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Book Review

Amanda Jansen, *Rough Draft Math: Revising to Learn*, Stenhouse Publishers, Portsmouth, New Hampshire, 2020

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This book is not about fuzzy, but it is about fuzzy. This a pedagogical book, it does not explicitly mention fuzzy logic – but its ideas are exactly the ideas underlying fuzzy logic, that everything is a matter of degree. Its ideas can therefore naturally be translated into the fuzzy language.

Moreover, these ideas add one more case to the – already large – list of cases in which fuzzy thinking and fuzzy techniques help solve important practical problems.

Teaching math is especially difficult. All teaching is difficult – and all learning is difficult.

There is a reason why teaching and learning math – especially in elementary and middle schools – is especially difficult: math is *crisp*. When a student is asked what is $90 - 74$, there is nothing imprecise of fuzzy about it: either the student gives a correct answer 16, or a wrong answer.

Why is crispness a problem? For example, in linear algebra, the instructor can ask students, on the test, to solve a system of two linear equations with two unknowns. A student writes down all the steps that lead to the answer. If the answer is correct, great. But even if the final answer is not correct, the instructor can check the steps, and if it turns out that the steps are correct but the student made a minor arithmetic mistake, this student still gets a partial credit. If a student missed or misunderstood some steps, the instructor points out what was wrong, so next time, the student will do better.

In a mechanics problem, the student may have forgotten to include one of the forces – this will be clear from the answer.

In all these cases, the student may go through intermediate stages of knowledge, in which his/her knowledge is not yet fully correct – it goes from ignorance to partial correctness and only then to full correctness.

In other words, in many other disciplines, students go from ignorance to full knowledge via stages of partial knowledge – and their grades corresponding to partial credit are the degrees to which the students learn the material. When re-scaled from, e.g., the usual 0 to 100 scale to a 0 to 1 scale, they become *fuzzy degrees* with which we are well accustomed.

A student who has not yet fully mastered a topic knows where he/she stands, and these degrees allow the student to track his/her progress.

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With simple math problems, there are no intermediate steps, so all the instructor sees is the answer. This answer is either right or wrong. The student may get a better understanding on the next quiz, but if the answer is still wrong, the student does not get any idea whether he/she is moving in the right direction. And when a student spends a lot of effort and still the answer is wrong, this discourages the student from future efforts – and often even alienates the student from mathematics as a whole.

We need fuzziness. The above comparison shows that it is desirable to introduce fuzziness. How can we do it?

How to add fuzziness: naive idea. A seemingly natural idea is simply to give partial credit when the student's answer is close to the correct one – and the closer it is, the more partial credit to give.

This idea may relieve some of the student's frustration, but it does not help to understand what was wrong in the student's thinking – and thus, it may not help to correct the student's incorrect ideas and hencd, it may not help to teach the student the correct solution.

Alternative idea described in the book: main part. An alternative idea is to ask the students not just to give the answer but to provide ideas and explanations of how they came up with this answer. In situations when a student cannot come up with an answer – we can ask the student to explain what ideas he/she thinks can be used to come up with a solution. This narrative will serve as a rough draft of the final answer – that is why it is called rough draft math.

Such narratives can be analyzed and – if needed – graded for partial credit, partial credit based on correctness of ideas and not just correctness of the answer. The book does not give any specific guidance on how to assign partial credit, i.e., how to evaluate the corresponding fuzzy degrees. However, we should not worry too much about the absence of these instructions: assigning partial credit is what most instructors do reasonably well already, they just do not have a chance to do it in elementary math classes.

Alternative idea: auxiliary part. Usually, we grade the students only when we are reasonably confident that they have mastered the desired skill. This makes sense when the answers are crisp: what is the purpose of having a test when students still did not learn how to subtract; students will only be discouraged by not-good grades.

But if we can give a fuzzy estimate, why not make this evaluation before the students mastered the material – so the students will get a good idea where they stand?

But is it practical? It can be made practical. Teachers are already overwhelmed with teaching and grading, is it realistic to expect them to grade and correct even more? Here is a simple solution promoted by the book: let other students correct. Instead of having each student individually doing the hard job of learning and avoiding misconceptions, but not encourage students to do it together, to help each other – then all the teacher would need is to observe, and to help and guide if needed. And hopefully, this will boost the students' learning – so that on the test, most of them will show not only correct thinking, but also correct answers.

But what if not everyone gets correct reasoning and correct answers? This is what *revising* – the process mentioned in the book's subtitle – is for. This is similar to repeating the material, a tired-but-true way to teach, but this time it is not exactly a repetition. Indeed, from the actual answers, we know what are the misconceptions, we know what exactly needs to be emphasized. So, this "revising" should work (and works) better than simply going over the same material again and again – which is, by the way, one of the reasons why students often find math boring.

Read the book. The main objective of this review was to show how the book is related to fuzzy ideas. Of course, to really apply these ideas, it is necessary to read the book. The book fleshes out the above general ideas into specific pedagogical recommendations.