\mathcal{S} \subseteq \mathcal{C}_\Phi$ be a subset of coalitions, and τ be a threshold. Then:

- $\mathcal{S}$ is a *strongly-conflict-free* set iff there is no $\mathcal{C}_1, \mathcal{C}_2 \in \mathcal{S}$ such that there exists a strong or weak attack from $\mathcal{C}_1$ to $\mathcal{C}_2$.
- $\mathcal{S}$ is a *τ-conflict-free* set iff there is no $\mathcal{C}_1, \mathcal{C}_2 \in \mathcal{S}$ such that there exists a strong attack from $\mathcal{C}_1$ to $\mathcal{C}_2$, and $\mathsf{Cont}^{\Phi}_{\mathcal{C}}(\mathcal{S}) \leqslant \tau$.
- $\mathcal{S}$ is a *weakly-conflict-free* set iff there is no $\mathcal{C}_1, \mathcal{C}_2 \in \mathcal{S}$ such that there exists a strong attack from $\mathcal{C}_1$ to $\mathcal{C}_2$.

The following proposition establishes the semantic connections between the meta-argumentation framework dealing with coalitions of arguments and the subjacent similarity-based argumentation framework.

Proposition 4. *Let $\Phi^{\mathcal{C}} = \langle \mathcal{C}_{\Phi}, \mathbb{R}_a^{\mathcal{C}_{\Phi}}, \mathsf{Str}^{\Phi}_{\mathcal{C}} \rangle$ be the meta-argumentation framework associated with Φ, $\mathsf{Cont}^{\Phi}_{\mathcal{C}}$ a controversy degree function defined over $\Phi^{\mathcal{C}}$, $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ be a finite set of coalitions, and $\tau \in [0, 1]$ be a threshold. Then:*

(i) *$\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is strongly-conflict-free for $\Phi^{\mathcal{C}}$ iff $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is strongly-conflict-free for Φ.*
(ii) *$\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is strongly-conflict-free for $\Phi^{\mathcal{C}}$ iff $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is strongly-safe for Φ.*
(iii) *If $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is τ-conflict-free for Φ, then $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is τ-conflict-free for $\Phi^{\mathcal{C}}$.*
(iv) *If $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is at least τ-safe for Φ then $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is τ-conflict-free for $\Phi^{\mathcal{C}}$.*
(v) *$\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is weakly-conflict-free for $\Phi^{\mathcal{C}}$ iff $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is weakly-conflict-free for Φ.*
(vi) *$\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is weakly-conflict-free for $\Phi^{\mathcal{C}}$ iff $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is at least weakly-safe for Φ.*

Previous examples have examined how to obtain the coalitions associated with an S-BAF, classifying them according to their degree of cohesion. The following example exercises the concepts just introduced by analyzing attacks between coalitions and the degree of controversy associated with them.

Example 8. Continuing the analysis of Example 7, and recalling that the threshold set is $\tau = 0.48$ and considering the meta-argumentation graph presented in Figure 10, we have that:

- There is a conflict point between the s-coalitions $\mathcal{C}_1$ and $\mathcal{C}_2$: the pair (B, D). In this case, $\mathsf{Str}^{\Phi}_{\mathcal{C}}(\mathcal{C}_1, \mathcal{C}_2) = 0.4 < \tau$, therefore, $\mathcal{C}_1$ weakly-attacks $\mathcal{C}_2$.
- There is a conflict point between the s-coalitions $\mathcal{C}_3$ and $\mathcal{C}_2$: the pair (G, E). In this case, $\mathsf{Str}^{\Phi}_{\mathcal{C}}(\mathcal{C}_3, \mathcal{C}_2) = 0.8 > \tau$, and $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_3) > \tau$. Thus, we can establish that $\mathcal{C}_3$ strongly-attacks $\mathcal{C}_2$.
- There is a conflict point between the s-coalitions $\mathcal{C}_2$ and $\mathcal{C}_4$: the pair (F, J) representing a weak attack, given that $\mathsf{Str}^{\Phi}_{\mathcal{C}}(\mathcal{C}_2, \mathcal{C}_4) = 0.5 > \tau$ but $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_2) < \tau$. However, in this case, there is a conflict point between the s-coalitions $\mathcal{C}_4$ and $\mathcal{C}_2$ too: the pair (J, F) identified a strong attack because $\mathsf{Str}^{\Phi}_{\mathcal{C}}(\mathcal{C}_4, \mathcal{C}_2) = 0.5 > \tau$ and $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_4) > \tau$.

The characterization of the attack relationship between coalitions and considering the associated strength of attacks allows us to establish the following property. This result will be relevant to characterize how an s-coalition "absorbs," or "assimilates" other s-coalitions.

Proposition 5. *Let $\Phi^{\mathcal{C}} = \langle \mathcal{C}_{\Phi}, \mathbb{R}_a^{\mathcal{C}_{\Phi}}, \mathsf{Str}^{\Phi}_{\mathcal{C}} \rangle$ be the meta-argumentation framework associated with Φ, $\mathsf{Cont}^{\Phi}_{\mathcal{C}}$ a controversy degree function defined over $\Phi^{\mathcal{C}}$, and $\mathcal{C}_1, \mathcal{C}_2 \in \mathcal{C}_{\Phi}$ be two s-coalitions. If $\mathcal{C}_1$ and $\mathcal{C}_2$ are two disjoint s-coalitions that are connected by the attack relation, then there exists at least a weak-attack between $\mathcal{C}_1$ and $\mathcal{C}_2$.*

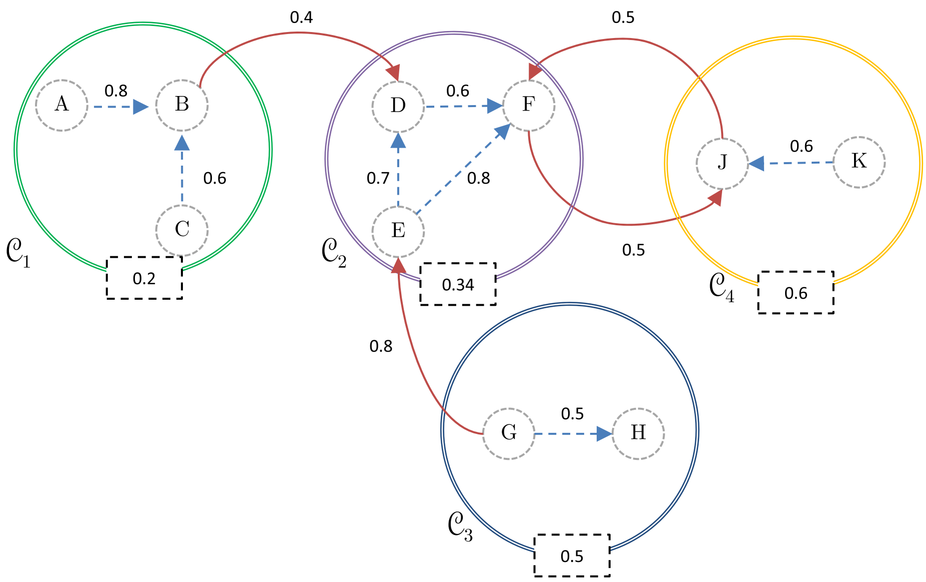

Fig. 10. Attacks between coalitions in a meta argumentation framework.

Now, we will introduce the notions of defense for coalitions by extrapolating from the defense relationship between the arguments gathered in the coalitions involved in the analysis. Furthermore, we present different definitions for admissibility, from the most general and strong to the most specific and weak.

Definition 29. Let $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$ be the meta-argumentation framework associated with Φ, $\mathsf{Cont}_{\mathcal{C}}^{\Phi}$ a controversy degree function defined over $\Phi^{\mathcal{C}}$, $\mathcal{S} \subseteq \mathcal{C}_\Phi$ be a set of coalitions over Φ, and $\mathcal{C}_1 \in \mathcal{C}_\Phi$ a s-coalitions. Then:

- The set $\mathcal{S}$ is a *strong defense* for $\mathcal{C}_1$ iff for all $\mathcal{C}_2 \in \mathcal{C}_\Phi$ such that if $\mathcal{C}_2$ is a strong or weak attacker of $\mathcal{C}_1$ then there exists $\mathcal{C}_3 \in \mathcal{S}$ where $\mathcal{C}_3$ is a strong attacker of $\mathcal{C}_2$.
- The set $\mathcal{S}$ is a *weak defense* for $\mathcal{C}_1$ iff for all $\mathcal{C}_2 \in \mathcal{C}_\Phi$ such that if $\mathcal{C}_2$ is a strong or weak attacker of $\mathcal{C}_1$ then there exists $\mathcal{C}_3 \in \mathcal{S}$ where $\mathcal{C}_3$ is a weak attacker of $\mathcal{C}_2$.

Once the attack and defense relationships are specified in the meta-argumentation framework, we can perform the semantic analysis over the argumentation model. The following definition establishes three levels of admissibility over the set of communities based on the level of tolerance to conflict between them and considering the quality of defense that the set provides to its elements.

Definition 30. Let $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$ be the meta-argumentation framework associated with Φ, $\mathsf{Cont}_{\mathcal{C}}^{\Phi}$ a controversy degree function defined over $\Phi^{\mathcal{C}}$, and $\mathcal{S} \subseteq \mathcal{C}_\Phi$ be a set of coalitions. Then:

- The set $\mathcal{S}$ is *strongly-admissible* iff $\mathcal{S}$ is strongly-conflict-free and strongly defends all its elements.
- The set $\mathcal{S}$ is τ*-admissible* iff $\mathcal{S}$ is τ-conflict-free and there exists a strong or weak defense for all its elements.
- The set $\mathcal{S}$ is *weakly-admissible* iff $\mathcal{S}$ is weakly-conflict-free and there exists a strong or weak defense for all its elements, or $\mathcal{S}$ is strongly-conflict-free and weakly defends all its elements.

From Definition 19, the following proposition establishes that any set of admissible coalitions is also closed by the notion of support in the underlying S-BAF.

Proposition 6. *Let* $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$ *be the meta-argumentation framework associated with* Φ*,* $\mathsf{Cont}_{\mathcal{C}}^{\Phi}$ *a controversy degree function defined over* $\Phi^{\mathcal{C}}$*,* $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ *be a finite set of coalitions, and* $\tau \in [0, 1]$ *be a threshold. Then,* $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ *is strongly-admissible for* $\Phi^{\mathcal{C}}$ *iff* $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ *is c-strongly-admissible for* Φ*.*

It is possible to determine different connections between the meta-argumentation framework and the underlying similarity-based argumentation framework. This connection is essential to carry out a semantic analysis of the relations between communities.

Proposition 7. *Let* $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$ *be the meta-argumentation framework associated with* Φ*,* $\mathsf{Cont}_{\mathcal{C}}^{\Phi}$ *a controversy degree function defined over* $\Phi^{\mathcal{C}}$*,* $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ *be a finite set of coalitions, and* $\tau \in [0, 1]$ *be a threshold.Then:*

(i) $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ *is strongly-admissible for* $\Phi^{\mathcal{C}}$ *iff* $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ *is s-strongly-admissible for* Φ*.*
(ii) *If* $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ *is s-*τ*-admissible for* Φ *then* $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ *is at least* τ*-admissible for* $\Phi^{\mathcal{C}}$*.*
(iii) $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ *is weakly-admissible for* $\Phi^{\mathcal{C}}$ *iff* $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ *is s-weakly-admissible for* Φ*.*

Finally, we present the preferred extensions for the meta-argumentation framework resulting from considering coalitions, where the notions of defense and conflict-freeness are put together to establish a set of communities with particular properties important to analyzing an argumentative discussion.

Definition 31. Let $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$ be the meta-argumentation framework associated with Φ, $\mathsf{Cont}_{\mathcal{C}}^{\Phi}$ a controversy degree function defined over $\Phi^{\mathcal{C}}$, $\mathcal{S} \subseteq \mathcal{C}_\Phi$ be a set of coalitions in $\Phi^{\mathcal{C}}$. Then:

(i) $\mathcal{S}$ is a strongly-preferred extension if $\mathcal{S}$ is a $\subseteq$-maximal among the strongly-admissible set of coalitions.
(ii) $\mathcal{S}$ is a τ-preferred extension if $\mathcal{S}$ is a $\subseteq$-maximal among the τ-admissible set of coalitions.
(iii) $\mathcal{S}$ is a weakly-preferred extension if $\mathcal{S}$ is a $\subseteq$-maximal among the weakly-admissible set of coalitions.

Naturally, Definitions 28 to 31 are extensions of those presented in Section 2.2 developed under the S-BAF approach since coalitions constitute a way to put enriched arguments together following specific criteria.

The following result establishes the connection between the meta-argumentation framework and the underlying similarity-based argumentation framework.

Proposition 8. *Let* $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$ *be the meta-argumentation framework associated with* Φ*,* $\mathsf{Cont}_{\mathcal{C}}^{\Phi}$ *a controversy degree function defined over* $\Phi^{\mathcal{C}}$*,* $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ *be a finite set of coalitions, and* $\tau \in [0, 1]$ *be a threshold.Then:*

- $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ *is strongly-preferred for* $\Phi^{\mathcal{C}}$ *iff* $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ *is s-strongly-preferred for* Φ*.*
- *If* $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ *is s-*τ*-preferred for* Φ *then* $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ *is at least* τ*-preferred for* $\Phi^{\mathcal{C}}$*.*
- $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ *is weakly-preferred for* $\Phi^{\mathcal{C}}$ *iff* $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ *is at least s-weakly-preferred for* Φ*.*

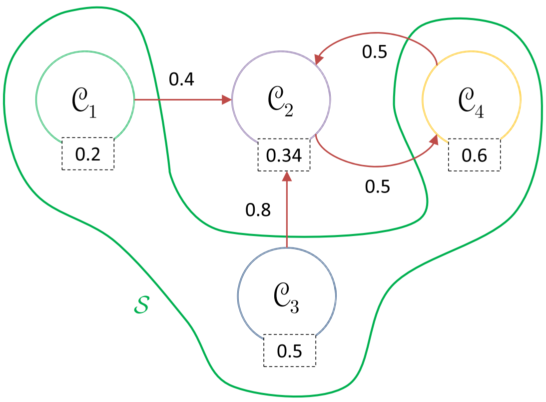

Fig. 11. Preferred semantics in a meta argumentation framework.

Example 9. Continuing with the running example and considering the elements provided by the Example 8, we observe that $\mathcal{C}_1$ does not receive any attack. Furthermore, there is no defense for the attacks of $\mathcal{C}_1$ and $\mathcal{C}_3$ to $\mathcal{C}_2$; on the other hand, $\mathcal{C}_3$ is free of attacks. The coalition $\mathcal{C}_4$ is strongly-defended by $\mathcal{C}_3$ from $\mathcal{C}_2$ attack. Thus, the $\mathcal{S} = \{\mathcal{C}_1, \mathcal{C}_3, \mathcal{C}_4\}$ is a $\subseteq$-maximal strongly-admissible set in $\mathcal{C}_\Phi$, therefore it is a strongly-preferred extension $w.r.t.$ $\mathcal{C}_\Phi$ (see Fig. 11). In this case, there are no τ-preferred nor weakly-preferred extensions. Remember that, in the graphs, the numbers attached to the arrows representing attacks convey the strength of these attacks, while those attached to the s-coalitions in dotted boxes indicate the cohesion of the s-coalition.

Under the classic approaches to modeling argumentative debate [3,16], even the introduction of a trivial opposing idea is treated as an attack undistinguishable from the other attacks resulting in that the attacked argument is effectively defeated or rebutted. However, following typical human behavior, it is natural to consider two coalitions that weakly attack each other as a unique set of stances, *i.e.*, a group of arguments with slight nuances that do not truly change their aim or the foundation of the community.

Definition 32 (Assimilated s-coalition)**.** Let $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}, \mathsf{Cont}_{\mathbb{C}} \rangle$ be an S-BAF where $\overline{\Theta}$ is the underlying bipolar argumentation framework described as $\overline{\Theta} = \langle \mathbb{A}\mathrm{rgs}, \mathbb{R}_s, \mathbb{R}_a \rangle$, and $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$ the meta-argumentation framework associated with Φ. Let $\mathcal{C}_1, \mathcal{C}_2 \in \mathcal{C}_\Phi$ be two s-coalitions. We will say that $\mathcal{C}_2$ is assimilated into $\mathcal{C}_1$ iff only weak attacks exist from $\mathcal{C}_1$ to $\mathcal{C}_2$ and $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_1) \geqslant \mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_2)$; the assimilated coalition will be denoted as $\mathcal{C}^{\dashv}{}_{1,2}$ and will result from the union of the two sets of arguments $\mathcal{C}_1$ and $\mathcal{C}_2$, *i.e.*, $\mathcal{C}^{\dashv}{}_{1,2} = \mathcal{C}_1 \cup \mathcal{C}_2$.

The definition of assimilation of s-communities comes naturally, introducing the potential of admitting a certain degree of controversy into a set of ideas or opinions. The intuition behind this tolerance is that two communities may have differences; however, these differences may become insignificant when we carefully study them in the context of a debate, so the stances supported in both sets can be considered as a single s-community with some internal, minor disagreements. In other words, a coalition coming from an assimilation is such that it has partially consistent beliefs, but the internal controversy can be tolerated. The new set of beliefs will be a possible weaker coalition but can evolve into more entrenched beliefs.

Proposition 9. *Given a meta-argumentation framework $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$, where $\mathcal{C}_\Phi$ is the set of s-coalitions, $\mathbb{R}_a^{\mathcal{C}_\Phi}$ is an attack relation between s-coalitions, and $\mathsf{Str}_{\mathcal{C}}^{\Phi}$ the strength of attack function. Let $\mathcal{C}_1, \mathcal{C}_2 \in \mathcal{C}_\Phi$ be two s-coalitions obtained from Φ. If $\mathcal{C}_1 \cup \mathcal{C}_2$ is a τ-conflict-free set (weak conflict-free set) in the underlying* S-BAF, *then the coalitions $\mathcal{C}_1$, $\mathcal{C}_2$ might be assimilated.*

Note that the attacks are maintained after the assimilation process, *i.e.*, if a coalition $\mathcal{C}_1$ assimilates a coalition $\mathcal{C}_2$, all the attacks that affect them are inherited by the new coalition, including the weak attacks between them. Furthermore, the attacks induced by the arguments in $\mathcal{C}_1$ and $\mathcal{C}_2$ are also maintained in the resulting assimilated coalition.

From another perspective, when a coalition assimilates another, the set of arguments expands by including other postures; thus, the resulting coalition has a more flexible view of the situation by admitting these alternatives. Nevertheless, the process results in a new coalition with a cohesion degree that cannot be greater than the assimilating coalition. The following proposition formalizes this result.

Proposition 10. *Given a meta-argumentation framework $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$, where $\mathcal{C}_\Phi$ is the set of s-coalitions, $\mathbb{R}_a^{\mathcal{C}_\Phi}$ is an attack relation between s-coalitions, and $\mathsf{Str}_{\mathcal{C}}^{\Phi}$ the strength of attack function. Let $\mathcal{C}_1, \mathcal{C}_2 \in \mathcal{C}_\Phi$ be two s-coalitions obtained from Φ. If $\mathcal{C}_2$ can be assimilated into $\mathcal{C}_1$ yielding $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_{1,2})$, then $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_1) \geqslant \mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_{1,2})$, i.e., the resulting coalition $\mathcal{C}_{1,2}$ cannot increase its cohesiveness.*

Example 10. Returning to Example 9, we observe that $\mathcal{C}_1$ weakly-attacks $\mathcal{C}_2$, using the (B, D) point attack, with $\mathsf{Str}_{\mathcal{C}}^{\Phi}(\mathcal{C}_1, \mathcal{C}_2) = 0.4 < \tau$. Besides the $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_2) = 0.34 > \mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_1) = 0.2$. In this way, and following the Definition 32, we can say that $\mathcal{C}_1$ can be assimilated into $\mathcal{C}_2$ conforming a new s-coalition

$$\mathcal{C}^{\dashv}{}_{1,2} = \mathcal{C}_1 \cup \mathcal{C}_2 = \{\mathrm{A, B, C, D, E, F}\}$$

as is represented in the Figure 12. Note that, the cohesion measure associated with the new s-coalition, using a product T-norm is 0.076, while the cohesion measure of $\mathcal{C}^{\dashv}{}_{1,2}$ through a Max T-conorm is 0.38. Furthermore, by the inheritance of attacks, we have that coalitions $\mathcal{C}_3$ and $\mathcal{C}_4$ attack the new coalition $\mathcal{C}^{\dashv}{}_{1,2}$, while $\mathcal{C}^{\dashv}{}_{1,2}$ attacks $\mathcal{C}_4$. However, a new analysis must be carried out to determine the attack types produced in this new scenario and, subsequently, the new extensions. Thus, the cohesion measure obtained through a Max T-conorm, we have that: $\mathcal{C}_4$ and $\mathcal{C}_3$ strongly-attacks $\mathcal{C}^{\dashv}{}_{1,2}$, while $\mathcal{C}^{\dashv}{}_{1,2}$ weakly-attacks $\mathcal{C}_4$. Then, we can say that the extension $\mathcal{S} = \{\mathcal{C}_3, \mathcal{C}_4\}$ is a $\subseteq$-maximal strongly-admissible set in $\mathcal{C}_\Phi$, therefore is a strongly-preferred extension $w.r.t.$ $\mathcal{C}_\Phi$ (see Fig. 13).

Note that, as a result of the assimilation process, the argumentation model can change. For example, when weak attacks are admitted as part of a coalition, attacks from one coalition to another can be transformed from weak to strong, among other situations. Thus, the changes in the argumentation model could impact the set of accepted arguments. In this sense, it would be reasonable to think that it will be necessary to compute the semantics again and detect the group of acceptable coalitions. However, one way to partially compute the semantics would be to detect the zones where changes occur in the argumentative framework, that is, which parts of the discussion are affected by the assimilation process, computing only the acceptability process over the coalitions affected. This issue exceeds the scope of this work, but we will explore it in future research.

Given the options now available for the preferred extensions (τ-preferred extension and weakly-preferred extension), we can ensure that it is always possible to carry out the coalition assimilation process within these extensions.

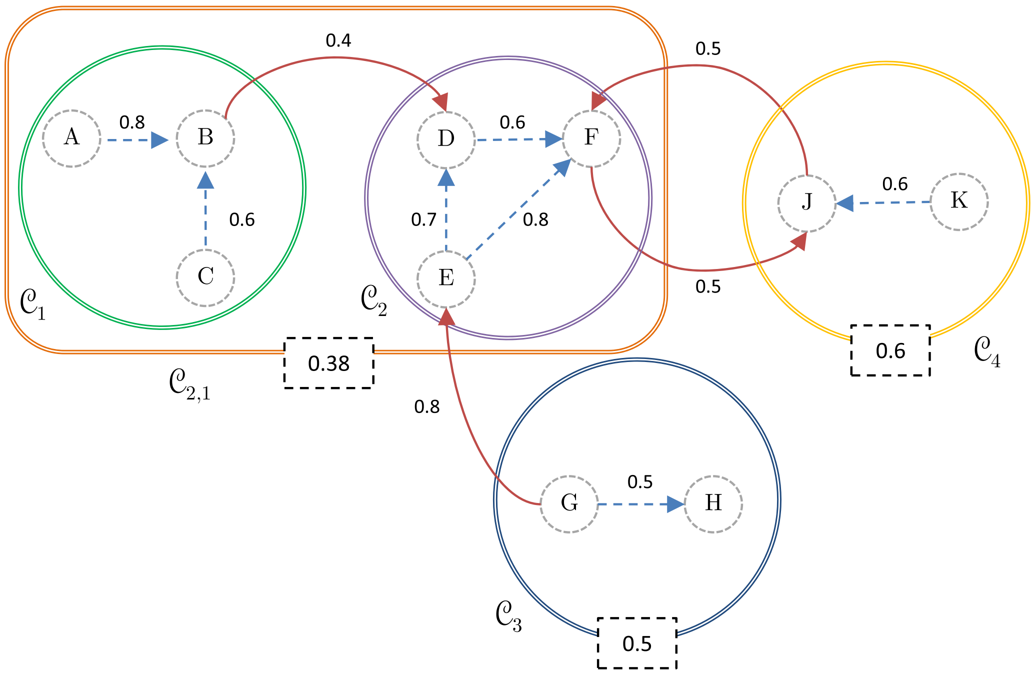

Fig. 12. Assimilated coalitions in a meta argumentation framework.

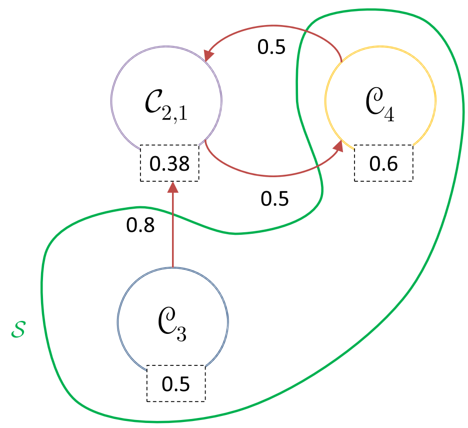

Fig. 13. Preferred semantics of the meta argumentation framework.

Proposition 11. *Given a meta-argumentation framework* $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$*, where* $\mathcal{C}_\Phi$ *is the set of s-coalitions,* $\mathbb{R}_a^{\mathcal{C}_\Phi}$ *is an attack relation between s-coalitions, and* $\mathsf{Str}_{\mathcal{C}}^{\Phi}$ *the strength of attack function. If* $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ *is a* τ*-preferred extension (weakly-preferred extension), then there exists at least two coalitions* $\mathcal{C}_1, \mathcal{C}_2 \in \{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ *that might be assimilated.*

To conclude, it is appropriate to observe that the notion of coalition introduced supports the definition of communities in the context of an argumentation-supported debate. We can also remark that the notions of *conflict-freeness* and *safety* can be replicated in this type of community, and these characteristics are especially interesting for analyzing complex debates, such as those that frequently occur in the social networks. The reason is that our expansion of formal argumentation theory provides tools to improve the analysis of a debate, like the situation when a community can assimilate another to widen the perspectives and establish a conflict characterization between disagreeing communities.

5. A case study

Now, let us consider the actual opinions in favor of (pro) or against (con) presented in Figure 14 about the following proposition "*Is Human Activity Primarily Responsible for Global Climate Change?*".

The case study can be represented as a BAF (see Fig. 15), characterized by $\Theta = \langle \mathtt{Args}, \mathrm{R}_a, \mathrm{R}_s \rangle$, where:

$$\mathtt{Args} = \{\mathtt{A}, \mathtt{B}, \mathtt{C}, \mathtt{D}, \mathtt{E}, \mathtt{F}, \mathtt{G}, \mathtt{H}\}$$

$$\mathrm{R}_a = \big\{(\mathtt{B}, \mathtt{C}), (\mathtt{F}, \mathtt{E}), (\mathtt{G}, \mathtt{F}), (\mathtt{G}, \mathtt{E})\big\}$$

$$\mathrm{R}_s = \big\{(\mathtt{D}, \mathtt{A}), (\mathtt{A}, \mathtt{B}), (\mathtt{F}, \mathtt{A}), (\mathtt{E}, \mathtt{C}), (\mathtt{H}, \mathtt{G})\big\}$$

A	**Con1:** More than one thousand scientists disagree that human activity is primarily responsible for global climate change.
B	**Con2:** The Cook review of 11,944 peer-reviewed studies found that 66.4% of the studies had no stated position on anthropogenic global warming, and while 32.6% of the studies implied or stated that humans are contributing to climate change, only 65 papers (0.5%) explicitly stated: "that humans are the primary cause of global warming."
C	**Pro1:** The rise in atmospheric CO2 over the last century was caused by human activity, as it occurred at a rate much faster than natural climate changes could produce.
D	**Con3:** Rising levels of atmospheric CO2 do not necessarily cause global warming, which contradicts the core thesis of human-caused climate change.
E	**Pro2:** A National Climate Assessment report said human-caused climate changes, such as increased heat waves and drought, "are visible in every state".
F	**Con4:** Human-produced CO2 is re-absorbed by oceans, forests, and other "carbon sinks," negating any climate changes.
G	**Undef1:** A 2012 Purdue University survey found that 47% of climatologists challenge the idea that humans are primarily responsible for climate change and instead believe that climate change is caused by an equal combination of humans and the environment (37%), mostly by the environment (5%), or that there's not enough information to say (5%).
H	**Undef2:** According to a report from the Tropical Meteorology Project at Colorado State University, specific hurricanes, were not a direct consequence of human-caused global warming.

Fig. 14. A set of arguments concerning the possible causes of climate change.

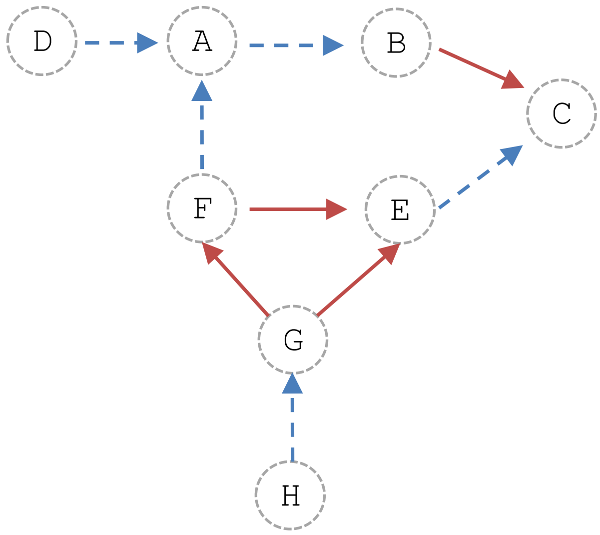

Fig. 15. Climate change argumentative process represented in a BAF.

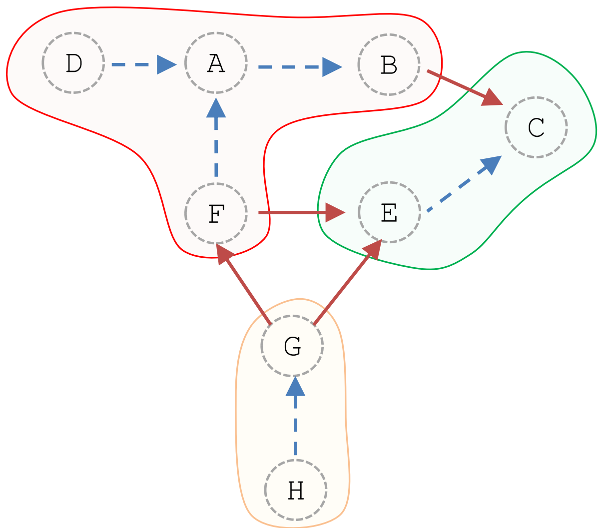

Fig. 16. Informal identification of stances concerning climate change.

From this set of arguments, informally, we can distinguish three general stances about climate change represented in Fig. 16: one of them (highlighted red) groups the arguments that disagree with the posture that human activity is the primary cause of climate change, another group (highlighted green) gathers the opinions that confront the previous one; the third (highlighted orange) collects the ones that adopt an intermediate posture. Now, we consider a specific context to perform a complete analysis, starting by computing the similarity between arguments, that allow us to detect how strong the relation between them are. So, we establish the following context of comparison:

$$\mathbb{C} = \{(climate\_change, 0.5); (human\_main\_cause, 0.3); (scientific\_evidence, 0.2)\}$$

and the following enriched arguments:

$$\delta_{\mathtt{A}} = \big\{\big(climate\_change, \{yes\}\big); \big(human\_responsability, \{yes\}\big); \big(human\_main\_cause, \{no\}\big); \big(other\_causes, \{no\}\big); \big(scientific\_evidence, \{yes\}\big)\big\}$$

$$\delta_{\mathtt{B}} = \big\{\big(climate\_change, \{yes\}\big); \big(human\_responsability, \{yes\}\big); \big(human\_main\_cause, \{no\}\big); \big(other\_causes, \{no\}\big); \big(scientific\_evidence, \{yes\}\big)\big\}$$

$$\delta_{\mathtt{C}} = \big\{\big(climate\_change, \{yes\}\big); \big(human\_responsability, \{yes\}\big); \big(human\_main\_cause, \{yes\}\big); \big(other\_causes, \{rise\_\mathrm{CO2}\}\big); \big(scientific\_evidence, \{no\}\big)\big\}$$

$$\delta_{\mathtt{D}} = \big\{\big(climate\_change, \{yes\}\big); \big(human\_responsability, \{yes\}\big); \big(human\_main\_cause, \{no\}\big); \big(other\_causes, \{rise\_\mathrm{CO2}\}\big); \big(scientific\_evidence, \{no\}\big)\big\}$$

$$\delta_{\mathtt{E}} = \big\{\big(climate\_change, \{yes\}\big); \big(human\_responsability, \{yes\}\big); \big(human\_main\_cause, \{no\}\big); \big(scientific\_evidence, \{yes\}\big)\big\}$$

$$\delta_{\mathtt{F}} = \big\{\big(climate\_change, \{yes\}\big); \big(human\_responsability, \{no\}\big); \big(human\_main\_cause, \{no\}\big); \big(other\_causes, \{rise\_\mathrm{CO2}\}\big); \big(scientific\_evidence, \{no\}\big)\big\}$$

$$\delta_{\mathtt{G}} = \big\{\big(climate\_change, \{yes\}\big); \big(human\_responsability, \{yes, no\}\big); \big(human\_main\_cause, \{yes, no\}\big); \big(other\_causes, \{human\_environmental\_mix, only\_environmental\}, \big); \big(scientific\_evidence, \{yes\}\big)\big\}$$

$$\delta_{\mathtt{H}} = \big\{\big(climate\_change, \{no\}\big); \big(human\_responsability, \{yes, no\}\big); \big(other\_causes, \{hurricanes\}, \big); \big(scientific\_evidence, \{yes\}\big)\big\}$$

It is important to note that enriched arguments G and H refer to human beings' partial responsibility for climate change. This type of situation can be represented given the descriptors the values "*yes*" and "*no*" simultaneously. It is possible to calculate the similarity degree between arguments using the T-conorm probabilistic sum $\alpha \odot \beta = \alpha + \beta - \alpha\beta$ obtaining the valuations for each relation showed in Fig. 17. Note that the attack between the arguments F and E is weaker than the attack between B and C, and the attack between G and E is weaker than the attack between G and F. It is also possible to determine the existing weakest support relationship in the model, which is the one between H and G. Now, focusing on the controversy associated with attacks and the cohesion related to the support relations, employing a *max T-conorm* to calculate the cohesion of the support and a *product T-norm* to obtain the controversy of the attack, we have:

$$\mathsf{Coh}_{\mathbb{C}}\big(\{(\mathrm{D}, \mathrm{A}), (\mathrm{A}, \mathrm{B})\}\big) = 0.72 \qquad \mathsf{Coh}_{\mathbb{C}}\big(\{(\mathrm{F}, \mathrm{A}), (\mathrm{A}, \mathrm{B})\}\big) = 0.72$$

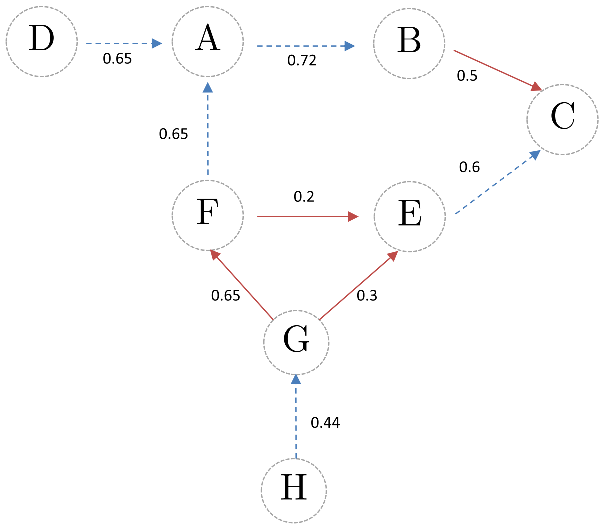

Fig. 17. Climate change stances represented as arguments in an S-BAF.

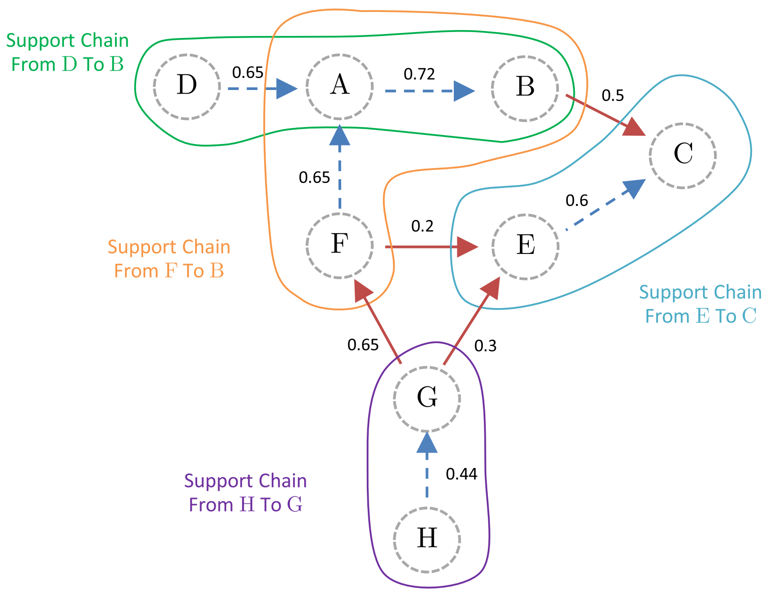

Fig. 18. Interpretation of the cohesion and controversial measures.

$$\mathsf{Coh}_{\mathbb{C}}\big(\{(\mathrm{H},\mathrm{G})\}\big) = 0.44 \qquad \mathsf{Coh}_{\mathbb{C}}\big(\{(\mathrm{E},\mathrm{C})\}\big) = 0.6$$
$$\mathsf{Cont}_{\mathbb{C}}\big(\{(\mathrm{B},\mathrm{C})\}\big) = 0.5 \qquad \mathsf{Cont}_{\mathbb{C}}\big(\{(\mathrm{F},\mathrm{E})\}\big) = 0.2$$
$$\mathsf{Cont}_{\mathbb{C}}\big(\{(\mathrm{G},\mathrm{F})\}\big) = 0.65 \qquad \mathsf{Cont}_{\mathbb{C}}\big(\{(\mathrm{G},\mathrm{E})\}\big) = 0.3$$

Note that, in this case, the cohesion associated with the support relation is examined considering the support chain presented in the argumentation model (see Fig. 18), while the controversy measure is obtained by analyzing the single pairs of attacking arguments. To continue our example analysis, we consider a threshold $\tau = 0.48$ to characterize the relations in our model. Thus, we have that:

- *Strong-direct attacks*:
  - ∗ from B to C, given that $\mathsf{Cont}_{\mathbb{C}}(\{(B, C)\}) > \tau$, and
  - ∗ from G to F, given that $\mathsf{Cont}_{\mathbb{C}}(\{(G, F)\}) > \tau$;
- *Weak-direct attacks*:
  - ∗ from F to E, due to $\mathsf{Cont}_{\mathbb{C}}(\{(F, E)\}) \leqslant \tau$, and
  - ∗ from G to E, given that $\mathsf{Cont}_{\mathbb{C}}(\{(G, E)\}) \leqslant \tau$;
- *Weak-supported attacks*:
  - ∗ between H and F since $\mathsf{Coh}_{\mathbb{C}}(\{(H, G)\}) \leqslant \tau$ while the $\mathsf{Cont}_{\mathbb{C}}(\{G, F\}) > \tau$,
  - ∗ between H and E since $\mathsf{Coh}_{\mathbb{C}}(\{(H, G)\}) \leqslant \tau$ while the $\mathsf{Cont}_{\mathbb{C}}(\{G, E\}) \leqslant \tau$,
- *Strong-secondary attacks*:
  - ∗ between G and B since $\mathsf{Cont}_{\mathbb{C}}(\{(G, F)\}) > \tau$ and $\mathsf{Coh}_{\mathbb{C}}(\{(F, A), ((A, B)\}) > \tau$,
  - ∗ between G and A since $\mathsf{Cont}_{\mathbb{C}}(\{(G, F)\}) > \tau$ and $\mathsf{Coh}_{\mathbb{C}}(\{(F, A)\}) > \tau$.
- *Weak-secondary attacks*:
  - ∗ between G and C since $\mathsf{Cont}_{\mathbb{C}}(\{(G, E)\}) \leqslant \tau$ while $\mathsf{Coh}_{\mathbb{C}}(\{(E, C)\}) > \tau$ and
  - ∗ between F and C since since $\mathsf{Cont}_{\mathbb{C}}(\{(F, E)\}) \leqslant \tau$ while $\mathsf{Coh}_{\mathbb{C}}(\{(E, C)\}) > \tau$;
- *Strong-supported attacks*:
  - ∗ between A and C since $\mathsf{Coh}_{\mathbb{C}}(\{(A, B)\}) > \tau$ and the $\mathsf{Cont}_{\mathbb{C}}(\{B, C\}) > \tau$,
  - ∗ between D and C since $\mathsf{Coh}_{\mathbb{C}}(\{(D, A), ((A, B)\}) > \tau$ and the $\mathsf{Cont}_{\mathbb{C}}(\{B, C\}) > \tau$,
  - ∗ between F and C since $\mathsf{Coh}_{\mathbb{C}}(\{(F, A), ((A, B)\}) > \tau$ and the $\mathsf{Cont}_{\mathbb{C}}(\{B, C\}) > \tau$.

Doing an extended analysis to determine the set of acceptable arguments (see Fig. 19), we have that:

- $S_1 = \{D, A, B, F\}$, is *strongly-conflict-free*, *strongly-safe*, because there are no elements in them that simultaneously support and attack external arguments. This set does not receive attacks, therefore is a d-strongly-admissible, s-strongly-admissible, and c-strongly-admissible set (since is closed under support relation); however, it is not a maximal set.
- $S_2 = \{E, C\}$ is *strongly-conflict-free* and *strongly-safe*; however, the set does not defend their elements from B and F attacks.
- $S_3 = \{H, G\}$ is *strongly-conflict-free* and *strongly-safe*; this set does not receives attacks, therefore is a d-strongly-admissible, s-strongly-admissible, and c-strongly-admissible set, however, it is not a maximal set.
- $S_4 = \{H, G, D, E\}$ is τ*-conflict-free*, is a maximal set, but it is not safe because the set supports and attacks the argument A simultaneously. Additionally, the set strongly-defends E from F attacks. So, S_4 is a d-τ-preferred extension.

We have analyzed the relationship between the arguments of our example as a BAF. Then, we weighted the framework using the similarity function to find the set of acceptable arguments according to S-BAF. Now, focusing on the concepts introduced here, using the *min T-conorm* to calculate the cohesion and considering a $\tau = 0.48$, we can distinguish three coalitions (see Fig. 20):

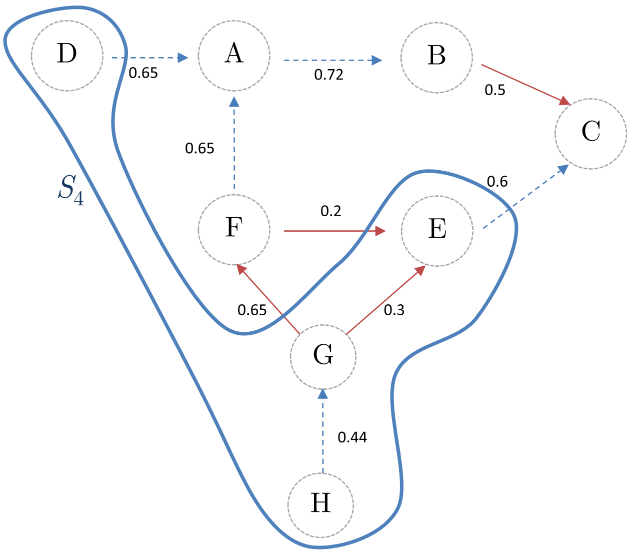

Fig. 19. Semantic extension in S-BAF.

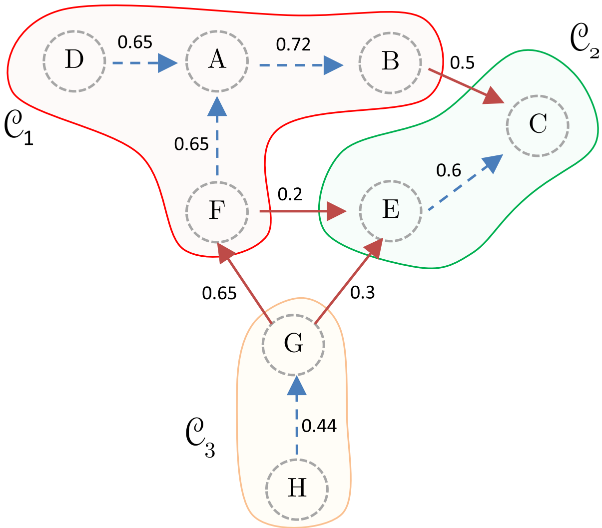

Fig. 20. Identification of coalitions in the climate change debate.

- $\mathcal{C}_1 = \{\text{D, F, A, B}\}$, highlighted green, that disagree with human activity as primary cause of climate change. The $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_1) = 0.65 \geqslant \tau$, so it is a τ*-coalition*.
- $\mathcal{C}_2 = \{\text{E, C}\}$, highlighted red that confronts the previous one with the opposite position, with $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_2) = 0.6 > \tau$ so it is a τ*-coalition*.
- $\mathcal{C}_3 = \{\text{G, H}\}$, highlighted orange that presents an intermediate posture between the other ones have a $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_3) = 0.44 < \tau$, so it is a *weak-coalition*.

Based on these cohesion values, we can classify the coalition as follows: $\mathcal{C}_1$ and $\mathcal{C}_2$ are *sd-moderate-communities* whereas $\mathcal{C}_3$ is a *sd-fragile-community*. Each of these three coalitions aggregates different

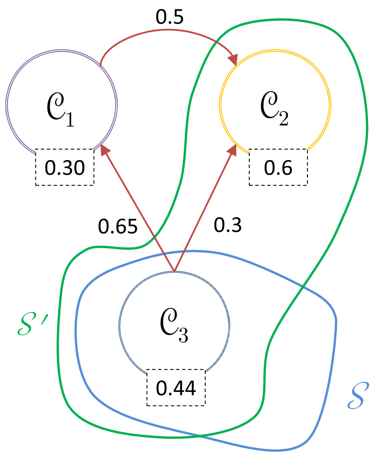

Fig. 21. Preferred extensions for the climate change debate.

points of view, providing various aspects in favor of the community's overall stance. If we analyze the two well-defined general postures, we will obtain the details of the beliefs backed by each community; but, by examining with more detail the opinions in each organization, we can determine the strength inherent to each community.

We have that, $\mathcal{C}_1$, $\mathcal{C}_2$, and $\mathcal{C}_3$ are maximal conflict-free sets, and maximal sets closed under $\mathtt{R}_s$. Then, continuing the analysis, recalling that our threshold is $\tau = 0.48$, taking into account the coherence of each coalition, and considering that we instantiate the controversial function $\mathsf{Cont}_{\mathcal{C}}^{\Phi}$ with the *product T-norm*, we have that:

- There are two conflict points between the s-coalitions $\mathcal{C}_1$ and $\mathcal{C}_2$: the pairs (B, C) and (F, E). There is a conflict point between $\mathcal{C}_3$ and $\mathcal{C}_1$ and between $\mathcal{C}_3$ and $\mathcal{C}_2$: the pairs (G, F) and (G, E), respectively;
- The strength associated with the attack between $\mathcal{C}_1$ and $\mathcal{C}_2$ is $\mathsf{Str}_{\mathcal{C}}^{\Phi}(\mathcal{C}_1, \mathcal{C}_2) = \mathit{prod}(0.5, 0.2) = 0.1$, where $\mathit{prod}(\cdot,\cdot)$ is the product T-norm, and that value is lower than the threshold τ. Besides the $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_1) = 0.65 > \tau$. Thus, $\mathcal{C}_1$ weakly-attacks $\mathcal{C}_2$. Furthermore, we know that $\mathcal{C}_3$ weakly-attacks $\mathcal{C}_1$ given that $\mathsf{Str}_{\mathcal{C}}^{\Phi}(\mathcal{C}_3, \mathcal{C}_1) = 0.6 > \tau$ but $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_3) = 0.44 < \tau$, while $\mathcal{C}_3$ weakly-attacks $\mathcal{C}_2$ because $\mathsf{Str}_{\mathcal{C}}^{\Phi}(\mathcal{C}_3, \mathcal{C}_2) = 0.3 < \tau$ and $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_3) = 0.44 < \tau$.

Analyzing the semantic level (see Fig. 21), we note that $\mathcal{C}_1$ and $\mathcal{C}_2$ do not defend themselves from $\mathcal{C}_3$ attacks, while the coalition $\mathcal{C}_3$ does not receive any attack, so $\mathcal{S} = \{\mathcal{C}_3\}$ is a strongly-admissible set and a strongly-preferred extension. On the other hand, the set of coalitions $\mathcal{S}' = \{\mathcal{C}_3, \mathcal{C}_2\}$ is a τ-admissible set and a τ-preferred extension. We notice that $\mathcal{C}_3$ weakly-attacks $\mathcal{C}_2$ but does not assimilate $\mathcal{C}_2$ given that $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_3) < \mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_2)$. In other words, $\mathcal{C}_3$ as the attacking coalition cannot incorporate the beliefs of $\mathcal{C}_2$ since the opinions advanced by coalition $\mathcal{C}_2$ are very close to a specific position over the discussion while those proposed by coalition $\mathcal{C}_3$ are more open. A similar analysis considers the $\mathcal{C}_3$ and $\mathcal{C}_1$. In this case, $\mathcal{C}_3$ strongly-attacks $\mathcal{C}_1$, for which the assimilation is impossible. However, we find an interesting result by analyzing the relation between $\mathcal{C}_1$ and $\mathcal{C}_2$. Given that $\mathcal{C}_1$ weakly-attacks $\mathcal{C}_2$ and the $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_1) > \mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_2)$, the assimilation is possible. Considering that *climate_change* is the prevalent descriptor in the context, the attacks between the coalitions have little importance, although the postures put forward by each coalition seem entirely different. This result reminds us that one of the main features of our framework is to be sensitive to the context.

In this way, we carried out a complete analysis of the example at three levels: BAF, S-BAF, and considering the meta-argumentation framework, where different extensions are obtained considering a range of aspects in an argumentation-based debate.

6. Related work

Regarding the concept of "community," although Fortunato [17] relates them with *clusters*, or *modules*, the author asserts the need to recognize that there is no accepted definition of what a community is. However, he notes that community detection is a significant issue in areas such as Biology, Sociology, and Computer Science. In that work, graphs are used as structures representing objects that coexist in order or disorder, and some valuable tools for finding clusters are mentioned, such as *graph partitioning*, *hierarchical clustering*, *partitional clustering*, and *spectral clustering*. However, no specific algorithm is provided in this regard. In our work, the objects that coexist in an argumentative discourse are the arguments supporting or attacking each other. Besides, we established the foundations to find communities (or coalitions), eliciting them from the enriched arguments and characterizing them by introducing computational models supporting our approach.

Concerning characterization of existing methods for community detection, Chochani et al. [14] present an extensive and comparative review of the detection methods of interesting communities in Online Social Networks, underlining the differences between several approaches based on specific criteria and classifying them. They assert the importance of this topic given the knowledge provided by these communities, highlighting the importance of recognizing how the groups exchange knowledge in the detection of shared interests and goals, among other elements that give the group the attribute of *cohesiveness*. Note that this work presents a comparative review of methods oriented to the search of communities represented with a graph structure. In addition, the authors mention a set of criteria to compare community detection approaches, like the following:

- *Structural features* or interactions between users.
- *Social activities* o behaviors inside the social network (for instance, comments on a post).
- *Attributes* that represent information about the nodes in the graph, which are embedded as labels in these nodes.
- *Content* provided by posts or multimedia publications.
- *Social influence* allowing the spread of information.

According to these criteria, our work considers structural features represented as the relationships of attack and support, attributes as descriptors in enriched arguments, and the degree of similarity between arguments. Based on these attributes, we introduced and exploited the degrees of controversy and cohesion to find communities as attributes inherent to the relation between the nodes in our argumentative graph structure. Furthermore, the process of assimilation between coalitions will be explored in future work as a possible measure of social influence.

More specifically, referring to communities as coalitions, and despite having referred to coalitions in Section 2, it is appropriate to mention the work of Amgoud et al. [1], where the term coalition is associated with collaborative and coordinated work among agents in order to accomplish a task. The work mentioned focuses on defining the structure of the coalition, determining which tasks can be executed independently, or analyzing the best way to distribute the tasks. Several works have been developed from a similar perspective, as detailed in [33]. Coalitions are studied in the decision-making process and multi-agent systems with different purposes. For example, in [42] the authors present an experimental work where coalitions are essential for the decision-making regarding classifiers, which can work collaboratively using dispersed knowledge based on friendly relationships; however, the classifiers can also have a conflictive or neutral relationship. In [4], the coalition represents cooperative tasks in a multi-agent system, where the agents share minimal private information about the other agents' preferences but jointly are capable of achieving their goals. In the same direction, in [8] the authors connect the abstract perspective of formal argumentation theories with the social theories of agent coalitions that offer a conceptual, less formal stance based on modeling languages that contribute more details to the representation. Coalitions are defined by "contracts" in which each agent contributes to the coalition and obtain benefits from it. For the argumentation process required for arguing about coalitions, three social viewpoints are defined with abstraction and refinement relations between them, adapting particular coalition argumentation theory to reason about the coalitions defined in the most abstract viewpoint, which is represented as a set of dynamic dependencies between agents. From an internal point of view, the agents inside a coalition can be described by viewpoints. Thus, a coalition is characterized as a set of agents with their goals and skills, as a set of agents related due to the notion of power, or as a set of dynamic dependencies. In our current approach, we have not analyzed the coalitions as a tool employed to coordinate work between agents.

Concerning the other topic inherent to our work, Furman et al. [19] present a method to find discursive *communities* in social media. The authors analyze small and comprehensive annotated datasets using standard tools like graphs, an algorithm for Modularity Maximization, and a supervised classifier. This approach has good practical performance when the source of the datasets is Twitter, and the topic is different from "*feminism*." Briefly, the proposed method consists of (i) obtaining tweets; (ii) constructing a graph of users where each node represents a user, and the edge is a retweet between them, (iii) detecting communities using an algorithm for Modularity Maximization, (iv) using of a supervised classifier to label communities, and (v) training of the classifier. In this approach, it is necessary to make assumptions about the number and distribution of stances. Another proposal is by Pamungkas et al. [37], which presents a method to classify tweets based on affective features, developing a tool to prevent spreading rumors. In this case, the rumors are provided as data inherent to an annotated dataset obtained from Twitter. The Jaccard similarity measure is used to characterize the conversational thread, measuring the similarity between a tweet source and the rest of the tweets in the thread. Then, each tweet is classified as *agree, accept, or support the rumor*, *reject or deny the rumor*, *request or questioning the rumor*, and *given an opinion or comment about the rumor*. This contribution is important, but it is limited to effective conversations. In previous work, but on the same line of research, in [56], the authors not only classified tweets but also presented a method to determine if each message is relevant or irrelevant for the considered target. The authors used supervised and weakly supervised tasks, considering five predefined targets with labeled training data. Although our research can be used to find communities in social media, we generalized a method to characterize these communities by understanding them as coalitions. The coalitions and their features are helpful in the examination of any websites where debates are raised and analyzed, *e.g.*, political debates. As we have described when developing our work, we have not used Modularity Maximization or any machine learning methods, although these techniques are not discarded in future work that will extend the current approach. Instead, we focus on the relationship between different pieces of knowledge and how these relationships help classify communities.

Continuing with work related to the process of identifying communities, Li [31] presents a comparison between three discovery algorithms, where a genetic algorithm is better than OCPLP (Overlapping Community Partitioning based on Label Propagation) and FSOCA (Footpad Skin Optical Clearing Agent) algorithms to find overlapping communities. An interesting point in this work is that the author does not use datasets like tweets; instead, simulated complex neural networks were used. This contribution naturally employs a very different approach from ours, not only because the AI techniques used to discover communities are different, but the work of Li [31] does not consider the coalition concept. Another interesting work in identifying communities is proposed by Puertas et al. [43] who presents an approach to detecting communities in social networks, analyzing several tweets from Colombian Universities' accounts, and finding sociolinguistic features in them. The proposed method considers three components: an expert, computational linguistics, and AI techniques. The authors: (i) extracted profiles and conversations from the mentioned sources and processed those conversations, examining the personal information of the users, the vocabulary employed in the communication between the users, the relation between them (followers or being followed), and their shared concepts, words, and interests; (ii) used techniques such as term-frequency, inverse-document frequency, and word frequency, identifying the features of the communities; in this step, the authors proposed determining the language of the content, applying some techniques such as the extraction of noun phrases, the analysis of the dependencies in a sentence, the finding of tokens in the sentence, and the reduction of the words to their roots; and, (iii) found the relation between words and categories of an area of interest and determined the relation between the words employed by the users and those used in the social network. Finally, this approach uses these results to detect social groups through their vocabulary. One of the most critical differences between the described work and ours is that the s-coalitions detection method presented here is not focused on the user that put forward an opinion; in fact, our work only considers the relationships among the opinions introduced in a debate to find communities or coalitions and to characterize them. However, both works consider some features depending on the language, *e.g.*, Puertas et al. use some techniques related to the frequency of the terms, and we based our method on the enriched arguments or arguments with additional information provided by a set of descriptors.

We can mention the work developed in [30] where the author presents a review of the literature concerning the research on different coalition categories: *conceptual* or based on mathematical models, *quasi-conceptual* or considering deducible empirical regularities, and *extrapolative*, that include experimentation with statistical models. Our work can be placed in the first category. In this direction, the proposed approach is based on S-BAF [11]. However, it is appropriate to mention the approach of Vassiliades et al. [49], where the authors developed an Abstract Argumentation Framework where each argument has a domain of application. With this formalism, it is possible to determine the scope of an argument that can be accepted or partially accepted. The main difference between the approach provided in [49] and the one presented in [11] is that the first contemplates only the attack relations between the arguments; meanwhile, the second incorporates the support relations too. Besides, the tools used to model the arguments in the approach of Vassiliades et al. are different from those used in [11], *e.g.*, an argument is understood as a set of elements, or entities, in the domain, or an argument is conceived as a decorated piece of information, respectively.

Finally, Budán et al. in [10], the authors offered the formalization of an abstract argumentation framework that considers a set of interrelated topics used to decorate arguments. One of the contributions is the examination of new argumentation semantics that consider these topics to obtain the accepted arguments. The topics are related to each other, leading to a graph structure representing that relationship; furthermore, from the graph, a notion of distance between topics is used to study proximity-based semantics. The main idea in these argumentation semantics is that an argument should be defended by arguments that are closely related to the addressed topics. In this sense, the authors explored this position by defining new elements, such as distance-bounded admissible sets and a new notion of skeptical semantics called focused extension. One of the main differences with our work is that, in our proposal, it is possible to analyze the bipolarity of human thought and model both support and attacks. Likewise, in Budán et al. proposal, only the effects of the distance between arguments are analyzed regarding notions of defense. At the same time, in our work, both attacks and supports can be dismissed or weakened according to similarity. Lastly, our work analyzes relationships to find communities that may have slightly conflicting thoughts.

7. Conclusion and future work

Community detection is an important research area in social networks analysis where we are concerned with discovering the network structure. The tendency of people with similar tastes, choices, and preferences to get associated in a network leads to virtual clusters or communities. Detection of these communities can benefit numerous applications, such as finding a common research area in collaboration networks, finding a set of like-minded users for marketing and recommendations, or finding protein interaction networks in biological networks. Detecting communities is essential in sociology, biology, and computer science, where often these communities are very complex, and only limited representation tools are available; in some cases, only a graph is used to represent the interchange of entities.

In a discussion, it is possible to find clusters or communities with a common point of view on a given topic or issue under discussion among debate participants. In this sense, finding these communities helps detect how many different points of view exist in a discussion, how strong they are, what relationships exist between them, and how they affect argumentative discourse.

In this sense, this work presented a novel mechanism to find meta-structures (coalitions) based on the similarity between the supported related arguments, using this measure as a sense of the coalition's cohesion. We used the similarity between supported related arguments to obtain the coherence of the coalition; this notion allowed us to rate them and find the trends of the conversation.

Furthermore, we employed the similarity degree to characterize the attacks between coalitions, advancing a controversy measure. Additionally, computing all the attacks received by an s-coalition, we proposed a mechanism to determine the weakening level over this set of arguments. However, these mechanisms have certain drawbacks. For example, they depend on the descriptors of the arguments, and obtaining these descriptors can rely on very specialized argument mining techniques.

Future work offers different lines of research, such as developing an implementation of S-BAFs with coalition detection by using the existing DeLP [21] system as an initial point. The resulting implementation will be applied to different domains requiring modeling decision support systems with representation and detection of communities where user preferences can be considered. Furthermore, it would be interesting to investigate how communities based on similarities can be used to group experts' opinions in different areas and then measure the impact and acceptance of those opinions on the community. In other words, we will try to use the proposed conceptualization to improve the Argument Scheme that appeals to Expert Opinions [51,52,54]. We want to use the formalisms developed to improve the computability of the argumentation scheme based on analogy [53]. The argumentation schemes are composed of premises and a conclusion expressed in semi-formal language. In particular, the schemata based on analogies consider two analogous cases. However, this scheme does not define the meaning of "*similar*" in a computable way. It would be possible to apply the similarity measures provided in this work to find a solution to this problem.

Appendix A. Proofs

Proposition 12. *Let* $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}, \mathsf{Cont}_{\mathbb{C}} \rangle$ *be a* S-BAF*, where* $\overline{\Theta}$ *is an enriched bipolar argumentation framework* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rgs}, \mathbb{R}_s, \mathbb{R}_a \rangle$*. Each enriched argument, which is not self-attacking, belongs to an s-coalition.*

Proof. Let A be an enriched argument, which is not self-attacking. Then {A} is strongly-conflict-free, and the subgraph induced by {A} is connected. So {A} satisfies the conditions of Definition 19. Either {A} is a coalition, or there exists a subset of enriched arguments $S \subseteq \mathbb{A}\mathrm{rgs}$ that contains A and satisfies the conditions of Definition 19. So, there exists a coalition $\mathcal{C}$ containing S and thus containing {A}. □

Proposition 13. *Let* $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}, \mathsf{Cont}_{\mathbb{C}} \rangle$ *be an* S-BAF*, where* $\overline{\Theta}$ *is the underlying bipolar argumentation framework* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rgs}, \mathbb{R}_s, \mathbb{R}_a \rangle$*, and let* A, B $\in \mathbb{A}\mathrm{rgs}$ *two enriched arguments such that* (A, B) $\in \mathbb{R}_s$*, and* $\mathbb{R}_a$ *does not contain* (A, B) *or* (B, A)*. If* A $\mathbb{R}_s$ B *is either a strong-support or weak-support relation, then there exists at least a weak-coalition containing both* A *and* B.

Proof. Suppose that (A, B) $\in \mathbb{R}_s$, $\mathbb{R}_a$ does not contain (A, B) or (B, A), and there is no coalition $\mathcal{C} \in \mathcal{C}_\Phi$ such that A, B $\notin \mathcal{C}$. By Definition 19, we know that a coalition is a maximal conflict-free set closed under support because a coalition induces a subgraph connected only by the support relation. Thus, if (A, B) $\in \mathbb{R}_s$ and $\mathbb{R}_a$ does not contain (A, B) or (B, A), it is clear that A and B should be part of some $\mathcal{C}$. Contradiction. Furthermore, by Definition 12, $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}) \geqslant 0$. Then, by Definition 20, we can conclude that $\mathcal{C}$ is at least a weak coalition. □

Proposition 14. *Given the* S-BAF $\Phi = \langle \overline{\Theta}, \mathsf{Sim}_{\mathbb{C}}, \mathsf{Coh}_{\mathbb{C}}, \mathsf{Cont}_{\mathbb{C}} \rangle$ *where* $\overline{\Theta}$ *is the underlying bipolar argumentation framework described as* $\overline{\Theta} = \langle \mathbb{A}\mathrm{rgs}, \mathbb{R}_s, \mathbb{R}_a \rangle$*, the meta-argumentation framework* $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$ *associated with* Φ*, and* $\mathsf{Cont}_{\mathcal{C}}^{\Phi}$ *a controversy degree function* $\mathsf{Cont}_{\mathcal{C}}^{\Phi}$ *defined over* $\Phi^{\mathcal{C}}$*. Let* $\mathcal{S} \subseteq \mathcal{C}_\Phi$ *be a set of coalitions, and* $\mathbb{S} \subseteq \mathbb{A}\mathrm{rgs}$ *be the enriched arguments involved in* $\mathcal{S}$*, then* $\mathsf{Cont}_{\mathbb{C}}(\mathbb{S}) = \mathsf{Cont}_{\mathcal{C}}^{\Phi}(\mathcal{S})$.

Proof. Consequence of Definition 12 and Definition 27. □

Proposition 15. *Let* $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$ *be the meta-argumentation framework associated with* Φ*,* $\mathsf{Cont}_{\mathcal{C}}^{\Phi}$ *a controversy degree function defined over* $\Phi^{\mathcal{C}}$*,* $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ *be a finite set of coalitions, and* $\tau \in [0, 1]$ *be a threshold. Then:*

(i) $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ *is strongly-conflict-free for* $\Phi^{\mathcal{C}}$ *iff* $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ *is strongly-conflict-free for* Φ.
(ii) $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ *is strongly-conflict-free for* $\Phi^{\mathcal{C}}$ *iff* $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ *is strongly-safe for* Φ.
(iii) *If* $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ *is* τ*-conflict-free for* Φ*, then* $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ *is* τ*-conflict-free for* $\Phi^{\mathcal{C}}$.
(iv) *If* $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ *is at least* τ*-safe for* Φ *then* $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ *is* τ*-conflict-free for* $\Phi^{\mathcal{C}}$.
(v) $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ *is weakly-conflict-free for* $\Phi^{\mathcal{C}}$ *iff* $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ *is weakly-conflict-free for* Φ.
(vi) $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ *is weakly-conflict-free for* $\Phi^{\mathcal{C}}$ *iff* $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ *is at least weakly-safe for* Φ.

Proof. we separate the proof in six parts:

(i) $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is strongly-conflict-free for $\Phi^{\mathcal{C}}$ iff $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is strongly-conflict-free for Φ.
We split this part of the proof in two:

$\Rightarrow$) $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is strongly-conflict-free for $\Phi^{\mathcal{C}}$ then $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is strongly-conflict-free for Φ. By hypothesis and Definition 28, we know that there are no $\mathcal{C}_i, \mathcal{C}_j \in \{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$, with $i \neq j$, such that $\mathcal{C}_i$ strongly or weakly attacks $\mathcal{C}_j$. That is, there is no $(\mathcal{C}_i, \mathcal{C}_j) \in \mathbb{R}_a^{\mathcal{C}_\Phi}$. Thus, by Definition 22, there is no arguments $\mathrm{A} \in \mathcal{C}_i$ and $\mathrm{B} \in \mathcal{C}_j$ such that $(\mathrm{A}, \mathrm{B}) \in \mathbb{R}_a$. Thus, by Definition 14, $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is strongly-conflict-free.

$\Leftarrow$) $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is strongly-conflict-free for Φ then $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is strongly-conflict-free for $\Phi^{\mathcal{C}}$. By hypothesis and Definition 14, we know that there are no arguments $\mathrm{A} \in \mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ and $\mathrm{B} \in \mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ such that A strongly or weakly attacks B. Then, by Definition 22 and Definition 19, there are no arguments $\mathrm{A} \in \mathcal{C}_i$ and $\mathrm{B} \in \mathcal{C}_j$ such that $(\mathrm{A}, \mathrm{B}) \in \mathbb{R}_a$ with $\mathcal{C}_i, \mathcal{C}_j \in \{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ and $i \neq j$. Thus, by Definition 28, $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is strongly-conflict-free.

(ii) $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is strongly-conflict-free for $\Phi^{\mathcal{C}}$ iff $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is strongly-safe for Φ.
We split this part of the proof in two:

$\Rightarrow$) If $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is strongly-conflict-free for $\Phi^{\mathcal{C}}$ then $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is strongly-safe for Φ. By hypothesis and Definition 28, we know that there are no $\mathcal{C}_i, \mathcal{C}_j \in \{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$, with $i \neq j$, such that $\mathcal{C}_i$ strongly or weakly-attacks $\mathcal{C}_j$, that is, there is no $(\mathcal{C}_i, \mathcal{C}_j) \in \mathbb{R}_a^{\mathcal{C}_\Phi}$. Therefore, there are no arguments $\mathrm{A} \in \mathcal{C}_i$ and $\mathrm{B} \in \mathcal{C}_j$ such that $(\mathrm{A}, \mathrm{B}) \in \mathbb{R}_a$. Thus, $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is a conflict-free set. Furthermore, by Definition 19, each coalition in $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is closed under the support relation since each of them represent a graph connected only by the supports relations. Finally, by Definition 14, we can conclude that $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is strongly-safe since there is no sequence of enriched arguments in $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ such that attacks and supports an element inside or outside of such set.

$\Leftarrow$) If $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is strongly-safe for Φ then $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is strongly-conflict-free for $\Phi^{\mathcal{C}}$. Suppose that $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is strongly-safe for Φ and $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is not strongly-conflict-free for $\Phi^{\mathcal{C}}$. By hypothesis, $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is not strongly-conflict-free for $\Phi^{\mathcal{C}}$. Thus, by Definition 28, we know that there must be two coalitions $\mathcal{C}_i, \mathcal{C}_j \in \{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$, with $i \neq j$, such that there exists a strong or weak attack from $\mathcal{C}_i$ to $\mathcal{C}_j$. But, we know that by Definition 6, an attack from $\mathcal{C}_i$ to $\mathcal{C}_j$ exists iff there exist enriched arguments $\mathrm{A} \in \mathcal{C}_i$ and $\mathrm{B} \in \mathcal{C}_j$ such that $(\mathrm{A}, \mathrm{B}) \in \mathbb{R}_a$. This leads us to a contradiction since our hypothesis is that $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is strongly-safe for Φ, which means by Definition 14, that there is no $\mathrm{A} \in \mathbb{A}\mathrm{rgs}$ and no pair $\mathrm{B}, \mathrm{C} \in \mathbb{S}$ such that there exists a strong or weak attack from B to A, and either there is a sequence of support from C to A, or $\mathrm{A} \in \mathbb{S}$.

(iii) If $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is τ-conflict-free for Φ, then $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is τ-conflict-free for $\Phi^{\mathcal{C}}$.
By Definition 14, $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is a *τ-conflict-free* set iff there are no arguments $\mathrm{A} \in \mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ and $\mathrm{B} \in \mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ such that A strongly-attacks B and $\mathsf{Cont}_{\mathbb{C}}(\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n) < \tau$. Then, by Definition 19, there are no arguments $\mathrm{A} \in \mathcal{C}_i$ and $\mathrm{B} \in \mathcal{C}_j$ such that A strongly-attacks B and $\mathsf{Cont}_{\mathbb{C}}(\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n) < \tau$, with $\mathcal{C}_i, \mathcal{C}_j \in \{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ and $i \neq j$. Thus, by Proposition 14, Definition 24, Definition 22 and 27, $\mathsf{Cont}^{\Phi}_{\mathcal{C}}(\mathcal{C}_1, \ldots, \mathcal{C}_n\ ) < \tau$ and there is no $\mathcal{C}_i, \mathcal{C}_j \in \{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ such that $\mathcal{C}_i$ strongly-attacks $\mathcal{C}_j$, with $i \neq j$. Then, $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is τ-conflict-free for $\Phi^{\mathcal{C}}$.

(iv) If $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is τ-safe for Φ then $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is τ-conflict-free for $\Phi^{\mathcal{C}}$.
Suppose that $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is not τ-conflict-free for $\Phi^{\mathcal{C}}$. Then, there exist $\mathcal{C}_i, \mathcal{C}_j \in \{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$, with $i \neq j$, such that $\mathcal{C}_i$ strongly-attacks $\mathcal{C}_j$ or $\mathsf{Cont}^{\Phi}_{\mathcal{C}}(\mathcal{C}_1, \ldots, \mathcal{C}_n\ ) \geqslant \tau$. That is, by Definition 25, $\mathcal{C}_i$ strongly-attacks $\mathcal{C}_j$ iff $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_i) \geqslant \tau$ and $\mathsf{Cont}^{\Phi}_{\mathcal{C}}(\mathcal{C}_i, \mathcal{C}_j) \geqslant \tau$. Furthermore, by Definition 24 and Definition 22, there exist an argument $\mathrm{A} \in \mathcal{C}_i$ and an argument $\mathrm{B} \in \mathcal{C}_j$ such that A strongly-

attacks B. However, by Proposition 14, we know that $\mathsf{Cont}^{\Phi}_{\mathcal{C}}(\mathcal{C}_1, \dots, \mathcal{C}_n) = \mathsf{Cont}_{\mathbb{C}}(\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n)$. Furthermore, our hypothesis is that $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is τ-safe for Φ. Thus, by Definition 14, we know that there are no $\mathrm{A} \in \mathbb{A}\mathrm{rgs}$ and no pair $\mathrm{B}, \mathrm{C} \in \mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ such that there exists a strong attack from B to A, $\mathsf{Cont}_{\mathbb{C}}(\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n \cup \mathrm{A}) > \tau$, and either there is a sequence of support from C to A such that $\mathsf{Coh}_{\mathbb{C}}(\{\mathrm{C}, \dots, \mathrm{A}\}) > \tau$, or $\mathrm{A} \in \mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$. Thus, there is no enriched argument B that strongly-attacks A. Leading us to conclude that $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is τ-conflict-free for $\Phi^{\mathcal{C}}$.

(v) $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is weakly-conflict-free for $\Phi^{\mathcal{C}}$ iff $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is at least weakly-conflict-free for Φ. We split this part of the proof in two:

- $\Rightarrow$) $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is weakly-conflict-free for $\Phi^{\mathcal{C}}$ then $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is at least weakly-conflict-free for Φ.
  We know, by hypothesis and Definition 28, that there are no $\mathcal{C}_i, \mathcal{C}_j \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$, with $i \neq j$, such that $\mathcal{C}_i$ strongly attacks $\mathcal{C}_j$. Thus, by Definition 22, there are no arguments $\mathrm{A} \in \mathcal{C}_i$ and $\mathrm{B} \in \mathcal{C}_j$ such that A strongly-attacks B. Thus, by Definition 14, $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is weakly-conflict-free.
- $\Leftarrow$) $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is weakly-conflict-free for Φ then $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is weakly-conflict-free for $\Phi^{\mathcal{C}}$.
  By hypothesis and Definition 14, we know that in the set $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ there are no arguments A and B such that A strongly-attacks B. Then, by Definition 22 and Definition 19, there are no arguments $\mathrm{A} \in \mathcal{C}_i$ and $\mathrm{B} \in \mathcal{C}_j$ such that A strongly-attacks B with $\mathcal{C}_i, \mathcal{C}_j \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ and $i \neq j$. Finally, $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is weakly-conflict-free for $\Phi^{\mathcal{C}}$.

(vi) $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is weakly-conflict-free for $\Phi^{\mathcal{C}}$ iff $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is weakly-safe for Φ. We split this part of the proof in two:

- $\Rightarrow$) If $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is weakly-conflict-free for $\Phi^{\mathcal{C}}$ then $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is weakly-safe for Φ. By hypothesis and (v) above we know that $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is at least weakly-conflict-free for Φ. Furthermore, by Definition 19, we know that each s-coalition in $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ induces a subgraph connected only by the support relation. Thus, we can deduce that each s-coalition is closed under the support relation and a weakly-conflict-free set for Φ.
- $\Leftarrow$) If $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is weakly-safe for Φ then $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is weakly-conflict-free for $\Phi^{\mathcal{C}}$. Suppose that $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is not weakly-conflict-free for $\Phi^{\mathcal{C}}$. Then, by Definition 28, there are two coalitions $\mathcal{C}_i, \mathcal{C}_j \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$, with $i \neq j$, such that there exists a strong-attack from $\mathcal{C}_i$ to $\mathcal{C}_j$. Thus, by Definition 22, there exist enriched arguments $\mathrm{A} \in \mathcal{C}_i$ and $\mathrm{B} \in \mathcal{C}_j$ such that A strongly-attacks B. That lead us to a contradiction, since by hypothesis $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is weakly-safe for Φ. Then, there is no $\mathrm{A} \in \mathbb{A}\mathrm{rgs}$ and no pair $\mathrm{B}, \mathrm{C} \in \mathbb{S}$ such that there is a strong-attack from B to A and either there is a sequence of support from C to A such that $\mathsf{Coh}_{\mathbb{C}}(\{\mathrm{C}, \dots, \mathrm{A}\}) > \tau$, or $\mathrm{A} \in \mathbb{S}$. □

Proposition 16. *Let $\Phi^{\mathcal{C}} = \langle \mathcal{C}_{\Phi}, \mathbb{R}_a^{\mathcal{C}_{\Phi}}, \mathsf{Str}^{\Phi}_{\mathcal{C}} \rangle$ be the meta-argumentation framework associated with Φ, $\mathsf{Cont}^{\Phi}_{\mathcal{C}}$ a controversy degree function defined over $\Phi^{\mathcal{C}}$, and $\mathcal{C}_1, \mathcal{C}_2 \in \mathcal{C}_{\Phi}$ be two s-coalitions. If $\mathcal{C}_1$ and $\mathcal{C}_2$ are two distinct s-coalitions that are connected, then there exists at least a weakly-attack between $\mathcal{C}_1$ and $\mathcal{C}_2$.*

Proof. By Definition 19, we know that the subgraph induced by $\mathcal{C}_1$ (resp. $\mathcal{C}_2$) is connected under the support relation, and $\mathcal{C}_1$ and $\mathcal{C}_2$ are connected. As Definition 19 states, an s-coalition is maximally connected under support and conflict-free, thus $\mathcal{C}_1 \cup \mathcal{C}_2$ cannot be conflict-free. But $\mathcal{C}_1$ and $\mathcal{C}_2$ are conflict-free. So, there exist arguments $\mathrm{A} \in \mathcal{C}_1$ and $\mathrm{B} \in \mathcal{C}_2$ such that at least A weakly-attacks B or vice versa. □

Proposition 17. *Let $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$ be the meta-argumentation framework associated with Φ, $\mathsf{Cont}_{\mathcal{C}}^{\Phi}$ a controversy degree function defined over $\Phi^{\mathcal{C}}$, $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ be a finite set of coalitions, and $\tau \in [0, 1]$ be a threshold. Then, $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is strongly-admissible for $\Phi^{\mathcal{C}}$ iff $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is c-strongly-admissible for Φ.*

Proof. $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is strongly-admissible for $\Phi^{\mathcal{C}}$ iff $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is c-strongly-admissible for Φ.

We break this part of the proof in two:

$\Rightarrow$) If $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is strongly-admissible for $\Phi^{\mathcal{C}}$ then $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is c-strongly-admissible for Φ.
Suppose that $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is strongly-admissible for $\Phi^{\mathcal{C}}$ and $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is not c-strongly-admissible for Φ. Then, by hypothesis and Definition 30, we know that $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is *strongly-admissible* since $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is strongly-conflict-free and strongly defends all its elements. Furthermore, by Definition 20, we can say that each s-coalition in $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is an s-coalition satisfying that is a maximal strongly-conflict-free set with the sub-graph G'_Φ induced by each s-coalition is connected only by support relations. Also, by hypothesis, $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is not c-strongly-admissible for Φ. Then, by Definition 16, if $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is not c-strongly-admissible for Φ, then $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ then it is not strongly-conflict-free or it is not closed under $\mathbb{R}_s$ or it is not strongly defending all its elements. However, each s-coalition satisfies closure under support. Also, by Proposition 15(i) and by hypothesis, if $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is maximally strongly-conflict-free then $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is a maximally strongly-conflict-free set.
On the other hand, if $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ does not strongly defend all its elements, there exist enriched arguments $\mathsf{A} \in \mathcal{C}_1$ and $\mathsf{B} \in \mathcal{C}_2$ such that there exists an attack from B to A and there is not exist an enriched argument C that belong to any s-coalition of $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ such that C strongly-attacks B, leading us to a contradiction since $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ strongly defends all its elements.

$\Leftarrow$) If $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is c-strongly-admissible for Φ, then $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is strongly-admissible for $\Phi^{\mathcal{C}}$.
Suppose that $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is c-strongly-admissible for Φ, and $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is not strongly-admissible for $\Phi^{\mathcal{C}}$. By hypothesis and Definition 16, we know that $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is *c-strongly-admissible* since $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is strongly-conflict-free and strongly defends all its elements. Also, by hypothesis and Definition 30, $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is not strongly-admissible for $\Phi^{\mathcal{C}}$, then or $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is not strongly-conflict-free or $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ does not strongly defend all its elements. However, by Proposition 15-i), if $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is strongly-conflict-free for Φ, then $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is strongly-conflict free for $\Phi^{\mathcal{C}}$, leading us to a Contradiction.
On the other hand, if $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ not strongly defends all its elements, there exist a coalition $\mathcal{C}_i \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ and $\mathcal{C}_j \in \mathcal{C}_\Phi$ such that there exists a weak or strong attack from $\mathcal{C}_j$ to $\mathcal{C}_i$ and does not exit an attack from $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ to $\mathcal{C}_j$. That is, there exist enriched arguments $\mathsf{A} \in \mathcal{C}_i$ and $\mathsf{B} \in \mathcal{C}_j$ such that there exists an attack from B to A and it does not exist an enriched argument C that belong to any s-coalition of $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ such that C strongly-attacks B, leading to a contradiction since $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ strongly defends all its elements. □

Proposition 18. *Let $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$ be the meta-argumentation framework associated with Φ, $\mathsf{Cont}_{\mathcal{C}}^{\Phi}$ a controversy degree function defined over $\Phi^{\mathcal{C}}$, $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ be a finite set of coalitions, and $\tau \in [0, 1]$ be a threshold.Then:*

(i) *$\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is strongly-admissible for $\Phi^{\mathcal{C}}$ iff $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is s-strongly-admissible for Φ.*
(ii) *If $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is s-τ-admissible for Φ then $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is at least τ-admissible for $\Phi^{\mathcal{C}}$.*
(iii) *$\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is weakly-admissible for $\Phi^{\mathcal{C}}$ iff $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is s-weakly-admissible for Φ.*

Proof. We break the proof in three parts:

(i) $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is strongly-admissible for $\Phi^{\mathcal{C}}$ iff $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is s-strongly-admissible for Φ.
We separate this part in two:

$\Rightarrow$) If $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is strongly-admissible for $\Phi^{\mathcal{C}}$ then $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is s-strongly-admissible for Φ. Suppose that $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is not s-strongly-admissible for Φ. Thus, by Definition 16, $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ either it is not strongly-safe or does not strongly defend all its elements. However, by hypothesis and Definition 30, we know that $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is strongly-conflict-free and strongly defends all its elements. Furthermore, by Proposition 6, if $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is strongly-admissible for $\Phi^{\mathcal{C}}$ then $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is c-strongly-admissible for Φ. Thus, $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is closed under support since each $\mathcal{C}_i \in \mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$, $1 \leqslant i \leqslant n$, induces a connected graph by the support relation by Definition 19, and strongly defends all its elements. Thus, there is no $\mathrm{A} \in \mathbb{A}\mathrm{rgs}$ and nor $\mathrm{B}, \mathrm{C} \in \mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ such that there exists a strong or weak attack from B to A, and either there is a sequence of support from C to A, or $\mathrm{A} \in \mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$. Contradiction.

$\Leftarrow$) If $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is s-strongly-admissible for Φ, then $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is strongly-admissible for $\Phi^{\mathcal{C}}$. Suppose that and $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is not strongly-admissible for $\Phi^{\mathcal{C}}$. Thus, by Definition 30, $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is not strongly-conflict-free or does not strongly defend all its elements. Thus, by Definition 28 and Definition 29, there are two coalitions $\mathcal{C}_i, \mathcal{C}_j \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$, with $i \neq j$, such that there exists a strong or weak attack from $\mathcal{C}_i$ to $\mathcal{C}_j$ or for all $\mathcal{C}_i \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ such that if $\mathcal{C}_j$ is a strong or weak attacker of $\mathcal{C}_i$ then there exists $\mathcal{C}_p \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ where $\mathcal{C}_p$ is a weak attacker of $\mathcal{C}_j$ or there does not exist a defender coalition. That is, by Definition 6, there exist enriched arguments $\mathrm{A} \in \mathcal{C}_i$ and $\mathrm{B} \in \mathcal{C}_j$ such that $(\mathrm{A}, \mathrm{B}) \in \mathbb{R}_a$, or for all enriched argument $\mathrm{A} \in \mathcal{C}_i$ with $\mathcal{C}_i \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$, such that if exist an enriched argument $\mathrm{B} \in \mathcal{C}_j$ that is a strong or weak attacker of $\mathrm{A} \in \mathcal{C}_i$, then there exists an enriched argument $\mathrm{C} \in \mathcal{C}_k$ with $\mathcal{C}_k \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ where $\mathrm{C} \in \mathcal{C}_k$ is a weak attacker of $\mathrm{B} \in \mathcal{C}_j$ or there is no defender for the enriched argument A. Which leads us to a contradiction, since by hypothesis, $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is s-strongly-admissible for Φ. Thus, $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is strongly-safe and strongly defends all its elements. Furthermore, by Proposition 15(ii) if $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is strongly-safe for Φ, then $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is strongly-conflict-free for $\Phi^{\mathcal{C}}$.

(ii) If $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is s-τ-admissible for Φ then $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is at least τ-admissible for $\Phi^{\mathcal{C}}$.
Suppose that $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is not τ-admissible for $\Phi^{\mathcal{C}}$. Then, by Definition 30, $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is not τ-conflict-free or does not exist a strong or weak defense for all its elements. That is, $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is not τ-conflict-free iff there is two coalitions $\mathcal{C}_i, \mathcal{C}_j \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ with $i \neq j$ such that there exists a strong attack from $\mathcal{C}_i$ to $\mathcal{C}_j$ or $\mathsf{Cont}^{\Phi}_{\mathcal{C}}(\{\mathcal{C}_1, \dots, \mathcal{C}_n\}) > \tau$, or there exists a coalition $\mathcal{C}_j \notin \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ such that there exists an attack from $\mathcal{C}_j$ to a coalition $\mathcal{C}_i \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ such that there does not exist a coalition $\mathcal{C}_k \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ verifying that $\mathcal{C}_k$ attacks $\mathcal{C}_j$. Thus, there exists an enriched argument $\mathrm{A} \in \mathcal{C}_i$ and $\mathrm{B} \in \mathcal{C}_j$ such that $(\mathrm{A}, \mathrm{B}) \in \mathbb{R}_a$ with $\mathcal{C}_i, \mathcal{C}_j \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ with $i \neq j$, or for all enriched argument $\mathrm{A} \in \mathcal{C}_i$ which $\mathcal{C}_i \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ such that if there exists an enriched argument $\mathrm{B} \in \mathcal{C}_j$ that is a strong or weak attacker of $\mathrm{A} \in \mathcal{C}_i$ then there not exist an enriched argument $\mathrm{C} \in \mathcal{C}_k$ that is a weak or strong attacker of B, with $\mathcal{C}_k \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$. However, by hypothesis, $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is s-τ-admissible for Φ. Then, $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is τ-safe and there exists a strong or weak defense for all its elements. Furthermore, by Proposition 15(iv), If $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is τ-safe for Φ, then $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is τ-conflict-free for $\Phi^{\mathcal{C}}$. Contradiction.

(iii) $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is weakly-admissible for $\Phi^{\mathcal{C}}$ iff $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is at least s-weakly-admissible for Φ.
We separate this part in two:

$\Rightarrow$) If $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is weakly-admissible for $\Phi^{\mathcal{C}}$ then $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is s-weakly-admissible for Φ. Suppose that $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is not s-weakly-admissible for Φ. Then, by Definition 16, $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is not weakly-safe and there not exists a strong or weak defense for all its elements, or $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is not strongly-safe and not exist a weak defend for all its elements. However, since $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is weakly-admissible for $\Phi^{\mathcal{C}}$, then $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is weakly-conflict-free and there exists a strong or weak defense for all its elements, or $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is strongly-conflict-free and weak defends all its elements.

On the one hand, by Proposition 15(i) and Proposition 15(v), if $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is weakly-conflict-free or strongly-conflict-free, then $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is weakly-conflict-free or strongly-conflict-free. Contradiction.

On the other, if there does not exist a strong or weak defense for all elements in $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$, then there exist enriched arguments $\mathrm{A} \in \mathcal{C}_1$ and $\mathrm{B} \in \mathcal{C}_2$ such that $(\mathrm{A}, \mathrm{B}) \in \mathbb{R}_a$. But, $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is weakly-admissible for $\Phi^{\mathcal{C}}$. Thus, by Definition 30, there exists a strong or weak defense for all elements belonging to $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$. That is, for all enriched arguments $\mathrm{A} \in \mathcal{C}_i$ with $\mathcal{C}_i \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ such that if there exist an enriched argument $\mathrm{B} \in \mathcal{C}_j$ that is a strong or weak attacker of $\mathrm{A} \in \mathcal{C}_i$ then there exists an enriched argument $\mathrm{C} \in \mathcal{C}_k$ with $\mathcal{C}_k \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ where $\mathrm{C} \in \mathcal{C}_k$ is a weak or strong attacker of $\mathrm{B} \in \mathcal{C}_j$. Contradiction.

$\Leftarrow$) If $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is s-weakly-admissible for Φ then $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is weakly-admissible for $\Phi^{\mathcal{C}}$. Suppose that $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is not weakly-admissible for $\Phi^{\mathcal{C}}$. Then, by Definition 30, $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is weakly-conflict-free and there exists a strong or weak defense for all its elements, or $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is strongly-conflict-free and weakly defends all its elements.

On the one hand, by Proposition 15(i) and Proposition 15(v), if $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is weakly-conflict-free or strongly-conflict-free, then $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is weakly-conflict-free or strongly-conflict-free. Contradiction.

On the other hand, if there is no strong or weak defense for all elements in $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$, then for all $\mathcal{C}_i \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ such that if $\mathcal{C}_j$ is a strong or weak attacker of $\mathcal{C}_i$ there exists $\mathcal{C}_k \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ where $\mathcal{C}_k$ is a weak attacker of $\mathcal{C}_j$ or there is no defender coalition. That is, by Definition 6, there exists enriched arguments $\mathrm{A} \in \mathcal{C}_1$ and $\mathrm{B} \in \mathcal{C}_2$ such that $(\mathrm{A}, \mathrm{B}) \in \mathbb{R}_a$, or for all enriched argument $\mathrm{A} \in \mathcal{C}_i$ with $\mathcal{C}_i \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ such that if there exists an enriched argument $\mathrm{B} \in \mathcal{C}_j$ that is a strong or weak attacker of $\mathrm{A} \in \mathcal{C}_i$ then there exists an enriched argument $\mathrm{C} \in \mathcal{C}_k$ with $\mathcal{C}_k \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ where $\mathrm{C} \in \mathcal{C}_k$ is a weak attacker of $\mathrm{B} \in \mathcal{C}_j$ or there is no defender for the enriched argument A, leading us to a contradiction, since by hypothesis, $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is s-weakly-admissible for Φ. Thus, $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is weakly-safe and there exists a strong or weak defense for all its elements. $\square$

Proposition 19. *Let* $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$ *be the meta-argumentation framework associated with* Φ, $\mathsf{Cont}_{\mathcal{C}}^{\Phi}$ *a controversy degree function defined over* $\Phi^{\mathcal{C}}$, $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ *be a finite set of coalitions, and* $\tau \in [0, 1]$ *be a threshold.Then:*

- $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ *is strongly-preferred for* $\Phi^{\mathcal{C}}$ *iff* $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ *is s-strongly-preferred for* Φ.
- *If* $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ *is s-*τ*-preferred for* Φ *then* $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ *is at least* τ*-preferred for* $\Phi^{\mathcal{C}}$.
- $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ *is weakly-preferred for* $\Phi^{\mathcal{C}}$ *iff* $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ *is at least s-weakly-preferred for* Φ.

Proof. We break the proof in three parts:

(i) $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is strongly-preferred for $\Phi^{\mathcal{C}}$ iff $\mathcal{C}_1 \cup \dots \cup \mathcal{C}_n$ is s-strongly-preferred for Φ.
We separate the proof in two parts:

$\Rightarrow$) If $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is strongly-preferred for $\Phi^{\mathcal{C}}$ then $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is s-strongly-preferred for Φ.
Suppose that $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is not s-strongly-preferred for Φ. Then, by Definition 17, $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is not $\subseteq$-maximal among the s-strongly-admissible subsets of $\mathbb{A}$rgs. However, by hypothesis, $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is strongly-preferred for $\Phi^{\mathcal{C}}$. Then, by Definition 31, $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is $\subseteq$-maximal among the strongly-admissible sets of coalitions. Furthermore, by Proposition 6(i), $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is strongly-admissible for $\Phi^{\mathcal{C}}$ iff $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is s-strongly-admissible for Φ. Which leads us to a contradiction.

$\Leftarrow$) If $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is s-strongly-preferred for Φ then $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is strongly-preferred for $\Phi^{\mathcal{C}}$.
Suppose that $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is not strongly-preferred for $\Phi^{\mathcal{C}}$. Then, by Definition 31, $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is not $\subseteq$-maximal among the strongly-admissible sets of coalitions. However, by hypothesis, $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is s-strongly-preferred for Φ. Then, by Definition 17, $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is s-strongly-admissible for Φ. Furthermore, by Proposition 6(i), $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is strongly-admissible for $\Phi^{\mathcal{C}}$ iff $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is s-strongly-admissible for Φ. Which leads us to a contradiction

(ii) If $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is s-τ-preferred for Φ then $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is at least τ-preferred for $\Phi^{\mathcal{C}}$.
Suppose that $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is not τ-preferred for $\Phi^{\mathcal{C}}$. Then, by Definition 31, $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is not $\subseteq$-maximal among the τ-admissible sets of coalitions. However, by hypothesis, $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is s-τ-preferred for Φ. Then, by Definition 17, $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is s-τ-admissible for Φ. Furthermore, by Proposition 6(ii), if $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is s-τ-admissible for Φ then $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is τ-admissible for $\Phi^{\mathcal{C}}$. Which leads us to a contradiction.

(iii) $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is weakly-preferred for $\Phi^{\mathcal{C}}$ iff $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is at least s-weakly-preferred for Φ.
We separate the proof in two parts:

$\Rightarrow$) If $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is weakly-preferred for $\Phi^{\mathcal{C}}$ then $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is at least s-weakly-preferred for Φ.
Suppose that $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is not s-weakly-preferred for Φ. Then, by Definition 17, $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is not $\subseteq$-maximal among the s-weakly-admissible subsets of $\mathbb{A}$rgs. However, by hypothesis, $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is weakly-preferred for $\Phi^{\mathcal{C}}$. Then, by Definition 31, $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is $\subseteq$-maximal among the weakly-admissible set of coalitions. Furthermore, by Proposition 6(iii), $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is strongly-admissible for $\Phi^{\mathcal{C}}$ iff $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is s-weakly-admissible for Φ. Which leads us to a contradiction.

$\Leftarrow$) If $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is at least s-weakly-preferred for Φ then $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is weakly-preferred for $\Phi^{\mathcal{C}}$.
Suppose that $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is not a weakly-preferred for $\Phi^{\mathcal{C}}$. Then, by Definition 31, $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is not $\subseteq$-maximal among the weakly-admissible sets of coalitions. However, by hypothesis, $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is s-weakly-preferred for Φ. Then, by Definition 17, $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is s-weakly-admissible for Φ. Furthermore, by Proposition 6(iii), $\{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$ is weakly-admissible for $\Phi^{\mathcal{C}}$ iff $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_n$ is s-weakly-admissible for Φ. Which leads us to a contradiction. $\square$

Proposition 20. *Given a meta-argumentation framework $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}_{\mathcal{C}}^{\Phi} \rangle$, where $\mathcal{C}_\Phi$ is the set of s-coalitions, $\mathbb{R}_a^{\mathcal{C}_\Phi}$ is an attack relation between s-coalitions, and $\mathsf{Str}_{\mathcal{C}}^{\Phi}$ the strength of attack function. Let $\mathcal{C}_1, \mathcal{C}_2 \in \mathcal{C}_\Phi$ be two s-coalitions obtained from Φ. If $\mathcal{C}_1 \cup \mathcal{C}_2$ is a τ-conflict-free set (weak conflict-free set) in* S-BAF, *then the coalitions $\mathcal{C}_1$, $\mathcal{C}_2$ that might be assimilated.*

Proof. We separate the proof in two parts:

- If $\mathcal{C}_1 \cup \mathcal{C}_2$ is a τ-conflict-free set in S-BAF, then the coalitions $\mathcal{C}_1, \mathcal{C}_2 \in \mathcal{C}_\Phi$ can be assimilated.
By hypothesis, $\mathcal{C}_1 \cup \mathcal{C}_2$ is a τ-conflict-free set in S-BAF. Then, by Proposition 15-iii), we know that $\{\mathcal{C}_1, \mathcal{C}_2\}$ is a τ-conflict-free set in $\Phi^{\mathcal{C}}$. Thus, by Definition 28 there are no $\mathcal{C}_1, \mathcal{C}_2 \in \mathcal{S}$ such that there exists a strong attack from $\mathcal{C}_1$ to $\mathcal{C}_2$, and $\mathsf{Cont}^{\Phi}_{\mathcal{C}}(\mathcal{S}) \leqslant \tau$. Finally, by Definition 32, we know that $\mathcal{C}_2$ is assimilated into $\mathcal{C}_1$ iff there exist a weak attack from $\mathcal{C}_1$ to $\mathcal{C}_2$ and $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_1) \geqslant \mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_2)$.
- If $\mathcal{C}_1 \cup \mathcal{C}_2$ is a weak-conflict-free set in S-BAF, then the coalitions $\mathcal{C}_1, \mathcal{C}_2 \in \mathcal{C}_\Phi$ can be assimilated.
By hypothesis, $\mathcal{C}_1 \cup \mathcal{C}_2$ is a weak-conflict-free set in S-BAF. Then, by Proposition 15-vi), we know that $\{\mathcal{C}_1, \mathcal{C}_2\}$ is a weak-conflict-free set in $\Phi^{\mathcal{C}}$. Thus, by Definition 28 there are no $\mathcal{C}_1, \mathcal{C}_2 \in \mathcal{S}$ such that there exists a strong attack from $\mathcal{C}_1$ to $\mathcal{C}_2$. Finally, by Definition 32, we know that $\mathcal{C}_2$ is assimilated into $\mathcal{C}_1$ iff there exist a weak attack from $\mathcal{C}_1$ to $\mathcal{C}_2$ and $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_1) \geqslant \mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_2)$. □

Proposition 21. *Given a meta-argumentation framework $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}^{\Phi}_{\mathcal{C}} \rangle$, where $\mathcal{C}_\Phi$ is the set of s-coalitions, $\mathbb{R}_a^{\mathcal{C}_\Phi}$ is an attack relation between s-coalitions, and $\mathsf{Str}^{\Phi}_{\mathcal{C}}$ the strength of attack function. Let $\mathcal{C}_1, \mathcal{C}_2 \in \mathcal{C}_\Phi$ be two s-coalitions obtained from Φ. If $\mathcal{C}_2$ can be assimilated into $\mathcal{C}_1$ yielding $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_{1,2})$, then $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_1) \geqslant \mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_{1,2})$, i.e., the resulting coalition $\mathcal{C}_{1,2}$ cannot increase its cohesiveness.*

Proof. Suppose that $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_{1,2}) > \mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_1)$. Then, By Definition 12, we know that there exist two enriched arguments A and B belonging to the assimilated coalition $\mathcal{C}_{1,2}$ such that $(\mathrm{A}, \mathrm{B}) \in \mathbb{R}_s^{\mathcal{C}_{1,2}}$ and $\mathsf{Sim}_{\mathbb{C}}(\mathrm{A}, \mathrm{B}) > \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}', \mathrm{B}')$ where A′ and B′ belonging to the coalition $\mathcal{C}_1$ with $(\mathrm{A}', \mathrm{B}') \in \mathbb{R}_s^{\mathcal{C}_1}$. Contradiction, since, by Definition 32, we know that $\mathcal{C}_1$ assimilate $\mathcal{C}_2$ if and only if $\mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_1) \geqslant \mathsf{Coh}_{\mathbb{C}}(\mathcal{C}_2)$ and there exists a weak attack from $\mathcal{C}_1$ to $\mathcal{C}_2$. Thus, there not exists a pair of arguments A″ and B″ belonging to the coalition $\mathcal{C}_2$ with $(\mathrm{A}'', \mathrm{B}'') \in \mathbb{R}_s^{\mathcal{C}_2}$ and a pairs of arguments A′ and B′ belonging to the coalition $\mathcal{C}_1$ with $(\mathrm{A}', \mathrm{B}') \in \mathbb{R}_s^{\mathcal{C}_1}$ verifying that $\mathsf{Sim}_{\mathbb{C}}(\mathrm{A}'', \mathrm{B}'') > \mathsf{Sim}_{\mathbb{C}}(\mathrm{A}', \mathrm{B}')$. □

Proposition 22. *Given a meta-argumentation framework $\Phi^{\mathcal{C}} = \langle \mathcal{C}_\Phi, \mathbb{R}_a^{\mathcal{C}_\Phi}, \mathsf{Str}^{\Phi}_{\mathcal{C}} \rangle$, where $\mathcal{C}_\Phi$ is the set of s-coalitions, $\mathbb{R}_a^{\mathcal{C}_\Phi}$ is an attack relation between s-coalitions, and $\mathsf{Str}^{\Phi}_{\mathcal{C}}$ the strength of attack function. If $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is a τ-preferred extension (weakly-preferred extension), then there exists at least two coalitions $\mathcal{C}_1, \mathcal{C}_2 \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ that might be assimilated.*

Proof. We separate the proof in two parts:

- Suppose that, $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is a τ-preferred extension. Then, by Definition 31, we know that $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is $\subseteq$-maximal among the τ-admissible sets of coalitions. That is, by Definition 30, $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is τ-conflict-free and there exists a strong or weak defense for all its elements. Thus, by Definition 28, there are $\mathcal{C}_i, \mathcal{C}_j \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ such that there exists a weak attack from $\mathcal{C}_i$ to $\mathcal{C}_j$, and $\mathsf{Cont}^{\Phi}_{\mathcal{C}}(\mathcal{C}_1, \dots, \mathcal{C}_n) \leqslant \tau$. Finally, by Definition 32, $\mathcal{C}_i, \mathcal{C}_j \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ can be assimilated.
- Suppose that, $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is a weakly-preferred extension. Then, by Definition 31, we know that $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is $\subseteq$-maximal among the weakly-admissible sets of coalitions. That is, by Definition 30, $\{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ is weakly-conflict-free and there exists a strong or weak defense for all its elements. Thus, by Definition 28, there are $\mathcal{C}_i, \mathcal{C}_j \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ such that there exists a weak attack from $\mathcal{C}_i$ to $\mathcal{C}_j$. Finally, by Definition 32, $\mathcal{C}_i, \mathcal{C}_j \in \{\mathcal{C}_1, \dots, \mathcal{C}_n\}$ can be assimilated. □

Appendix B. Coalitions algorithm

Algorithm 1, presents a procedure to obtain the s-coalitions given an s-valued bipolar argumentation graph. The algorithm is straightforward, and its computational complexity highly depends on how the

Algorithm 1: Obtain s-coalitions

Input: *s-valued argumentation graph* G_Φ, *nodeX*, *nextX* *Threshold* $\tau \in [0, 1]$, *func*_coh.
Output: array: *coalition*.

Initialize *coalition* as an empty array;
Initialize each *nodeX* of G_Φ with a *not visited status*;

Mark the *nodeX* as visited;
for *each nodeX in* G_Φ *that is not visited* **do**
  if *Relation between nodeX and nextX is a support*
   then Add *nodeX* and *nextX* to *coalition*; *nodeX* = *nextX*; obtain *nextX*;

Calculate *func*_coh over *coalition*;
if *func*_coh = 1 **then**
  Display "This is a strong-coalition:" array *coalition*
else
  if ($\tau \leqslant$ *func*_coh < 1) **then**
    Display "This is a τ-*coalition*:" array *coalition*
  else
    if ($0 <$ *func*_coh $< \tau$)
     then Display "This is a *weak-coalition*:" array *coalition*;

nodes are visited; it can vary widely according to how the searching algorithm is implemented [36,47]. According to Niewola and Podsedkowski [36], the L^* algorithm, which improves the common heuristics used in the search algorithm, can lower the cost of search to $O(n)$.

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