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## Book Review

**Variational Methods and Complementary Formulations In Dynamics**, by B. Tabarrok and F. P. J. Rimrott. Published by Kluwer Academic Publishers, Dordrecht/Boston/London, 1994, \$156.00, 380 pp.

The book reviewed is the 31st volume of the series *Solid Mechanics and Its Application*, edited by G. M. L. Gladwell. As the editor promises, fundamental questions of *Why?*, *How?*, and *How much?* across the spectrum of solid mechanics are addressed. In this reviewer's opinion, the authors did an excellent job of selecting a variety of these questions, putting them in a logical order, and presenting answers intended for senior undergraduate or for graduate students, as well as for engineers, physicists, and mathematicians active in dynamics.

The basic idea of the book is outlined in the preface "In analytical mechanics the fundamental laws are expressed in terms of work done and energy exchanged. The extensive use of mathematics is a consequence of the fact that in analytical mechanics problems can be expressed by variational statements, thus giving rise to the employment of variational methods. Further it can be shown that the independent variables may be either displacements or impulses, thus providing . . . the possibility of two complementary formulations . . . this duality is an important characteristic of mechanics problems and is given special emphasis in the presented book."

In Chapter 1, after a historical review of important contributions to the field of mechanics (including one of the authors), fundamental definitions and equations are named and explained, such as generalized displacement variables, degrees of freedom, holonomic and non-holonomic constraints, virtual displacements and virtual work, potential energy, stability, energy, and

complementary energy. There are over 20 example calculations in this chapter and an equal number of problems for the reader to solve. This underlines the textbook character and is helpful for further understanding of the material. In the first chapter, as well as in the following chapters, the examples and problems are intentionally chosen to give the reader an idea about typical situations, results, and values.

The second chapter introduces *differential* variational formulations; the third chapter presents *integral* variational formulations. These two chapters form the main part of the book. The reader is expected to be familiar with variational methods. If this is not the case, Appendix A, which introduces the calculus of variation, provides some background material.

Differential variational formulations are defined as formulations that make independent statements at each instant of time during a motion. The second chapter includes D'Alembert's Principle, Gauss' Principle of Least Constraint, and Lagrange's Equation and its complementary form. Integral variational formulations are defined as formulations of motion over a period of time, considered as a whole. In the third chapter, Hamilton's Principle and its complementary form is introduced, the use of the Lagrange's Multiplier Method to include holonomic and non-holonomic constraints is described, and the concept of ignorable coordinates is discussed. The duality of displacement formulations and its complementary formulation is emphasized in this chapter. The authors emphasize that the various

principles serve different purposes, for example, movie film that records motion at discrete points in time is used to distinguish between the Principle of Least Action and the Hamilton's Principle. With this example, the authors illustrate how these principles can be applied to determine motion, if for instance a piece of the film is missing. This comparison, although simplistic, is helpful to understand the principles completely.

Although the book is generally focused on principles and formulations in dynamics, it does address other related issues in mechanics. One example is the description of the similarity between Jacobi's Principle of Least Action and Fermat's Principle of Least Time in geometrical optics and the way the knowledge of this similarity eventually led to the wave equation for quantum mechanics. These digressions broaden the perspective within the fields of mechanics and the relationships between them.

The fourth chapter is on canonical transformations as a way to solve the equations of motion. The fifth chapter deals with rigid body dynamics. It is focused on rotary motions of rigid bodies (gyrodynamics). Good examples and figures clearly present the material. The sixth and last

chapter challenges the reader with special applications of the formulations made in the previous chapters. It deals basically with two areas of applications: small amplitude oscillations about a position of equilibrium or about a predetermined steady motion, and spike loads and spike speeds and the changes in the energies of a system when it is subjected to such a load. This is of special interest for anyone who wants to relate the material of the book to real world problems, as these two areas cover a range of issues commonly found in dynamics problems.

The last chapter concludes a comprehensive text book on variational methods in dynamics. However, one might look at Appendix B with a critical eye. It is about some celebrities in mechanics who seem to be rather randomly picked and also very well known. How about some biographies about contemporary scientists in the more recent research on complementary formulations?

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