Book Review

Introduction to Finite and Boundary Element Methods for Engineers, by G. Beer and J. O. Watson. Published by John Wiley & Sons, Inc., Chichester, UK, 1992. $73, 95, 509 pp.

The finite element method (FEM) has been widely used in stress analysis in structural and continuum mechanics since the early 1950s. With the advent of high speed computers and affordable computer mass storage devices, the use of FEM, both in thermomechanics and computational electromagnetics, exploded in the late 1970s. Many excellent books have been written to expound on the theory and implementation of FEM in diverse engineering applications. The boundary element method (BEM) and other computational techniques gained popularity when it became clear that certain deficiencies in FEM cannot be overcome with the use of faster computers and more computer memory. Although many research articles have appeared in professional journals, to date there exist few introductory text and reference books on BEM. The publication of Beer and Watson's book is timely in bridging this gap.

Chapter 1 describes the organization of the book, and introduces the reader to the idea of computational stress analysis using the finite and the boundary element methods. An overview of the procedures involved in FEM and BEM, and the mathematical notations used in the rest of the book are presented. The reference section contains an extensive and useful list of books for both methods for readers who wish to go beyond the introductory level of Beer and Watson's book.

Chapter 2 introduces the most fundamental concept used in both FEM and BEM: spatial discretization. Starting from a simple 1-D object, and graduating to the more complex 3-D case, the shape functions, which are the building blocks of the numerical technique used in FEM and BEM, are introduced. The authors have done excellent job in showing the functional dependence of the various shape functions. The chapter also contains a brief introduction to coordinate transformation and differential geometry.

Chapter 3 first introduces some basic equations governing heat conduction and seepage in solid and porous media, and the fundamental concepts of stress and strain in continuum mechanics. It then discusses the elastic and inelastic material models. The simple Hooke's law that governs the functional dependence between stresses and strains for elastic materials is discussed. For inelastic materials, the material response at the yield point of isotropic and anisotropic models are described in some detail. The last section is rather unusual in that it contains a short discussion on time-dependent plasticity, which is not a commonly used material model due to the lack of material property data.

Chapter 4 gives a brief discussion of the theoretical background of the FEM and its numerical implementation. The fundamental matrix equations and the numerical techniques to solve these equations are presented. Readers will be introduced to a number of new concepts: Gauss elimination to solve a system of simultaneous linear equations, sparse matrices, waveband minimization, the frontal solver, Gauss quadrature, Gauss points, boundary conditions, etc. As one should expect, based on the title of the book, the treatment is by no means rigorous: equations are presented "as is" with relatively few derivations.

Shock and Vibration, Vol. 2, No. 4, pp. 351–353 (1994)

CCC 1070-9622/95/040351-03

The readers who desire a more formal and complete treatment of numerical techniques involved will have to consult some of the books listed in the reference section. The last third of the chapter contains a few sample problems to illustrate the use of the FEM. Starting from the classical cantilever beam to the more complicated analysis of sequential mining. These problems clearly demonstrate the wealth of information an experienced analyst can get out of an FE analysis. Simple problems are given at the end of the chapter for readers who wish to understand better and to exercise the FEM.

Solid elements are introduced in **Chapter 4.** In **Chapter 5,** the readers are introduced to the rather unique concept of plate/shell and joint elements. Shell and plate elements, sometimes referred to as 2-D elements, are used when the dimension of a 3-D object along one direction is much smaller than those in the other two orthogonal directions, such that the strain and stress normal to the element surfaces can be assumed to be small. The difference between plates and shells, and the numerical techniques needed to compute the response of 2-D elements accurately, especially for thin elements, are briefly discussed. Joints, used when discontinuity in the computed response is expected across the element boundaries, are given a short treatment. Two-thirds of this chapter is devoted to examples that illustrate where 2-D and joint elements should be used in a stress analysis.

Chapter 6 introduces the basic principles of the BEM. In BEM, the approximations are made only on the surface of the object to be analyzed; every point in the interior is assumed to be in a state of equilibrium. Only surface meshes are needed for BE analyses, in contrast to the discretization of the entire object in an FE analysis. This results in a reduction in computer memory requirements, and a smaller number of equations to be solved. A more fundamental difference between the two numerical techniques is that in BEM, a fundamental solution to the governing partial differential equations must be known. This restriction, plus the fact that the method is applicable mostly to phenomena localized to the surface and edges of the physical object, partially explains why BEM has not enjoyed the same wide applicability as FEM. The authors confine the discussion of solutions to those of Laplace's equation and for isotropic elasticity. Different methods are introduced and compared. The last section of the chapter contains a succinct discussion of the state and limitations of the BEM.

Chapter 7 is the longest chapter in the book. It discusses the practical implementation of the BEM. A brief overview of the procedures involved in setting up a BE analysis is presented. The matrix equations resulting from the isoparametric representation of finite systems and the coupling to infinite elements for infinite domains are introduced. The examples given in the second half of the chapter are divided into two sets. The first set of examples is for problems that either possess analytical solutions such as the cantilevers, or involve cut-out or localized crack in geometry with infinite extent. The simple geometry in these examples are well chosen to illustrate clearly the principles of BEM. The second set of three examples are closer to "real-life" problems where complex geometries are encountered. These examples are the stress analysis in a crankshaft, the analyses of perforated cased deviated wellbore and mine excavation. A rather extensive set of problems are given at the end of the chapter as an exercise for readers who wish to understand the BEM better by using the computer codes supplied with the book.

Chapter 8 shows how the FEM and the BEM can be coupled to become a versatile and powerful tool in engineering analysis. The coupling is basically done by transforming the BE formalism into a form analogous to that used in FE analysis such that the BE region can be treated as a "super finite element," and assembled with the regular FEs in the usual way. Simple examples are given at the end of the chapter to illustrate the usefulness of such coupling.

Chapter 9 discusses the procedures for solving nonlinear problems. Only material nonlinearity is presented; geometric nonlinearity and time-dependent boundary conditions are omitted in the discussion. The two material models covered are the elastic-plastic model and time-dependent plasticity. The computational steps in both FEM and BEM are briefly reviewed. Simple examples are given at the end of the chapter to illustrate nonlinear stress–strain relation for the two material models.

Chapter 10 discusses two important aspects of numerical analyses: the quality of a mesh and error analysis. Meshing, the process by which a physical domain is discretized into FEs and/or BEs, ultimately determines the accuracy and precision of the solution. Numerical solutions from too coarse a mesh will most probably miss most of the highly localized phenomena, whereas analyses with unnecessarily fine mesh incur unnecessarily high computational cost, and, in the worst case scenario, the problem may become too big to be solved. Finally, the analyst has to decide which of the two numerical methods, FEM or BEM, is the more appropriate choice for a given problem. The authors give a brief overview on the factors to be considered when designing a mesh, and provide some simple tips on the choice of numerical method. The second half of the chapter is devoted to a discussion of the various sources of numerical errors and their estimation and mesh refinement. The authors should be complemented for their discussion on error analysis as this aspect of numerical analyses is usually ignored in most books on the same subject. The *h*- and the *p*-type elements are mentioned in the context of adaptive mesh refinement.

Pre- and postprocessing are discussed in the last chapter, **Chapter 11.** The requirements in mesh generation procedures, computer graphics, user interfaces, and the display of computed quantities are presented.

There are altogether five appendices. **Appendix A** introduces the computer code, DEMON, for the analysis of plane strain by the FEM. **Appendix B** introduces the computer code, BOUND, for the analysis of plane strain by the BEM. **Appendix C** discusses the fundamental solutions for point sources and displacement discontinuities in an infinite domain. **Appendix D** discusses the Green's symmetry identity and the boundary integral equation of the direct method for Laplace's equation. Finally, **Appendix E** discusses the Betti's theorem and the boundary integral equation of the direct method for elasticity. The discussions in both Appendices D and E are a supplement to Chapter 6. Note that the computer codes DEMON and BOUND were not reviewed.

Overall, Beer and Watson's introductory level book fills the need for a book in computational methods rather well. It is most suitable for engineers and senior engineering students who want to gain a basic understanding of the finite and boundary element methods. I should add that familiarity with the matrix notation and elementary knowledge in linear algebra is assumed. For the readers who find the introduction of some of the underlying equations too abrupt, and the lack of derivation of some important results unsatisfactory, ample references are given at the end of each chapter to fill these gaps.

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