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Darwinian adverse selection[1]

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Abstract. We develop a model to study the role of individual rationality in economics and biology. The model's agents differ continuously in their ability to make rational choices. The agents' objective is to ensure their individual survival over time or, equivalently, to maximize profits. In equilibrium, however, individually rational agents who maximize their objective survival probability are, individually and collectively, eliminated by the forces of competition. Instead of individual rationality, there emerges a unique distribution of irrational players who are individually not fit for the struggle of survival. The selection of irrational players over rational ones relies on the fact that all rational players coordinate on the same optimal action, which leaves them collectively undiversified and thus vulnerable to aggregate risks.

Keywords: Maximization, individual rationality, economics, biology

JEL: D81, D01, D03, G11

1. Introduction

In economics it is commonplace that "man's ability to operate as a logical animal capable of systematic empirical induction was itself the direct outcome of the Darwinian struggle for survival"[2]. Most economic models therefore assume that the forces of competition ensure the elimination of agents who are not capable of profit-maximizing behavior. One advantage of such rational maximizing behavior is that it is complementary to mathematical optimization techniques which accommodate systematic model building. Moreover, it follows from the laws of diminishing marginal returns that most maximization problems feature unique solutions. Accordingly, rational choice yields unique predictions which serve as a natural reference point.

For the present purpose, we summarize the case for rational choice models as follows: (i) *The competitive struggle for survival selects rational agents over those who make mistakes.* Moreover, (ii) *unlike rational maximizing behavior, which is well defined in the context of mathematical models, irrational "choice" is inherently hard to define and thus cannot serve as a benchmark.*

In this paper, we develop a simple dynamic neoclassical model of "Darwinian adverse selection". While the model's assumptions are neoclassical, its theoretical predictions invert our previous theses (i) and (ii). The model economy is inhabited by agents who differ continuously in their ability to make rational choices and the agents' objective is to ensure their individual survival over time or, equivalently, to maximize profits. And, indeed, the rational choice by which individuals maximize their individual survival probability will be uniquely defined. The fact that maximizing behavior is well defined, however, has the implication that rational players are eliminated by the forces of competition, while irrational player types, not capable of maximizing, survive.We derive this result for two standard economic model environments.

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[2] Samuelson (1972, p. 249). See Friedman (1953, p.22), and Schumpeter (1933) for similar arguments. See Nelson and Winter (1985) and Winter (1971) for more differentiated references.

The key intuition is very simple: Suppose agents have to choose between two ferry boats. One is well maintained, the other is leaking. All rational agents will therefore board the well-maintained ferry, while irrational agents, not capable of maximizing their individual survival probability, may be found on either boat. Consequently, to ensure that some irrational players survive, it suffices that one of the two boats does not sink. Rational players, on the contrary, individually and collectively depend on only one boat. And thus the probability of some rational players surviving is strictly lower than the probability of some irrational players surviving. Put differently, individual rationality has the consequence that it aligns the choices of rational players. Accordingly, rational players as a group suffer from under-diversification.

Compared to our initial theses (i) and (ii), we therefore find that our model yields rather different results. Rationality with its unique predictions is the very reason for the extinction of rational players. Irrational agents, on the contrary, survive not despite, but because of their individual mistakes. Accordingly, in relation to our first thesis (i), we derive a simple antithesis (i'): *The struggle for survival selects irrational agents, who make mistakes over those who make no mistakes in the struggle for survival.* Moreover, irrationality serves as the reference point: there exists only one unique distribution of irrational behavior, which is well defined in the context of analytical model building.

In Section 2, we introduce the model and present the main result. In our baseline model, players are faced with a simple binary choice problem. In Section 3, we drive our results for a capital market setting where agents can choose from an arbitrary number of alternatives. Finally, we discuss assumptions, compare our results to the literature, and suggest different interpretations. Section 4 concludes.

2. Model

At the beginning of time, entrepreneurs choose between two actions/"production technologies" A and B. After this choice is made, nature randomly selects between two states 0 and 1. State 0 is selected with probability $P > 1/2$. State 1 is less likely and occurs with the residual probability $(1 - P) < 1/2$. If state 0 (1) is selected, then technology A (B) is more productive than technology B (A). Accordingly, if state 0 is selected, the competitive market system eliminates those firms that chose the inefficient technology B.[3] Likewise in state 1, where players with technology B drive those out of the market who chose A. Agents who used the wrong technology are eliminated. Successful entrepreneurs move on to the next period where they have to choose once again between two technologies A and B. Time t goes from $t = 0, 1, 2..., \infty$.

Entrepreneurs differ in types $\phi \in [0, 1]$. These types vary continuously from rational to completely irrational. More precisely, an agent i who is of type ϕ plays the following strategies:

$$S_i = \begin{cases} A & \text{with probability } \phi \\ B & \text{with probability } 1 - \phi \, . \end{cases}$$

In the following, we identify players' rationality by their innate probability, $\phi \in [0, 1]$, to play strategy A, which maximizes the individual survival probability.[4] Individuals who maximize their survival probability therefore play A all the time and hence $\phi = 1$ agents are rational. As ϕ decreases, agents put less and less emphasis on the dominant action and eventually agents with $\phi = 0$ reliably play B, which minimizes their survival probability. At the beginning of time, there exists a density one of each player type ϕ. Moreover, players of type ϕ randomize independently over the two actions A and B such that we can use the weak law of large numbers to calculate the share of type ϕ agents who play A and B as ϕ and $1 - \phi$, respectively.

Proposition 1. *Over time, rational player types $\phi = 1$ who maximize their individual survival probability, are eliminated with probability one. All agents $\phi = 0$ who minimize their probability of survival are also eliminated. The probability of some agents of all other types $\phi \in (0, 1)$, who play strategies that are neither consistent with maximization nor minimization, surviving to any point in time is equal to one.*

Proof. Rational agents ($\phi = 1$), who maximize their individual survival probability, choose strategy A,

[3] An economy consistent with such an assumption would, for example, be one where (i) goods can be sold at a price $p \geq 1$, (ii) each entrepreneur has one unit of labor L and needs one "unit" of income for subsistence/survival, and (iii) production is $Y = \tau_i L, i = A, B$ with $\tau_A = 1, \tau_B = 0$, in state 0 and $\tau_A = 0, \tau_B = 1$ in state 1. This environment can be generalized incorporating elastic demand, varying returns to scale, and production involving several inputs.

[4] That is, choosing A maximizes both the probability of surviving to the next period since the probability $P > 1/2$ with which A is the superior technology exceeds the probability $1 - P < 1/2$ with which B is the superior technology.

with its superior survival probability $P > 1/2$, all the time. Hence, P^t is the probability of a rational agent surviving to period t. Indeed rationality induces all rational agents to play A such that the *collective survival probability* for the entire group of rational agents coincides with the individual survival probability P. The long-run survival probability of rational agents, individually and collectively, is therefore $\lim_{t\to\infty} P^t = 0$. Second, the individual and collective survival probability for minimizing agents ($\phi = 0$) is $(1 - P)$. Hence, over time, we have $\lim_{t\to\infty}(1 - P)^t = 0$. Finally, irrational agents $\phi \in (0, 1)$, who do not maximize: The individual survival probability of a type ϕ agent is given by the convex combination $P\phi + (1 - P)(1 - \phi) \in (1 - P, P)$. That is, as individuals, irrational agents survive with a probability that falls short of the rational agent's probability P. However, a share ϕ of irrational type ϕ players survives in state 0, and a share $1 - \phi$ in state 1. The probability of some type ϕ players surviving from one period to the next is therefore $P + (1 - P) = 1$. □

Proposition 1 reflects that the extinction of rational agents is an immediate consequence of the very fact that rational agents maximize their individual survival probabilities. In doing so, they coordinate on the one rational answer that exists to the struggle for survival that nature presents them with. However, since individual rationality is well-defined in the present model, rational players all take the same bet and are thus eliminated collectively in the rare event where technology B dominates A. Put differently, nature rewards a diversification, which is individually irrational.

3. Discussion

The previous result suggests that the failure of rationality originates from a lack of diversification. In this section, we reflect on this finding. At first sight, the binary choice model from the previous section raises the question whether rational agents might find some way to diversify once they can choose from more than just two actions. To address this concern, we show that our findings also obtain in a standard capital market environment where rational agents can choose from an arbitrary number of different assets to build diversified portfolios. Again, we find that rational choice is not an outcome once we add an evolutionary component to our otherwise standard model. Second, we emphasize the fact that the present model implies the emergence of equilibrium biases. Third, we relate our conclusions to the literature on evolutionary biology.

3.1. Diversification

Proposition 1 clearly shows that the survival of irrational players relies on the fact that rational players indeed perform to the best of their ability. Accordingly, for every given task, they come to the same optimal conclusion. And all rational players will choose the *same* action A. Everyone choosing A, however, implies that rational players' choices are perfectly correlated. This aligns their actions perfectly, which in the presence of aggregate risks, means that they perish simultaneously. Hence, rational players, *as a collective*, do not diversify.

To emphasize this point, we develop a simple dynamic version of the workhorse capital asset pricing model (CAPM)[5] in Appendix A. In this model investors choose repeatedly an asset portfolio. If an investor's portfolio return Y_j falls below a minimum Y^{min} he is removed from the game. In this model, there are again rational and irrational investors. The rational investors behave according to the predictions of the standard CAPM. That is, they choose the same diversified "market portfolio" which maximizes their individual survival probability. That is, even though investors can choose from infinitely many different assets, they all choose the same diversified "market portfolio" which we call A. Moreover, since all rational agents choose the same optimal portfolio A, their *collective survival probability* is once again identical to their individual survival probability. Irrational players play B. That is, they invest all their wealth into one asset which they pick at random. Hence, their individual survival probability falls short of that of the rational players. However, as a collective, they are fully diversified: Regardless of how deep the overall market falls, a positive measure of irrational agents will survive. Put differently, it turns out that this model's predictions are in line with the simple binary choice model from Section 2. In Appendix A, we substantiate the foregoing claims and prove

Proposition 2. *All rational investors who maximize their survival probability choose the market portfolio A. And, over time, the probability of rational investors' funds being closed due to catastrophic losses goes to* 1. *The probability with which a posi-*

[5] See Sharpe (1991).

tive measure of irrational investors, who individually invest all of their wealth into one randomly drawn asset, survive to any point in time is one.

Put differently, those investors who behave according to the predictions of the capital asset pricing model will eventually go bankrupt in one unlikely market downturn. In this downturn, however, there are always some outlier assets, and those irrational investors who chose to invest their entire wealth into these outlier assets survive the downturn.

3.1.1. Preferences and additional strategies

From the previous analysis it is clear that the population of players declines over time. Moreover, there is no safe act that would allow agents to survive until the next period with probability one. If we were to introduce safe acts alongside rewards to risk-taking, which would come in the form of increased fertility in case the risky strategy pays off, we would obtain the same results as before if rational players maximize the number of expected offspring by playing one risky act, "A", all the time. If rational players were to maximize their survival probability instead, they would eventually be marginalized in a population where other player types individually randomize over the safe and the risky option where expected fertility is higher. Once again, the key to this finding would lie in the observation that rational players, regardless of what their aim is, would not have an incentive to randomize individually. Similar arguments would apply if there were two groups of rational players, one maximizing the expected number of children, the other maximizing the probability of survival. However, the current findings, which suggest that certain types of agents survive with probability 1, do hinge on the assumption that there are infinitely many players. Finite populations of players would always face a non-zero probability of going extinct.

3.2. Trembling hand and heterogenous priors

The foregoing model has shown that an endogenous population of players evolves over time. And the probability of rational players being included in this distribution goes to zero as time evolves, leaving only agents playing trembling hand. The present model thus provides a framework where the often criticized trembling hand assumption of Selten (1975) is a model outcome rather than an assumption.

In an alternative interpretation, the present model can explain the emergence of heterogenous priors. Suppose that all players die after one period and only those who made the right choice have one "child". In turn, the child of a type ϕ agent will, with probability ϕ (respectively $1-\phi$), hold the prior belief that the probability P with which action A dominates B is $P > 1/2$ ($P < 1/2$). By the law of large numbers, we would have a stable "sex ratio" and a share ϕ play A while a share $1-\phi$ play B. For each history of events, agents will therefore know that they live in a society that agrees to disagree. In this interpretation, the present model explains the empirical fact that agents "agree to disagree", which - as Aumann (1976) points out - is otherwise hard to justify.

3.3. Related literature

Our model adds an evolutionary component to a standard neoclassical model environment. The main conclusion suggests that rational choice, which is usually assumed in mainstream neoclassical economics, is not a model outcome. That is, the group of agents who are rational in the sense that they maximize their individual survival probability/profit, perish collectively with probability one. On the contrary, players who play "trembling hand" strategies, resulting in lower individual survival probabilities, prevail.

This finding is very similar to the results that obtain in the literature on "bet-hedging", which finds that phenotypic heterogeneity is necessary to ensure that a population is well prepared for environmental changes.[6] In this interpretation, our models from Sections 2, 3.1, and 3.2 show that populations of rational economic agents, who maximize their objective individual survival probability, fail to hedge their bets on aggregate. That is, in our capital market context in Section 3.1, rational agents hedge their individual bet and choose the diversified market portfolio which maximizes their individual survival probability. This however, implies that the entire species of rational investors perishes in the rare event when the market portfolio drops. On the contrary, irrational investors, who invest all their money in one randomly picked asset, do not hedge their individual bet. This has the consequence that the entire population of irrational traders is perfectly diversified and some traders survive even the worst market crash/drought. Similarly, Lewontin and Cohen (1969) show that a population's expected size can go to infinity while the probability of

[6] de Jong et al. (2011) and Simons (2011) for two recent surveys. Cohen (1966), MacArthur (1972) and Venable and Lawlor (1980) discuss the case of delayed germination in an intertemporal setting with stochastic rainfall.

its extinction goes to one.[7] A related observation holds in section 2's binary choice setting: The expected size of the rational players' population, which faces extinction, is higher for every future period t than that of the players who do not maximize.

4. Conclusion

Individual rationality with its unambiguous predictions makes model building operational. Moreover, since rational agents outperform irrational ones, it is commonly assumed that the forces of competition will eliminate irrational agents over time. In the current model, rational players indeed perform to the best of their ability. Accordingly, for every given task, they come to the same optimal conclusion. As discussed, if A is the well diversified "market portfolio" and B the choice to invest all funds into one randomly chosen asset, then all rational players choose the *same* diversified market portfolio A. Everyone holding A, however, implies that rational players' choices are *collectively* perfectly correlated. This aligns their actions perfectly, which means in the presence of aggregate risks, that they perish simultaneously. Hence, rational players, *as a collective*, do not diversify. Accordingly, over time, they will eventually be washed away by one unforseen regime change.

The opposite is true for irrational agents. The choice to invest the entire individual wealth into one randomly chosen asset minimizes the individual probability of success. At the same time, this behavior ensures that some irrational agents survive even the worst crises. Hence, even though irrational players may be outperformed by their rational counterparts for long periods of time, they survive those unlikely events where uncomprehensible choices pay off.

[7]Samuelson (1977) reviews the related literature on the St. Petersburg paradox, which explains why a gamble with an infinite expected payoff may trade at a very low price. Related, Ulanowicz (2008) argues that ecological systems become unstable once they have too much structure to them. Borrelli et al. (2015) discuss the metaproblem of system selection.

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Appendix A. Proof of Proposition 2

In this appendix, we first derive the main prediction of the CAPM, namely that all rational players will hold the same "market portfolio", which we call A. This portfolio minimizes the probability with which investors suffer a "catastrophic loss". As in the baseline model, investors suffering catastrophic losses are eliminated from the capital market. Second, we show that rational investors who diversify optimally have a "survival probability" that exceeds the survival probability of irrational investors who do not diversify. Finally, we show that a positive share of irrational investors survives even the worst market downturns with probability one. Rational investors on the contrary are eliminated in severe downturns.

Market: There exists a continuum of assets that yield y_i:

$$y_i = \theta + \xi_i, \quad i \in [0, 1], \quad \theta \sim \mathcal{N}(\mu, \sigma^2), \quad (1)$$

where ξ_i is i.i.d. white noise $\xi_i \sim \mathcal{N}(0, \sigma_\xi^2)$. Clearly θ represents the general "market risk", which is common to all assets, and ξ_i is the idiosyncratic risk associated with a particular asset i. In the following, each agent j can choose a portfolio to minimize the probability that returns Y_j fall short of a minimum requirement $Y^{min} < \mu$. Investors receiving $Y_j < Y^{min}$ go bankrupt. Respectively, if we think of the investor as a fund manager, the fund is closed due to poor performance if $Y_j < Y^{min}$. Investors receiving $Y_j > Y^{min}$ move on to the next period where the game is repeated... We normalize all assets' prices to one, and each investor can invest one unit of currency. To substantiate the claims from the main text, we have to show that there exists one unique optimal "market portfolio" A, which maximizes the objective survival probability $P(Y_A > Y^{min})$. To derive the optimal portfolio, we note that the investor j can buy the following portfolios:

$$Y_{j1} = y_1, \quad Y_{j1} \sim \mathcal{N}(\mu, \sigma^2 + \sigma_\xi^2) \quad (2)$$

$$Y_{j2} = \frac{1}{2}y_1 + \frac{1}{2}y_2, \quad Y_{j2} \sim \mathcal{N}(\mu, \sigma^2 + \frac{1}{2}\sigma_\xi^2) \quad (3)$$

$$Y_{j3} = \frac{1}{3}y_1 + \frac{1}{3}y_2 + \frac{1}{3}y_3, \; Y_{j3} \sim \mathcal{N}(\mu, \sigma^2 + \frac{1}{3}\sigma_\xi^2) \quad (4)$$

$$Y_{jN} = \sum_{n=1}^{N} \frac{1}{N} y_n, \quad Y_{jN} \sim \mathcal{N}(\mu, \sigma + \frac{1}{N}\sigma_\xi^2). \quad (5)$$

Rational Choice: It follows from (2)–(5) that $P(Y_{jN} > Y^{min}) = \Phi\left(\sqrt{\frac{1}{\sigma^2 + \frac{1}{N}\sigma_\xi^2}}(\mu - Y^{min})\right)$, where $\Phi()$ is the cumulative density function of the standard normal distribution. Thus, the survival probability of a fund manager is monotonously increasing in N since we assumed that $\mu > Y^{min}$. Accordingly, rational investors will choose $N = \infty$ to maximize their survival probability. That is, they include a small amount of every asset $y_i, i \in [0, 1]$ in their portfolio to achieve maximum diversification. This means that *all rational fund managers* choose the same "market portfolio" as predicted by the standard capital asset pricing model. Once we call this market portfolio portfolio A, we have substantiated the claim from the main text that all rational managers choose A, which puts their individual and collective survival probability to $P_A = \Phi\left(\sqrt{\frac{1}{\sigma^2}}(\mu - Y^{min})\right)$.

Irrational Agents: As before, there exists a measure one of irrational players who do not diversify. They simply invest their total wealth into one asset which they pick at random from the continuum of assets $j \in [0, 1]$. We call this choice B. Accordingly, the survival probability of an individual irrational manager is $P_B(Y_j > Y^{min}) = \Phi\left(\sqrt{\frac{1}{\sigma^2 + \sigma_\xi^2}}(\mu - Y^{min})\right) < P_A$. The *collective* survival probability of a mass one of irrational players, however, is again equal to one. That is, for every given draw θ, we have the distribution of individual asset returns $y_i|\theta \sim \mathcal{N}(\theta, \sigma_\xi^2)$, and there is always a positive mass $P(y > Y^{min}|\theta) = \Phi\left(\sqrt{\frac{1}{\sigma_\xi^2}}(\theta - Y^{min})\right) > 0$ of irrational players, who receive a return in excess of the required minimum.

Put differently, regardless of how deep the general "market", θ, falls, there are always some outlier assets, which deliver a return sufficient to ensure that $Y_i = y_i > Y^{min}$. Hence, there are always surviving irrational agents. The same is not true for rational agents, who hold the market portfolio A, which yields $Y_A = \int_{[0,1]} x_i di = \theta + \int_{[0,1]} \xi_i di = \theta$. That is, once $\theta < Y^{min}$, which happens with probability $1 - P_A > 0$, all rational agents perish simultaneously.