

## Letter to the editor

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*Comment on the paper: A. M. Yu, Y. Hao, Improved Riccati transfer matrix method for free vibration of non-cylindrical helical springs including warping, Shock and Vibration, Volume 19, number 6, 2012, pp. 1167–1180, DOI 10.3233/SAV-2012-0677*

Manuel Paredes

Université de Toulouse; INSA, UPS, Mines Albi, ISAE; ICA (Institut Clément Ader); 135, avenue de Rangueil, F-31077 Toulouse, France

Tel.: +33 561 55 9956/+33 561 55 9700; E-mail: manuel.paredes@insa-toulouse.fr

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In this paper, the authors present the Riccati transfer matrix method applied to the study of springs of arbitrary shape and also provide FE results. Work on this subject would be a useful addition to the literature but only if it is done to a sufficient standard.

I would like to point out that this paper contains an error in the parametrization of the helix. It is easy to understand why the authors made this mistake as the parametrization and examples used in this work appear in previously published articles [1–7] that mostly suffer from the same deficiencies. It is important to note that the paper published by Nagaya [1] was severely criticized by Pearson [8] in 1988, who found several deficiencies. As things stand now, we can only regret that this early warning was not taken onboard by subsequent authors.

The following details should clarify the error encountered in the paper.

The authors present a parametric equation of a non-cylindrical helix which is correct.

$$x = R(\beta) \cos \beta, \quad y = R(\beta) \sin \beta, \quad z = h(\beta)\beta \quad (1)$$

where  $R(\beta)$  denotes the centerline radius and  $h(\beta)$  the step per unit angle of the helix as a function of the horizontal angle.

They also consider Eq. (2), which is a particular case where the pitch angle  $\bar{\alpha}$  is taken as constant. Note that, in the general case,  $\bar{\alpha}$  could also be described as a function of the horizontal angle.

$$h(\beta) = R(\beta) \tan \bar{\alpha} \quad (2)$$

The authors then treat the cases of conical springs and hyperboloidal springs.

The case of the conical spring is now analysed precisely to highlight the error.

To describe the conical spring the authors exploit Eq. (7).

$$R(\beta) = R_1 + \frac{(R_2 - R_1)\beta}{2\pi n} \quad (7)$$

where  $(0 \leq \beta \leq 2\pi n)$  and  $n$  is the number of active turns of the helix.

At this stage, we can note that the shape of a conical spring is described by an equation in which the radius is calculated as a linear function of the axial position, whereas the authors take the radius as a function of the horizontal angle.

In this case, Eq. (7) represents the exact description of an Archimedes' spiral (spiral with a constant radial pitch). Projecting this spiral on to a conical shape induces a conical helix with a constant axial pitch.

Unfortunately, the authors also exploit Eq. (2), in which the helix angle is considered as constant. When a conical spring with a constant helix angle is considered, a logarithmic spiral is obtained.

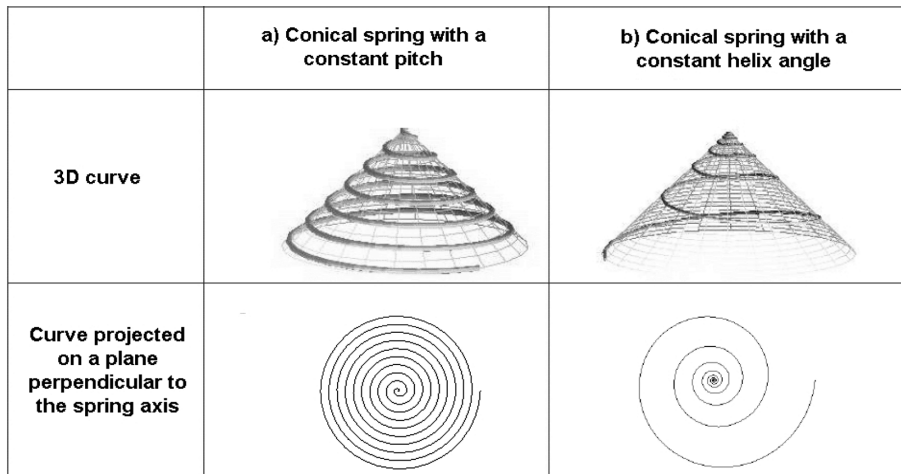


Fig. 1. Examples of spirals.

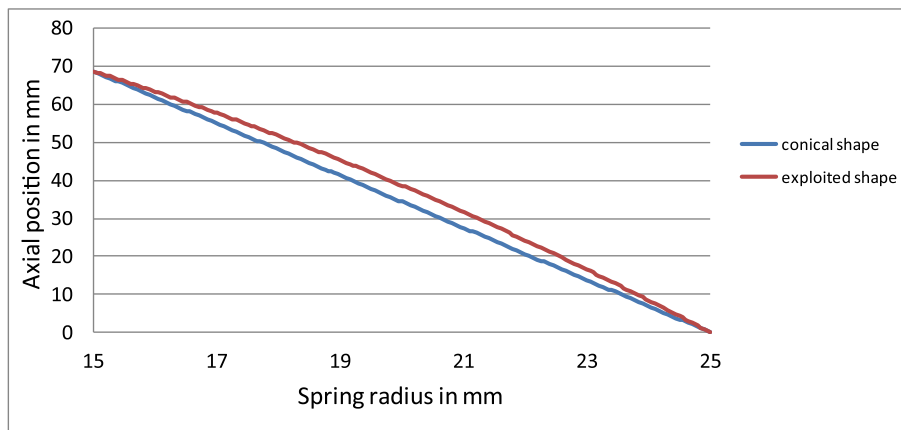


Fig. 2. Comparison of the shape exploited in the paper and the related conical shape.

In fact, there are an infinity of possibilities for describing a spiral that moves from one radius to another in a given number of turns [9]. Archimedean and logarithmic spirals are two distinct possibilities, which are illustrated in Figs (1a) and (1b) respectively.

Thus a conical spring cannot be described by Eqs (2) and (7) at the same time.

Equation (7) can be used to describe a conical spring with a constant pitch. This kind of conical spring has been exploited by several authors [10–12].

Equation (2) can be used to describe a conical spring with a constant helix angle. In their study, Liu et al. [13] present a precise description of the geometry of this spring.

So far, Eqs (2) and (7) describe a spring that could be built but that is definitely not a conical spring.

In order to illustrate this point, the shape obtained and exploited by the authors in example 1 has been calculated and compared to the expected conical shape. The result is presented in Fig. 2 and shows the difference between the two shapes.

The same approach can be applied to the case of hyperboloidal springs. The shape of the spring exploited in example 1 was calculated and compared to the expected hyperboloidal shape. Figure 3 presents the two shapes obtained.

In the paper, the authors also make a comparison between the results obtained by using their analytical equations and FE results. Here, it would be interesting to know how the FE models were built. On the one hand, if the helices

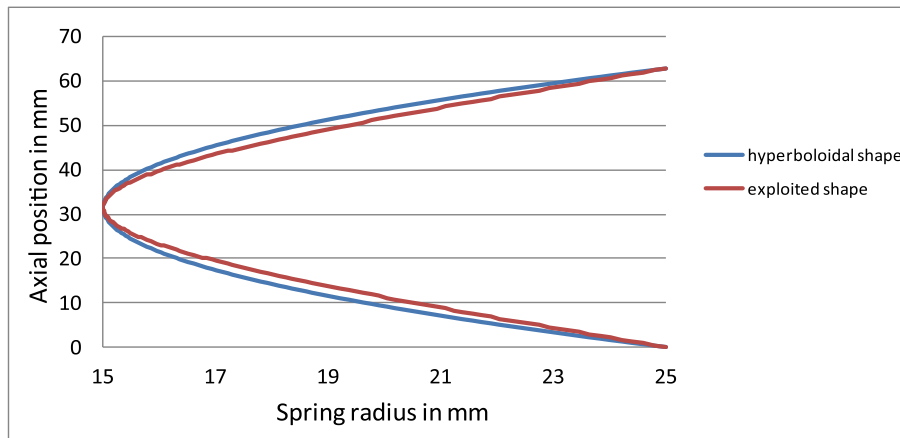


Fig. 3. Comparison of the shape exploited in the paper and the related hyperboloidal shape.

of the FE models were constructed point by point using the equations presented, then the springs are not the ones that were expected but the results can be compared as they involve the same geometries. On the other hand, if the helix of the FE models was made using a dedicated tool in the software, then the shapes obtained may represent the expected springs but the results should no longer be compared.

## Conclusion

This demonstration proves that combining the use of Eqs (2) and (7) for the study of a conical spring is a scientific error. It is also shown that the parameterization of hyperboloid springs presented by Yu and Hao is false.

I hope that these few comments will help the scientific community not to exploit inconsistent equations in the future to describe springs with arbitrary shapes.

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