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q-Rung Orthopair Fuzzy Improved Power Weighted Operators For Solving Group Decision-Making Issues

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Abstract. This paper proposes a new multi-criteria group decision-making (MCGDM) method utilizing q-rung orthopair fuzzy (qROF) sets, improved power weighted operators and improved power weighted Maclaurin symmetric mean (MSM) operators. The power weighted averaging operator and power weighted Maclaurin symmetric mean (MSM) operator used in the existing MCGDM methods have the drawback of being unable to distinguish the priority order of alternatives in some scenarios, especially when one of the qROF numbers being considered has a non-belongingness grade of 0 or a belongingness grade of 1. To address this limitation of existing MCGDM methods, four operators, namely qROF improved power weighted averaging (qROFIPWA), qROF improved power weighted geometric (qROFIPWG), qROF improved power weighted averaging MSM (qROFIPWAMSM) and qROF improved power weighted geometric MSM (qROFIPWGMSM), are proposed in this paper. These operators mitigate the effects of erroneous assessment of information from some biased decision-makers, making the decision-making process more reliable. Following that, a group decision-making methodology is developed that is capable of generating a reasonable ranking order of alternatives when one of the qROF numbers considered has a non-belongingness grade of 0 or a belongingness grade of 1. To investigate the applicability of the proposed approach, a case study is also presented and a comparison-based investigation is used to demonstrate the superiority of the approach.

Key words: q-rung orthopair fuzzy sets, improved power weighted operators, improved power weighted Maclaurin symmetric mean (MSM) operators, group decision-making.

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1. Introduction

Fuzzy sets (FSs) (Zadeh, 1965) were initiated primarily due to the consideration of ambiguous human evaluations when dealing with realistic problems. Alternatively, the FSs doctrine can manage the reality beyond computational observation and comprehension, that is ambiguity, partial belongingness, inaccuracy and sharpless limits (Zhan *et al.*, 2022). Later on, intuitionistic FSs (Atanassov, 1986) were introduced as a generalization of FSs to tackle problems that have deficient data. Since IFSs were proposed, many scholars have conducted in-depth studies (Chen and Chang, 2016; Chen *et al.*, 2016; Garg, 2017b, 2019; Kumar and Chen, 2021; Mishra *et al.*, 2019, 2020; Zeng *et al.*, 2019; Zou *et al.*, 2020). Yet, the relevance of intuitionistic FS is restricted because of given limitation, which is the sum of belongingness grade (BG) μ and non-belongingness grade (NBG) ν cannot surpass one, that is $\mu + \nu \leqslant 1$. However, it was later discovered that the aforesaid limitation is not satisfied based on expert preferences for complicated decision-making difficulties. For example, if an expert favours BG 0.7 and NBG 0.5 during the usage of IFSs, at that point, obviously, their sum surpasses 1. To overcome this sort of circumstance, Yager (2013a; 2013b) pioneered the notion of Pythagorean FSs with BG μ and NBG ν complying with the condition $\mu^2 + \nu^2 \leqslant 1$. As a result, Pythagorean FSs are preferred over intuitionistic FSs for expressing ambiguous data. Numerous studies have been conducted based on Pythagorean FSs. Yager and Abbasov (2013) developed a decision-making (DM) approach with Pythagorean fuzzy (PF) sets. An extension of TOPSIS tool under PF sets setting was realized by Zhang and Xu (2014) to resolve DM issues. Ma and Xu (2016) presented the symmetric PF weighted aggregation operators (AOs). Garg (2016a) defined some new generalized PF information AOs and applied them to DM problems. Garg (2016b) also presented a DM process based on correlation coefficients of PF sets. A confidence level-based methodology was presented by Garg (2017a) with PF information. Peng *et al.* (2017) built up a series of PF information measures and applied them in DM. Garg (2018) developed generalized geometric interactive AOs based on PF sets and Einstein operations. Mardani *et al.* (2018) extensively reviewed decision making methods based on fuzzy aggregation operators. Nguyen *et al.* (2019) introduced PF exponential similarity measures to tackle pattern recognition problems. Nie *et al.* (2019) provided a DM strategy with PFSs using Shapley fuzzy measures and the partitioned normalized weighted Bonferroni mean operator. Jana *et al.* (2019a) used PF Dombi operators to tackle MADM problems. Rani *et al.* (2019) presented a VIKOR approach with entropy and divergence measures of PFSs. To assess waste treatment technologies, Rani *et al.* (2020) again proposed a new DM framework. Ejegwa (2021) proposed a generalized tri-parametric correlation coefficient for PF sets.

The q-rung orthopair fuzzy (qROF) set, introduced by Yager (2017), reserves the constraint that the sum of qth power of the BG and the NBG must be the value in [0, 1], i.e. $0 \leqslant \mu^q + \nu^q \leqslant 1$. Clearly, qROF sets are extended versions of intuitionistic FSs (for $q = 1$) and PF sets (for $q = 2$). For the last couple of years, information aggregation has been a popular topic due to its significance and close connection to the issues of multi-criteria group decision-making (MCGDM) under the qROF setting. The qROF weighted averaging and geometric (qROFWA and qROFWG, respectively) were announced by Liu and Wang (2018). Based on the ideas of certainty and possibility, Yager and Alajlan (2017) recommended approximate reasoning on qROFSs. Peng *et al.* (2018) proposed exponential operators and acquired satisfactory outputs after using them in the assessment of the teaching management system. In a qROFSs context, Yager *et al.* (2018) managed strong coordination between probability, certainty, believability, and faith. Liu and Liu (2018) proposed qROF weighted BM operators and used them for MCGDM problems. Wei *et al.* (2018) introduced qROF weighted Heronian mean AOs in DM issues. Jana *et al.* (2019b) proposed qROF Dombi weighted averaging and geometric operators for aggregating criteria values. Mi *et al.* (2019) settled a multi-criteria DM (MCDM) issue utilizing a qROFS-VIKOR strategy. Xing *et al.* (2019a) proposed a new group of weighted AOs to amassed qROF data which takes part in the rearrangement of BG and NBG in qROFNs as per different principles. Qin *et al.* (2019a) presented Archimedean Muirhead mean (MM) AOs of qROFNs and furthermore indicated its conceivable application in settling MCGDM problem. In view of association operations and dual Hamy mean (HM) operation, Xing *et al.* (2019b) introduced qROF interaction dual HM AO to solve a MCGDM problem. Qin *et al.* (2019b) built up the Archimedean power partitioned MM of qROFNs to tackle the MCGDM strategy. Zhong *et al.* (2019) introduced qROF Dombi power partitioned weighted Heronian mean (HEM) AO to decrease the negative impact of some criteria degrees during the aggregation process. Darko and Liang (2020) built up some qROF Hamacher AOs to extend EDAS technique for solving MCDM concern. Yang and Pang (2020) developed qROF Bonferroni mean (BM) Dombi operators for a site selection problem. Yang *et al.* (2020) developed an online shopping structure for utilizing the qROF interaction weighted HEM operator. Joshi and Gegov (2020) conveyed the commonality level of DEs with considered elements for starting appraisals on qROF setting and suggested some AOs to combine the required information. Liu and Wang (2020) introduced qROF generalized MSM operator (qROFGMSM) and qROF generalized geometric MSM operator (qROFGGMSM), which might access BGs and NBGs in the range [0, 1], respectively, and admit different criteria. Using qROF-MULTIMOORA methodology and qROF Dombi-Prioritized weighted AOs, Aydemir and Gunduz (2020) solved a MCDM problem. Garg and Chen (2020) presented qROF weighted neutrality operators by using the notion of proportional distribution procedures of the BGs and NBGs. Liu *et al.* (2022a) developed group decision-making tool using linguistic qROF generalized point weighted AOs.

1.1. *Research Motivation*

The interrelationship between multiple criteria can be seen in different realistic situations. Many of the existing studies (Liu and Wang, 2018; Jana *et al.*, 2019b; Liu and Wang, 2019; Garg and Chen, 2020) cannot tackle this situation. Although few operators (Liu and Liu, 2018; Yang and Pang, 2020) have been developed earlier, none of them is capable of handling this situation as they consider dependency between two criteria only. Although the Archimedean Muir-head mean operator (Qin *et al.*, 2019a) and generalized MSM operator (Liu and Wang, 2018) can meet this requirement, they fail to eliminate the impact of extreme evaluating criteria values from some biased experts with diverse preference attitudes. To address such circumstances, Liu *et al.* (2020) proposed qROF power MSM operator. The method of Liu *et al.* (2020) has the constraint that it fails to distinguish the priority orders of alternatives in certain cases, specifically when among the qROF numbers considered one qROF number has a non-belongingness grade that equals to 0 (or a belongingness grade that equals to 1). Thus, it is essential to develop a novel MCGDM approach to overcome the limitation of the existing method (Liu *et al.*, 2020) and the existing power weighted MSM operator (Liu *et al.*, 2020).

1.2. *Contributions*

To overcome the shortcomings of Liu *et al.*'s (2020) method, in this paper, the followings have been incorporated:

1. Some new operational laws are presented in order to fair treatment of belongingness and non-belongingness grades.
2. Four new operators, namely qROF improved power weighted averaging and geometric (qROFIPWA and qROFIPWG, resp.) operators, qROF improved power weighted averaging and geometric MSM (qROFIPWAMSM and qROFIPWGMSM, resp.) operators are developed.
3. A novel DM approach is developed based on the proposed operators. This proposed approach can resolve the limitations of Liu *et al.* (2020).
4. To show the efficiency of the proposed methodology, a personnel selection problem is considered under qROF setting.
5. A detailed comparative investigation is demonstrated to validate the superiority of the proposed model.

The rest of the paper is arranged as given below:

Some essential concepts related to qROF sets are briefly discussed in Section 2. Section 3 presents some new operations between qROF numbers. This section also puts forward the qROFPWA operator, qROFIPWG operator, qROFIPWAMSM operator and qROFIPWGMSM operator along with their characteristics. In Section 4, a decision-making methodology using the developed operators is provided. A case study of personnel selection problem is demonstrated in Section 5 to show the applicability of the developed approach. The solution of the case study, effect of the parameter and comparative study are also demonstrated in this section. Section 6 concludes the paper along with future research directions.

2. Preliminaries

2.1. *q-Rung Orthopair Fuzzy Sets (qROFSs)*

Some important concepts on qROFNs, basic operations between qROFNs and qROF weighted neutral AOs are highlighted as follows:

Definition 1 (Yager, 2017). Let U be the discourse set. Then a qROFS Θ on U is given by

$$\Theta = \{\langle t, \Delta(t), \nabla(t)\rangle : t \in U\},$$

where $\Delta(t)$ and $\nabla(t)$ represent the BG and NBG, respectively, of $t \in U$ with the constraint $0 \leqslant \Delta(t), \nabla(t) \leqslant 1$ and $0 \leqslant (\Delta(t))^q + (\nabla(t))^q \leqslant 1$, $(q \geqslant 1)$.

Next, the hesitancy grade of $t \in U$ in Θ is given by $\pi(t) = \sqrt[q]{1 - (\Delta(t))^q - (\nabla(t))^q}$. Obviously, $0 \leqslant \pi(t) \leqslant 1$. Also, Yager (2017) called the pair $\langle \Delta(t), \nabla(t)\rangle$ a qROFN. For easiness, the symbol $\Theta = \langle \Delta, \nabla\rangle$ is used to signify a qROFN. Suppose Σ^U denotes the collection of all qROFNs over U.

Definition 2 (Liu and Wang, 2018). Let $\Theta = \langle \Delta, \nabla\rangle$ be a qROFN. Then the score value of Θ is defined by

$$V(\Theta) = \Delta^q - \nabla^q. \tag{1}$$

Clearly, $-1 \leqslant S_c(\Theta) \leqslant 1$. It should be mentioned that the score value cannot be effectively utilized to separate numerous qROFNs for the situation when score values become identical. As a result, when comparing qROFNs, it is not recommended to rely solely on their score values. To manage such an issue, Liu and Wang (2018) proposed the idea of accuracy value of a qROFN.

Definition 3 (Liu and Wang, 2018). Let $\Theta = \langle \Delta, \nabla\rangle$ be a qROFN. Then the accuracy value of Θ is given by

$$A(\Theta) = \Delta^q + \nabla^q. \tag{2}$$

According to the score function and accuracy function, a comparison scheme of qROFNs is given as follows:

Definition 4 (Liu and Wang, 2018). Let $\Theta_1 = \langle \Delta_1, \nabla_1\rangle$ and $\Theta_2 = \langle \Delta_2, \nabla_2\rangle$ be two qROFNs. Then

(1) If $V(\Theta_1) > V(\Theta_2)$, then $\Theta_1 \succ \Theta_2$;
(2) If $V(\Theta_1) = V(\Theta_2)$, then
 (i) if $A(\Theta_1) > A(\Theta_2)$, then $\Theta_1 \succ \Theta_2$;
 (ii) if $A(\Theta_1) = A(\Theta_2)$, then $\Theta_1 = \Theta_2$.

Definition 5 (Liu and Wang, 2018). Let $\Theta_1 = \langle \Delta_1, \nabla_1\rangle$ and $\Theta_2 = \langle \Delta_2, \nabla_2\rangle$ be two qROFNs and $\lambda > 0$. Then the basic operations are defined by

(i) $\Theta_1 \otimes \Theta_2 = \langle \sqrt[q]{1-(1-\Delta_1^q)(1-\Delta_2^q)}, \nabla_1\nabla_2 \rangle$, (3)

(ii) $\Theta_1 \otimes \Theta_2 = \langle \Delta_1\Delta_2, \sqrt[q]{(1-(1-\nabla_1^q)(1-\nabla_2^q))} \rangle$, (4)

(iii) $\lambda\Theta_1 = \langle \sqrt[q]{1-(1-\Delta_1^q)^\lambda}, \nabla_1^\lambda \rangle$, (5)

(iv) $\Theta_1^\lambda = \langle \Delta_1^\lambda, \sqrt[q]{1-(1-\nabla_1^q)^\lambda} \rangle$. (6)

Definition 6. (Liu *et al.*, 2020). Let $\Theta_1 = \langle \Delta_1, \nabla_1 \rangle$ and $\Theta_2 = \langle \Delta_2, \nabla_2 \rangle$ be two qROFNs. Then the normalized Hamming distance between them is expressed as:

$$Dist(\Theta_1, \Theta_2) = \frac{1}{2}\left(\left|\Delta_1^q - \Delta_2^q\right| + \left|\nabla_1^q - \nabla_2^q\right| + \left|\pi_1^q - \pi_2^q\right|\right). \tag{7}$$

2.2. *Power Averaging Operator (PAO)*

The PAO, discovered by Yager (2001), can relegate weights to the aggregated elements' values by means of processing the degree of support among the elements. The conventional definition of PAO is given by:

Definition 7 (Yager, 2001). Let $b_1, b_2, \dots, b_n$ be a collection of crisp values. Then the power averaging operator (PAO) of these numbers is defined as follows:

$$PA(b_1, b_2, \dots, b_n) = \frac{\sum_{i=1}^n (1+\psi(b_i))b_i}{\sum_{i=1}^n (1+\psi(b_i))}, \tag{8}$$

where $\psi(b_i) = \sum_{j=1, j\neq i}^n Supp(b_i, b_j)$.

Here, $Supp(b_i, b_j)$ denotes the support of b_i from b_j and has the three axioms as

(i) $0 \leqslant Supp(b_i, b_j) \leqslant 1$,
(ii) $Supp(b_i, b_j) = Supp(b_j, b_i)$,
(iii) $Supp(b_i, b_j) \geqslant Supp(b_k, b_r)$ provided $|b_i - b_j| < |b_k - b_r|$, where $1 \leqslant i, j, k, r \leqslant n$.

3. qROF Improved Power Weighted Operators

3.1. *New Operations Between qROFNs*

A few new operations are introduced between qROFNs and the basic laws are investigated.

Definition 8. Let $\Theta_1 = \langle \Delta_1, \nabla_1 \rangle$ and $\Theta_2 = \langle \Delta_2, \nabla_2 \rangle$ be two qROFNs and $\lambda > 0$. Then we define:

$$\text{(i) } \Theta_1 \tilde{\oplus} \Theta_2 = \left\langle \sqrt[q]{1 - \prod_{r=1}^{2} (1 - \Delta_r^q)}, \sqrt[q]{\prod_{r=1}^{2} (1 - \Delta_r^q) - \prod_{r=1}^{2} (1 - \Delta_r^q - \nabla_r^q)} \right\rangle; \tag{9}$$

$$\text{(ii) } \Theta_1 \tilde{\otimes} \Theta_2 = \left\langle \sqrt[q]{\prod_{r=1}^{2} (1 - \nabla_r^q) - \prod_{r=1}^{2} (1 - \Delta_r^q - \nabla_r^q)}, \sqrt[q]{1 - \prod_{r=1}^{2} (1 - \nabla_r^q)} \right\rangle; \tag{10}$$

$$\text{(iii) } \lambda \Theta_1 = \left\langle \sqrt[q]{1 - (1 - \Delta_1^q)^\lambda}, \sqrt[q]{(1 - \Delta_1^q)^\lambda - (1 - \Delta_1^q - \nabla_1^q)^\lambda} \right\rangle; \tag{11}$$

$$\text{(iv) } \Theta_1^\lambda = \left\langle \sqrt[q]{(1 - \nabla_1^q)^\lambda - (1 - \Delta_1^q - \nabla_1^q)^\lambda}, \sqrt[q]{1 - (1 - \nabla_1^q)^\lambda} \right\rangle. \tag{12}$$

To understand the superiority of the developed operations, four examples are considered as follows:

Example 1. Let us consider two qROFNs $\Theta_1 = \langle 0.4, 0.7 \rangle$ and $\Theta_2 = \langle 0.8, 0 \rangle$. Then using the basic operational laws (Liu and Wang, 2018) of qROFNs, we have $\Theta_1 \oplus \Theta_2 = \langle 0.8352, 0 \rangle$, which means that the non-zero non-belongingness grade has no impact on the output. This makes the operation '$\oplus$' unreasonable. But based on the proposed operations '$\tilde{\oplus}$' and '$\tilde{\otimes}$', we have $\Theta_1 \tilde{\oplus} \Theta_2 = \langle 0.8352, 0.42 \rangle$ and $\Theta_1 \tilde{\otimes} \Theta_2 = \langle 0.6196, 0.7 \rangle$.

Example 2. Let us consider two qROFNs $\Theta_1 = \langle 1, 0 \rangle$ and $\Theta_2 = \langle 0.8, 0.5 \rangle$. Then using the basic operational laws (Liu and Wang, 2018) of qROFNs, we have $\Theta_1 \oplus \Theta_2 = \langle 1, 0 \rangle$, which means that the belongingness grade which is not equals to '1' has no impact on the output. This again makes the operation '$\oplus$' unreasonable. But based on the proposed operation '$\tilde{\otimes}$', we get $\Theta_1 \tilde{\otimes} \Theta_2 = \langle 0.8660, 0.5 \rangle$.

Example 3. Let us consider two qROFNs $\Theta_1 = \langle 0, 0.7 \rangle$ and $\Theta_2 = \langle 0.8, 0.3 \rangle$. Then using the basic operational laws (Liu and Wang, 2018) of qROFNs, we have,$\Theta_1 \otimes \Theta_2 = \langle 0, 0.7320 \rangle$, which means that the non-zero belongingness grade has no impact on the output. This makes the operation '$\otimes$' unreasonable. But based on the proposed operations '$\tilde{\oplus}$' and '$\tilde{\otimes}$', we have $\Theta_1 \tilde{\oplus} \Theta_2 = \langle 0.8, 0.4714 \rangle$ and $\Theta_1 \tilde{\otimes} \Theta_2 = \langle 0.5713, 0.7320 \rangle$.

Example 4. Let us consider two qROFNs $\Theta_1 = \langle 0, 1 \rangle$ and $\Theta_2 = \langle 0.6, 0.6 \rangle$. Then using the basic operational laws (Liu and Wang, 2018) of qROFNs, we have $\Theta_1 \otimes \Theta_2 = \langle 0, 1 \rangle$, which means that the non-belongingness grade which is not equals to '1' has no impact on the output. This again makes the operation '$\otimes$' unreasonable. But based on the proposed operation '$\tilde{\oplus}$', we have $\Theta_1 \tilde{\oplus} \Theta_2 = \langle 0.6, 0.8 \rangle$.

From the above four examples, it is clear that our proposed operations are more sensible.

Theorem 1. *Let* $\Theta_1 = \langle \Delta_1, \nabla_1 \rangle$ *and* $\Theta_2 = \langle \Delta_2, \nabla_2 \rangle$ *be two* qROFNs *and* $\lambda, \lambda_1, \lambda_2 > 0$. *Then*:

(i) $\Theta_1 \tilde{\oplus} \Theta_2 = \Theta_2 \tilde{\oplus} \Theta_1$;

(ii) $\Theta_1 \tilde{\otimes} \Theta_2 = \Theta_2 \tilde{\otimes} \Theta_1$;
(iii) $\lambda(\Theta_1 \tilde{\oplus} \Theta_2) = \lambda\Theta_1 \tilde{\oplus} \lambda\Theta_2$;
(iv) $(\Theta_1 \tilde{\otimes} \Theta_2)^{\lambda} = \Theta_1^{\lambda} \tilde{\otimes} \Theta_2^{\lambda}$;
(v) $(\lambda_1 + \lambda_2)\Theta_1 = \lambda_1\Theta_1 \tilde{\oplus} \lambda_2\Theta_1$;
(vi) $\Theta_1^{\lambda_1+\lambda_2} = \Theta_1^{\lambda_1} \tilde{\otimes} \Theta_1^{\lambda_2}$.

Proof. Follows from Definition 8. □

3.2. *qROF Improved Power Weighted Averaging Operators*

In this paper, qROF improved power weighted averaging (qROFIPWA) and qROF improved power weighted averaging MSM (qROFIPWAMSM) operators are developed as follows.

Definition 9. Let $\Theta_r = \langle \Delta_r, \nabla_r \rangle (r = 1(1)n) \in \Sigma^U$. Then the qROFIPWA operator is defined by:

$$\text{qROFIPWA}(\Theta_1, \Theta_2, \ldots, \Theta_n) = \widetilde{\bigoplus_{r=1}^{n}} \frac{\varpi_r(1 + \psi(\Theta_r))}{\sum_{r=1}^{n} \varpi_r(1 + \psi(\Theta_r))} \Theta_r. \tag{13}$$

In Eq. (13), $\frac{\varpi_r(1+\psi(\Theta_r))}{\sum_{r=1}^{n} \varpi_r(1+\psi(\Theta_r))}$ is called the power weight of Θ_r, where ϖ_r is the weight of Θ_r satisfying $\varpi_r \geqslant 0$ and $\sum_{r=1}^{n} \varpi_r = 1$. To keep things simple, Ω_r is used denote the power weight of Θ_r. Then Eq. (13) can be re-written as:

$$\text{qROFIPWA}(\Theta_1, \Theta_2, \ldots, \Theta_n) = \widetilde{\bigoplus_{r=1}^{n}} \Omega_r \Theta_r. \tag{14}$$

Theorem 2. *Let* $\Theta_r = \langle \Delta_r, \nabla_r \rangle (r = 1(1)n) \in \Sigma^U$. *Then the aggregated value* qROFIPWA$(\Theta_1, \Theta_2, \ldots, \Theta_n)$ *is also a* qROFN *and*

$$\begin{aligned} &\text{qROFIPWA}(\Theta_1, \Theta_2, \ldots, \Theta_n) \\ &= \left\langle \left(1 - \prod_{r=1}^{n}(1 - \Delta_r^q)^{\Omega_r}\right)^{\frac{1}{q}}, \left(\prod_{r=1}^{n}(1 - \Delta_r^q)^{\Omega_r} - \prod_{r=1}^{n}(1 - \Delta_r^q - \nabla_r^q)^{\Omega_r}\right)^{\frac{1}{q}} \right\rangle. \end{aligned} \tag{15}$$

Proof. Straightforward. □

The following Theorems readily follow from Theorem 2.

Theorem 3 (Idempotency). *Let* $\Theta_r = \langle \Delta_r, \nabla_r \rangle (r = 1(1)n) \in \Sigma^U$ *with* $\Theta_r = \Theta_0 (r = 1(1)n)$. *Then,* qROFIPW$(\Theta_1, \Theta_2, \ldots, \Theta_n) = \Theta_0$.

Theorem 4 (Boundedness). *Let* $\Theta_r = \langle \Delta_r, \nabla_r \rangle (r = 1(1)n) \in \Sigma^U$. *If* $\Theta^- = \langle \min_r \Delta_r, \sqrt[q]{\max_r(\Delta_r^q + \nabla_r^q) - \min_r \Delta_r^q} \rangle$ *and* $\Theta^+ = \langle \max_r \Delta_r, \hbar \rangle$, *then we have* $\Theta^- \prec$ qROFIPWA$(\Theta_1, \Theta_2, \ldots, \Theta_n) \prec \Theta^+$, *where*

$$\hbar = \begin{cases} 0, & \text{if } \min_r(\Delta_r^q + \nabla_r^q) \leqslant \max_r \Delta_r^q, \\ \sqrt[q]{\min_r(\Delta_r^q + \nabla_r^q) - \max_r \Delta_r^q}, & \text{if } \min_r(\Delta_r^q + \nabla_r^q) \geqslant \max_r \Delta_r^q. \end{cases}$$

Theorem 5 (Monotonicity). *Let* $\Theta_r = \langle \Delta_r, \nabla_r \rangle$ *and* $\Theta'_r = \langle \Delta'_r, \nabla'_r \rangle (r = 1(1)n) \in \Sigma^U$ *such that* $\Delta_r \leqslant \Delta'_r$, $\nabla_r \geqslant \nabla'_r$. *Then,*

$$\text{qROFIPWA}(\Theta_1, \Theta_2, \ldots, \Theta_n) \prec \text{qROFIPWA}(\Theta'_1, \Theta'_2, \ldots, \Theta'_n).$$

Definition 10. Let $\Theta_r = \langle \Delta_r, \nabla_r \rangle (r = 1(1)n) \in \Sigma^U$. Then the qROFIPWAMSM operator is defined by:

$$\text{qROFIPWAMSM}^{(p)}(\Theta_1, \Theta_2, \ldots, \Theta_n) = \left(\frac{1}{{}^n c_p} \bigoplus_{1 \leqslant t_1 < t_2 < \cdots < t_p \leqslant n} \left(\bigotimes_{j=1}^{p} (n\Omega_{t_j} \Theta_{t_j}) \right) \right)^{\frac{1}{p}}, \tag{16}$$

where $t_1, t_2, \ldots, t_p \geqslant 0$, p is a parameter, ${}^n c_p$ stands for binomial coefficient, $(t_1, t_2, \ldots, t_p)$ denotes a p-tuple combination of $(1, 2, \ldots, n)$.

In Eq. (16), $\Omega_{t_j} = \frac{\varpi_{t_j}(1+\psi(\Theta_{t_j}))}{\sum_{t=1}^{n} \varpi_{t_j}(1+\psi(\Theta_{t_j}))}$ is called the power weight of Θ_{t_j}, where ϖ_k is the weight of Θ_k satisfying $\varpi_k \geqslant 0$ and $\sum_{k=1}^{n} \varpi_k = 1$.

Theorem 6. *Let* $\Theta_r = \langle \Delta_r, \nabla_r \rangle (r = 1(1)n) \in \Sigma^U$. *Then the aggregated value* qROFIPWAMSM$^{(p)}(\Theta_1, \Theta_2, \ldots, \Theta_n)$ *is also a* qROFN *and*

$$\begin{aligned} &\text{qROFIPWAMSM}^{(p)}(\Theta_1, \Theta_2, \ldots, \Theta_n) \\ &= \Bigg\langle \Bigg(\Bigg(\Bigg(1 - \Bigg(\prod_{1 \leqslant t_1 < t_2 < \cdots < t_p \leqslant n} \Bigg(1 - \prod_{j=1}^{p} \Big(1 - \big(1 - (\Delta_{t_j})^q\big)^{n\Omega_{t_j}} \\ &\quad + \big(1 - (\Delta_{t_j})^q - (\nabla_{t_j})^q\big)^{n\Omega_{t_j}} \Big) + \prod_{j=1}^{p} \big(1 - (\Delta_{t_j})^q - (\nabla_{t_j})^q\big)^{n\Omega_{t_j}} \Bigg) \Bigg)^{\frac{1}{{}^n c_p}} \\ &\quad + \Bigg(\prod_{1 \leqslant t_1 < t_2 < \cdots < t_p \leqslant n} \Bigg(\prod_{j=1}^{p} \big(1 - (\Delta_{t_j})^q - (\nabla_{t_j})^q\big)^{n\Omega_{t_j}} \Bigg) \Bigg)^{\frac{1}{{}^n c_p}} \Bigg)^{\frac{1}{r}} \\ &\quad - \Bigg(\Bigg(\prod_{1 \leqslant t_1 < t_2 < \cdots < t_p \leqslant n} \Bigg(\prod_{j=1}^{p} \big(1 - (\Delta_{t_j})^q - (\nabla_{t_j})^q\big)^{n\Omega_{t_j}} \Bigg) \Bigg)^{\frac{1}{{}^n c_p}} \Bigg)^{\frac{1}{p}} \Bigg)^{\frac{1}{q}}, \end{aligned}$$

$$\left(1-\left(1-\left(\prod_{1\leqslant t_1<t_2<\cdots<t_p\leqslant n}\left(1-\prod_{j=1}^{p}\left(1-\left(1-(\Delta_{t_j})^q\right)^{n\Omega_{t_j}}\right.\right.\right.\right.\right.$$
$$\left.\left.\left.+\left(1-(\Delta_{t_j})^q-(\nabla_{t_j})^q\right)^{n\Omega_{t_j}}\right)+\prod_{j=1}^{p}\left(1-(\Delta_{t_j})^q-(\nabla_{t_j})^q\right)^{n\Omega_{t_j}}\right)\right)^{\frac{1}{n_{c_p}}}$$
$$\left.+\left(\prod_{1\leqslant t_1<t_2<\cdots<t_p\leqslant n}\left(\prod_{j=1}^{p}\left(1-(\Delta_{t_j})^q-(\nabla_{t_j})^q\right)^{n\Omega_{t_j}}\right)\right)^{\frac{1}{n_{c_p}}}\right)^{\frac{1}{p}}\right)^{\frac{1}{q}}\Bigg\rangle. \tag{17}$$

Proof. Added in the Supplementary material. □

The following Theorems readily follow from Theorem 6.

Theorem 7 (Idempotency). *For* $\Theta_r = \langle\Delta_r, \nabla_r\rangle(r = 1(1)n) \in \Sigma^U$ *with* $\Theta_r = \Theta_0(r = 1(1)n)$, qROFIPWAMSM$^{(p)}(\Theta_1, \Theta_2, \ldots, \Theta_n) = \Theta_0$.

Theorem 8 (Boundedness). *Let* $\Theta_r = \langle\Delta_r, \nabla_r\rangle(r = 1(1)n) \in \Sigma^U$. *If* $\Theta^- = \langle\min_r \Delta_r, \sqrt[q]{\max_r(\Delta_r^q + \nabla_r^q) - \min_r \Delta_r^q}\rangle$ *and* $\Theta^+ = \langle\max_r \Delta_r, \hbar\rangle$, *then we have,* $\Theta^- \prec$ qROFIPWAMSM$(\Theta_1, \Theta_2, \ldots, \Theta_n) \prec \Theta^+$, *where*

$$\hbar = \begin{cases} 0, & \text{if } \min_r(\Delta_r^q + \nabla_r^q) \leqslant \max_r \Delta_r^q, \\ \sqrt[q]{\min_r(\Delta_r^q + \nabla_r^q) - \max_r \Delta_r^q}, & \text{if } \min_r(\Delta_r^q + \nabla_r^q) \geqslant \max_r \Delta_r^q. \end{cases}$$

Theorem 9 (Monotonicity). *Let* $\Theta_r = \langle\Delta_r, \nabla_r\rangle$ *and* $\Theta_r' = \langle\Delta_r', \nabla_r'\rangle(r = 1(1)n) \in \Sigma^U$ *such that* $\Delta_r \leqslant \Delta_r'$, $\nabla_r \geqslant \nabla_r'$. *Then,* qROFIPAWMSM$^{(p)}(\Theta_1, \Theta_2, \ldots, \Theta_n) \prec$ qROFIPWAMSM$^{(p)}(\Theta_1', \Theta_2', \ldots, \Theta_n')$.

3.3. *qROF Improved Power Weighted Geometric Operators*

This paper develops qROF improved power weighted geometric (qROFIPWG) operator and qROF improved power weighted geometric MSM (qROFIPWGMSM) operator as follows:

Definition 11. Let $\Theta_r = \langle\Delta_r, \nabla_r\rangle(r = 1(1)n) \in \Sigma^U$. Then the qROFIPWG operator is defined by:

$$\text{qROFIPWG}(\Theta_1, \Theta_2, \ldots, \Theta_n) = \widetilde{\bigotimes_{r=1}^{n}} \Theta_r^{\Omega_r}. \tag{18}$$

In Eq. (18), $\Omega_r = \frac{\varpi_r(1+\psi(\Theta_r))}{\sum_{r=1}^{n}\varpi_r(1+\psi(\Theta_r))}$ is called the power weight of Θ_r, where ϖ_r is the weight of Θ_r satisfying $\varpi_r \geqslant 0$ and $\sum_{r=1}^{n}\varpi_r = 1$.

Theorem 10. *Let* $\Theta_r = \langle \Delta_r, \nabla_r \rangle (r = 1(1)n) \in \Sigma^U$. *Then the aggregated value* qROFIPWG$(\Theta_1, \Theta_2, \dots, \Theta_n)$ *is also a* qROFN *and*

$$\text{qROFIPWG}(\Theta_1, \Theta_2, \dots, \Theta_n) = \left\langle \left(\prod_{r=1}^{n} (1 - \nabla_r^q)^{\Omega_r} - \prod_{r=1}^{n} (1 - \Delta_r^q - \nabla_r^q)^{\Omega_r} \right)^{\frac{1}{q}}, \left(1 - \prod_{r=1}^{n} (1 - \nabla_r^q)^{\Omega_r} \right)^{\frac{1}{q}} \right\rangle. \tag{19}$$

Proof. Straightforward. □

The following Theorems readily follow from Theorem 10.

Theorem 11. (Idempotency) *Let* $\Theta_r = \langle \Delta_r, \nabla_r \rangle (r = 1(1)n) \in \Sigma^U$ *such that* $\Theta_r = \Theta_0 (r = 1(1)n)$, *then we have* qROFIPWG$(\Theta_1, \Theta_2, \dots, \Theta_n) = \Theta_0$.

Theorem 12. (Boundedness) *Let* $\Theta_r = \langle \Delta_r, \nabla_r \rangle (r = 1(1)n) \in \Sigma^U$. *If* $\Theta^- = \langle \min_r \nabla_r, \sqrt[q]{\max_r (\Delta_r^q + \nabla_r^q) - \min_r \nabla_r^q} \rangle$ *an* $\Theta^+ = \langle \max_r \nabla_r, \hbar \rangle$, *then we have* $\Theta^- \prec$ qROFIPWG$(\Theta_1, \Theta_2, \dots, \Theta_n) \prec \Theta^+$, *where*

$$\hbar = \begin{cases} 0, & \text{if } \min_r (\Delta_r^q + \nabla_r^q) \leqslant \max_r \nabla_r^q, \\ \sqrt[q]{\min_r (\Delta_r^q + \nabla_r^q) - \max_r \nabla_r^q}, & \text{if } \min_r (\Delta_r^q + \nabla_r^q) \geqslant \max_r \nabla_r^q. \end{cases}$$

Theorem 13. (Monotonicity) *Let* $\Theta_r = \langle \Delta_r, \nabla_r \rangle$ *and* $\Theta'_r = \langle \Delta'_r, \nabla'_r \rangle (r = 1(1)n) \in \Sigma^U$ *such that* $\Delta_r \leqslant \Delta'_r$, $\nabla_r \geqslant \nabla'_r$. *Then,* qROFIPWG$(\Theta_1, \Theta_2, \dots, \Theta_n) \prec$ qROFIPWG$(\Theta'_1, \Theta'_2, \dots, \Theta'_n)$.

Definition 12. Let $\Theta_r = \langle \Delta_r, \nabla_r \rangle (r = 1(1)n) \in \Sigma^U$. Then the qROFIPWGMSM operator is defined by:

$$\text{qROFIPWGMSM}^{(p)}(\Theta_1, \Theta_2, \dots, \Theta_n) = \frac{1}{p} \left(\widetilde{\bigotimes_{1 \leqslant t_1 < t_2 < \dots < t_p \leqslant n}} \left(\widetilde{\bigoplus_{j=1}^{p}} \Theta_{t_j}^{\Omega_{t_j}} \right) \right)^{\frac{1}{^n c_p}}, \tag{20}$$

where $t_1, t_2, \dots, t_p \geqslant 0$, p is a parameter, $^n c_p$ stands for binomial coefficient, $(t_1, t_2, \dots, t_p)$ denotes a p-tuple combination of $(1, 2, \dots, n)$.

In Eq. (20), $\Omega_{t_j} = \frac{\varpi_{t_j}(1+\psi(\Theta_{t_j}))}{\sum_{t=1}^{n} \varpi_{t_j}(1+\psi(\Theta_{t_j}))}$ is called the power weight of Θ_{t_j}, where ϖ_k is the weight of Θ_k satisfying $\varpi_k \geqslant 0$ and $\sum_{k=1}^{n} \varpi_k = 1$.

Theorem 14. *Let* $\Theta_r = \langle \Delta_r, \nabla_r \rangle (r = 1(1)n) \in \Sigma^U$. *Then the aggregated value* qROFIPWGMSM$^{(p)}(\Theta_1, \Theta_2, \dots, \Theta_n)$ *is also a* qROFN *and*

$$\begin{aligned}
&\mathrm{qROFIPWGMSM}^{(p)}(\Theta_1, \Theta_2, \ldots, \Theta_n)\\
&= \Bigg\langle \Bigg(\Bigg(1 - \Bigg(1 - \Bigg(\prod_{1\leqslant t_1 < t_2 < \cdots < t_p \leqslant n} \Bigg(1 - \prod_{j=1}^{p}\big(1 - \big(1 - (\nabla_{t_j})^q\big)^{n\Omega_{t_j}}\\
&\quad + \big(1 - (\Delta_{t_j})^q - (\nabla_{t_j})^q\big)^{n\Omega_{t_j}}\big) + \prod_{j=1}^{p}\big(1 - (\Delta_{t_j})^q - (\nabla_{t_j})^q\big)^{n\Omega_{t_j}}\Bigg)\Bigg)^{\frac{1}{n_{c_p}}}\\
&\quad + \Bigg(\prod_{1\leqslant t_1 < t_2 < \cdots < t_p \leqslant n}\Bigg(\prod_{j=1}^{p}\big(1 - (\Delta_{t_j})^q - (\nabla_{t_j})^q\big)^{n\Omega_{t_j}}\Bigg)\Bigg)^{\frac{1}{n_{c_p}}}\Bigg)^{\frac{1}{p}}\Bigg)^{\frac{1}{q}},\\
&\quad \Bigg(\Bigg(1 - \Bigg(\prod_{1\leqslant t_1 < t_2 < \cdots < t_p \leqslant n} \Bigg(1 - \prod_{j=1}^{p}\big(1 - \big(1 - (\nabla_{t_j})^q\big)^{n\Omega_{t_j}}\\
&\quad + \big(1 - (\Delta_{t_j})^q - (\nabla_{t_j})^q\big)^{n\Omega_{t_j}}\big) + \prod_{j=1}^{p}\big(1 - (\Delta_{t_j})^q - (\nabla_{t_j})^q\big)^{n\Omega_{t_j}}\Bigg)\Bigg)^{\frac{1}{n_{c_p}}}\\
&\quad + \Bigg(\prod_{1\leqslant t_1 < t_2 < \cdots < t_p \leqslant n}\Bigg(\prod_{j=1}^{p}\big(1 - (\Delta_{t_j})^q - (\nabla_{t_j})^q\big)^{n\Omega_{t_j}}\Bigg)\Bigg)^{\frac{1}{n_{c_p}}}\Bigg)^{\frac{1}{p}}\\
&\quad - \Bigg(\Bigg(\prod_{1\leqslant t_1 < t_2 < \cdots < t_p \leqslant n}\Bigg(\prod_{j=1}^{p}\big(1 - (\Delta_{t_j})^q - (\nabla_{t_j})^q\big)^{n\Omega_{t_j}}\Bigg)\Bigg)^{\frac{1}{n_{c_p}}}\Bigg)^{\frac{1}{p}}\Bigg)^{\frac{1}{q}}\Bigg\rangle. \qquad (21)
\end{aligned}$$

Proof. Similar to Theorem 6. □

The following Theorems readily follow from Theorem 14.

Theorem 15. (Idempotency) *Let* $\Theta_r = \langle \Delta_r, \nabla_r \rangle (r = 1(1)n) \in \Sigma^U$ *such that* $\Theta_r = \Theta_0 (r = 1(1)n)$*, then we have* $\mathrm{qROFIPWGMSM}^{(p)}(\Theta_1, \Theta_2, \ldots, \Theta_n) = \Theta_0$.

Theorem 16. (Boundedness) *Let* $\Theta_r = \langle \Delta_r, \nabla_r \rangle (r = 1(1)n) \in \Sigma^U$. *If* $\Theta^- = \langle \min_r \nabla_r, \sqrt[q]{\max_r(\Delta_r^q + \nabla_r^q) - \min_r \nabla_r^q} \rangle$ *and* $\Theta^+ = \langle \max_r \nabla_r, \hbar \rangle$*, then we have* $\Theta^- \prec \mathrm{qROFIPWGMSM}^{(p)}(\Theta_1, \Theta_2, \ldots, \Theta_n) \prec \Theta^+$*, where*

$$\hbar = \begin{cases} 0, & \text{if } \min_r(\Delta_r^q + \nabla_r^q) \leqslant \max_r \nabla_r^q, \\ \sqrt[q]{\min_r(\Delta_r^q + \nabla_r^q) - \max_r \nabla_r^q}, & \text{if } \min_r(\Delta_r^q + \nabla_r^q) \geqslant \max_r \nabla_r^q. \end{cases}$$

Theorem 17 (Monotonicity). *Let* $\Theta_r = \langle \Delta_r, \nabla_r \rangle$ *and* $\Theta_r' = \langle \Delta_r', \nabla_r' \rangle (r = 1(1)n) \in \Sigma^U$ *such that* $\Delta_r \leqslant \Delta_r'$, $\nabla_r \geqslant \nabla_r'$. *Then* $\mathrm{qROFIPWGMSM}^{(p)}(\Theta_1, \Theta_2, \ldots, \Theta_n) \prec \mathrm{qROFIPWGMSM}^{(p)}(\Theta_1', \Theta_2', \ldots, \Theta_n')$.

4. Group Decision Making Methodology

Suppose m different alternatives $X_i(i = 1(1)m)$ need to be assessed over n distinct attributes $L_j(j = 1(1)n)$. Assume a set of l experts $D_d(d = 1(1)l)$ with weights $\eta_d(d = 1(1)l)$ with $\eta_d \geqslant 0$ and $\sum_{d=1}^{l} \eta_d = 1$ for the assessment of considered alternatives. The initial assessment result of the expert $D_d(d = 1(1)l)$ is specified in terms of qROFNs $\Theta_{ij}^{(d)} = \langle \Delta_{ij}^{(d)}, \nabla_{ij}^{(d)} \rangle$ subject to $0 \leqslant \Delta_{ij}^{(d)}, \nabla_{ij}^{(d)} \leqslant 1$ and $0 \leqslant (\Delta_{ij}^{(d)})^q + (\nabla_{ij}^{(d)})^q \leqslant 1$.

To find the best-suited alternative(s), the introduced operators are applied to propose a MCGDM methodology relating to the qROF data with the steps acquired as follows:

Step 1: The initial assessment results of experts are: $\Im_d = [\Theta_{ij}^{(d)}]_{m\times n} = [\langle \Delta_{ij}^{(d)}, \nabla_{ij}^{(d)} \rangle]_{m\times n}$ $(d = 1(1)l)$.

Step 2: Normalize the decision matrices $\Im_d = [\Theta_{ij}^{(d)}]_{m\times n}(d = 1(1)l)$.

The Normalized decision matrix is: $\tilde{\Im}_d = [\tilde{\Theta}_{ij}^{(d)}]_{m\times n} = [\langle \tilde{\Delta}_{ij}^{(d)}, \tilde{\nabla}_{ij}^{(d)} \rangle]_{m\times n}(d = 1(1)l)$ where:

$$\tilde{\Theta}_{ij}^{(d)} = \begin{cases} \langle \Delta_{ij}^{(d)}, \nabla_{ij}^{(d)} \rangle & \text{if } C_j \text{ is of benefit-type,} \\ \langle \nabla_{ij}^{(d)}, \Delta_{ij}^{(d)} \rangle & \text{if } C_j \text{ is of cost-type.} \end{cases} \tag{22}$$

Step 3: Find $Supp(\tilde{\Theta}_{ij}^{(d)}, \tilde{\Theta}_{ij}^{(s)})(d, s = 1(1)l; d \neq s)$ based on the following formula:

$$Supp\big(\tilde{\Theta}_{ij}^{(d)}, \tilde{\Theta}_{ij}^{(s)}\big) = 1 - Dist\big(\tilde{\Theta}_{ij}^{(d)}, \tilde{\Theta}_{ij}^{(s)}\big)\big(d, s = 1(1)l; d \neq s\big), \tag{23}$$

where $Dist(\tilde{\Theta}_{ij}^{(d)}, \tilde{\Theta}_{ij}^{(s)})$ is the Hamming distance between the qROFNs [20].

Step 4: Compute $\psi(\tilde{\Theta}_{ij}^{(d)})$ by

$$\psi\big(\tilde{\Theta}_{ij}^{(d)}\big) = \sum_{s=1, s\neq d}^{l} Supp\big(\tilde{\Theta}_{ij}^{(d)}, \tilde{\Theta}_{ij}^{(s)}\big)\big(i = 1(1)m;\ j = 1(1)n;\ d = 1(1)l\big). \tag{24}$$

Step 5: Calculate the power weights $\Omega_{ij}^{(d)}(i = 1(1)m;\ j = 1(1)n;\ d = 1(1)l)$ associated with the qROFNs $\tilde{\Theta}_{ij}^{(d)}$ by utilizing the weights η_d of DEs $D_d(d = 1(1)l)$, where

$$\Omega_{ij}^{(d)} = \frac{\eta_d(1 + \psi(\tilde{\Theta}_{ij}^{(d)}))}{\sum_{d=1}^{l} \eta_d(1 + \psi(\tilde{\Theta}_{ij}^{(d)}))}. \tag{25}$$

Step 6: Construct aggregated normalized qROF decision matrix $\Im* = [\Theta_{ij}]_{m\times n} = [\langle \Delta_{ij}, \nabla_{ij} \rangle]_{m\times n}$.

The operator qROFIPWA or qROFIPWG can be applied for aggregating normalized qROFNs.

$$\begin{aligned}
&\mathrm{qROFIPWA}\big(\tilde{\Theta}_{ij}^{(1)}, \tilde{\Theta}_{ij}^{(2)}, \dots, \tilde{\Theta}_{ij}^{(l)}\big) \\
&= \Bigg\langle \Bigg(1 - \prod_{d=1}^{l}\big(1 - (\tilde{\Delta}_{ij}^{(d)})^q\big)^{\Omega_{ij}^{(d)}}\Bigg)^{\frac{1}{q}}, \\
&\Bigg(\prod_{d=1}^{l}\big(1 - (\tilde{\Delta}_{ij}^{(d)})^q\big)^{\Omega_{ij}^{(d)}} - \prod_{d=1}^{l}\big(1 - (\tilde{\Delta}_{ij}^{(d)})^q - (\tilde{\nabla}_{ij}^{(d)})^q\big)^{\Omega_{ij}^{(d)}}\Bigg)^{\frac{1}{q}}\Bigg\rangle, \qquad (26)
\end{aligned}$$

$$\begin{aligned}
&\mathrm{qROFIPWG}\big(\tilde{\Theta}_{ij}^{(1)}, \tilde{\Theta}_{ij}^{(2)}, \dots, \tilde{\Theta}_{ij}^{(l)}\big) \\
&= \Bigg\langle \Bigg(\prod_{d=1}^{l}\big(1 - (\tilde{\nabla}_{ij}^{(d)})^q\big)^{\Omega_{ij}^{(d)}} - \prod_{d=1}^{l}\big(1 - (\tilde{\Delta}_{ij}^{(d)})^q, -(\tilde{\nabla}_{ij}^{(d)})^q\big)^{\Omega_{ij}^{(d)}}\Bigg)^{\frac{1}{q}}, \\
&\Bigg(1 - \prod_{d=1}^{l}\big(1 - (\tilde{\nabla}_{ij}^{(d)})^q\big)^{\Omega_{ij}^{(d)}}\Bigg)^{\frac{1}{q}}\Bigg\rangle. \qquad (27)
\end{aligned}$$

Step 7: Calculate the supports $Supp(\Theta_{ij}, \Theta_{iy})(j, y = 1(1)n;\ j \neq y)$ based on the following formula:

$$Supp(\Theta_{ij}, \Theta_{iy}) = 1 - Dist(\Theta_{ij}, \Theta_{iy})\big(j, y = 1(1)n;\ j \neq y\big), \qquad (28)$$

where $Dist(\Theta_{ij}, \Theta_{iy})$ is the Hamming distance between the qROFNs (Liu *et al.*, 2020).

Step 8: Compute the values $\psi(\Theta_{ij})$ using the formula given by:

$$\psi(\Theta_{ij}) = \sum_{y=1, y\neq j}^{n} Supp(\Theta_{ij}, \Theta_{iy})\big(i = 1(1)m;\ j = 1(1)n\big). \qquad (29)$$

Step 9: Calculate the power weights $\Omega_{ij}(i = 1(1)m;\ j = 1(1)n)$ associated with the qROFNs Θ_{ij} by utilizing the weights η_d of DEs $D_d(d = 1(1)l)$, where

$$\Omega_{ij} = \frac{\varpi_j(1 + \psi(\Theta_{ij}))}{\sum_{j=1}^{n}\varpi_j(1 + \psi(\Theta_{ij}))}\big(i = 1(1)m;\ j = 1(1)n\big). \qquad (30)$$

Step 10: Construct the final aggregated qROF decision matrix $\hat{\Im} = [\Theta_i]_{m\times 1} = [\langle \Delta_i, \nabla_i\rangle]_{m\times 1}$.

The final aggregated qROF decision matrix is constructed based on the qROFIPWAMSM or qROFIPWGMSM operator.

$$\Theta_i = \text{qROFIPWAMSM}^{(p)}(\Theta_{i1}, \Theta_{i2}, \dots, \Theta_{in}) = \left(\frac{1}{{}^{n}c_p} \bigoplus_{1 \leqslant t_1 < t_2 < \cdots < t_p \leqslant n} \left(\bigotimes_{j=1}^{p} (n\Omega_{it_j}\Theta_{it_j})\right)\right)^{\frac{1}{p}}, \tag{31}$$

$$\Theta_i = \text{qROFIPWGMSM}^{(p)}(\Theta_{i1}, \Theta_{i2}, \dots, \Theta_{in}) = \frac{1}{p}\Big(\widetilde{\bigotimes_{1 \leqslant t_1 < t_2 < \cdots < t_p \leqslant n}} \Big(\widetilde{\bigoplus_{j=1}^{p}} \Theta_{it_j}^{n\Omega_{it_j}} \Big)\Big)^{\frac{1}{{}^{n}c_p}}. \tag{32}$$

Step 11: Estimate the score values of $\Theta_i (i = (1)m)$ by utilizing Definition 2.

If two score values $S_c(\Theta_i)$ and $S_c(\Theta_u)$ are same, then accuracy values (Definition 3) should be computed.

Step 12: Obtain the priority order of alternatives $A_i (i = 1(1)m)$ indicated by the Definition 4 and subsequently choose the optimal one.

5. Application of the Proposed Methodology

5.1. *Problem Description*

Personnel selection plays a significant role for tracking down the adequate information quality for an organization/industry. Personnel selection is the most common way of picking the people who match the capabilities needed to play out a characterized work in the most ideal manner. A personnel selection problem can be viewed as a MCGDM problem due to the fact that a group of experts and many attributes are considered in the selection process of suitable personnel. qROFS theory can be considered as an essential tool to provide an efficient decision framework to tackle personnel selection problems. Now, let's think about an Engineering Institute (Under Graduate level), which desires to appoint a Placement officer for 'Training and Placement Cell'. Suppose five candidates X_i $(i = 1(1)5)$ are shortlisted for personal interview based on their scores of written tests. A team of three experts (Principal, Director and HR manager) is formed to assess the five candidates on the grounds of industry experience (L_1), communication skill (L_2), networking skill (L_3), and academic qualifications (L_4).

5.2. *Problem Solution*

Step 1: Present the initial assessments of each expert as: $\Im_d = [\Theta_{ij}^{(d)}]_{5\times4} = [\langle \Delta_{ij}^{(d)}, \nabla_{ij}^{(d)} \rangle]_{5\times4} (d = 1(1)3)$ (Table 1).

For each of the remaining steps, $q = 2$ is taken since the least value of q that satisfies $(\Delta_{ij}^{(d)})^q + (\nabla_{ij}^{(d)})^q \leqslant 1$ is '2'.

Step 2: Since all the criteria are of benefit type, normalization is not required. Hence, $\Im_d = [\Theta_{ij}^{(d)}]_{5\times4} = [\langle \Delta_{ij}^{(d)}, \nabla_{ij}^{(d)} \rangle]_{5\times4} = [\langle \tilde{\Delta}_{ij}^{(d)}, \tilde{\nabla}_{ij}^{(d)} \rangle]_{5\times4} = [\tilde{\Theta}_{ij}^{(d)}]_{5\times4} = \tilde{\Im}_d$ $(d = 1(1)l)$.

Table 1
Initial assessment results of the experts.

Expert	Alternative	L_1	L_2	L_3	L_4
D_1	X_1	⟨0.2, 0.6⟩	⟨0.4, 0.6⟩	⟨0.4, 0.3⟩	⟨0.5, 0.6⟩
	X_2	⟨0.6, 0.5⟩	⟨0.6, 0.5⟩	⟨0.5, 0.4⟩	⟨0.5, 0.2⟩
	X_3	⟨0.8, 0.2⟩	⟨0.5, 0.2⟩	⟨0.5, 0.3⟩	⟨0.4, 0.3⟩
	X_4	⟨0.5, 0.6⟩	⟨0.3, 0.5⟩	⟨0.5, 0.2⟩	⟨0.5, 0.2⟩
	X_5	⟨0.5, 0.3⟩	⟨0.4, 0.7⟩	⟨0.6, 0.4⟩	⟨0.6, 0.6⟩
	X_1	⟨0.4, 0.6⟩	⟨0.2, 0.2⟩	⟨0.5, 0.4⟩	⟨0.4, 0.6⟩
D_2	X_2	⟨0.5, 0.1⟩	⟨0.6, 0.4⟩	⟨0.5, 0.5⟩	⟨0.4, 0.3⟩
	X_3	⟨0.7, 0.3⟩	⟨0.4, 0.2⟩	⟨0.4, 0.1⟩	⟨0.5, 0.4⟩
	X_4	⟨0.5, 0.4⟩	⟨0.5, 0.7⟩	⟨0.5, 0.6⟩	⟨0.3, 0.8⟩
	X_5	⟨0.6, 0.4⟩	⟨0.3, 0.3⟩	⟨0.6, 0.3⟩	⟨0.4, 0.2⟩
	X_1	⟨0.7, 0.7⟩	⟨0.5, 0.4⟩	⟨0.2, 0.4⟩	⟨0.4, 0.6⟩
D_3	X_2	⟨0.4, 0.2⟩	⟨0.5, 0.4⟩	⟨0.6, 0.3⟩	⟨0.5, 0.1⟩
	X_3	⟨0.5, 0.3⟩	⟨0.4, 0.2⟩	⟨0.4, 0.3⟩	⟨0.6, 0.4⟩
	X_4	⟨0.3, 0.5⟩	⟨0.5, 0.4⟩	⟨0.5, 0.2⟩	⟨0.8, 0.2⟩
	X_5	⟨0.4, 0.6⟩	⟨0.6, 0.3⟩	⟨0.4, 0.4⟩	⟨0.6, 0.1⟩

Step 3: The supports are calculated as $Supp(\tilde{\Theta}_{ij}^{(d)}, \tilde{\Theta}_{ij}^{(s)})$ $(d, s = 1(1)l; d \neq s)$ using Eq. (23). For sake of simplicity, $S_{ds}(d \neq s; d, s = 1(1)3)$ is used here to represent $Supp(\tilde{\Theta}_{ij}^{(d)}, \tilde{\Theta}_{ij}^{(s)})$ $(d, s = 1(1)l; d \neq s)$ and, consequently, the following matrices are obtained:

$$[S_{12}]_{5\times4} = [S_{21}]_{5\times4} = \begin{bmatrix} 0.88 & 0.56 & 0.84 & 0.91 \\ 0.65 & 0.91 & 0.91 & 0.91 \\ 0.85 & 0.91 & 0.83 & 0.84 \\ 0.8 & 0.6 & 0.68 & 0.4 \\ 0.82 & 0.53 & 0.93 & 0.48 \end{bmatrix},$$

$$[S_{13}]_{5\times4} = [S_{31}]_{5\times4} = \begin{bmatrix} 0.42 & 0.8 & 0.88 & 0.91 \\ 0.59 & 0.8 & 0.89 & 0.97 \\ 0.61 & 0.91 & 0.91 & 0.73 \\ 0.73 & 0.84 & 1 & 0.61 \\ 0.73 & 0.6 & 0.8 & 0.65 \end{bmatrix},$$

$$[S_{23}]_{5\times4} = [S_{32}]_{5\times4} = \begin{bmatrix} 0.54 & 0.67 & 0.79 & 1 \\ 0.91 & 0.89 & 0.84 & 0.91 \\ 0.76 & 1 & 0.92 & 0.89 \\ 0.84 & 0.67 & 0.68 & 0.4 \\ 0.8 & 0.73 & 0.8 & 0.8 \end{bmatrix}.$$

Step 4: According to Eq. (24), the values $\psi(\tilde{\Theta}_{ij}^{(d)})$ $(i = 1(1)5; j = 1(1)4; d = 1(1)3)$ are calculated as given by the following matrices:

$$[\psi(\tilde{\Theta}_{ij}^{(1)})]_{5\times4} = \begin{bmatrix} 1.3 & 1.36 & 1.72 & 1.82 \\ 1.24 & 1.71 & 1.8 & 1.88 \\ 1.46 & 1.82 & 1.74 & 1.57 \\ 1.53 & 1.44 & 1.68 & 1.01 \\ 1.55 & 1.13 & 1.73 & 1.13 \end{bmatrix},$$

$$[\psi(\tilde{\Theta}_{ij}^{(2)})]_{5\times4} = \begin{bmatrix} 1.42 & 1.23 & 1.63 & 1.91 \\ 1.56 & 1.8 & 1.75 & 1.82 \\ 1.61 & 1.91 & 1.75 & 1.73 \\ 1.64 & 1.27 & 1.36 & 0.8 \\ 1.62 & 1.26 & 1.73 & 1.28 \end{bmatrix},$$

$$[\psi(\tilde{\Theta}_{ij}^{(3)})]_{5\times4} = \begin{bmatrix} 0.96 & 1.47 & 1.67 & 1.91 \\ 1.5 & 1.69 & 1.73 & 1.88 \\ 1.37 & 1.91 & 1.83 & 1.62 \\ 1.57 & 1.51 & 1.68 & 1.01 \\ 1.53 & 1.33 & 1.6 & 1.45 \end{bmatrix}.$$

Step 5: According to Eq. (25), the weights $\eta_d (d = 1(1)3)$ of DEs are utilized to calculate the power weights $\Omega_{ij}^{(d)}$ $(i = 1(1)5;\ j = 1(1)4;\ d = 1(1)3)$ associated with the qROFNs and the following matrices are obtained:

$$[\Omega_{ij}^{(1)}]_{5\times4} = \begin{bmatrix} 0.355722 & 0.353671 & 0.356354 & 0.342887 \\ 0.322236 & 0.346041 & 0.354751 & 0.352941 \\ 0.344746 & 0.342887 & 0.346647 & 0.339883 \\ 0.342685 & 0.357397 & 0.367555 & 0.365265 \\ 0.346871 & 0.334005 & 0.354217 & 0.328414 \end{bmatrix},$$

$$[\Omega_{ij}^{(2)}]_{5\times4} = \begin{bmatrix} 0.427751 & 0.381931 & 0.393786 & 0.404378 \\ 0.420879 & 0.408609 & 0.398190 & 0.394958 \\ 0.418018 & 0.404378 & 0.397614 & 0.412620 \\ 0.408669 & 0.379996 & 0.369906 & 0.373832 \\ 0.407307 & 0.405018 & 0.404819 & 0.401762 \end{bmatrix},$$

$$[\Omega_{ij}^{(3)}]_{5\times4} = \begin{bmatrix} 0.216527 & 0.264397 & 0.249859 & 0.252736 \\ 0.256884 & 0.245348 & 0.247059 & 0.252101 \\ 0.237237 & 0.252736 & 0.255738 & 0.247497 \\ 0.248645 & 0.262607 & 0.262539 & 0.260903 \\ 0.245822 & 0.260977 & 0.240964 & 0.269824 \end{bmatrix}.$$

Step 6: According to the qROFIPWA operator expressed by Eq. (26), the matrices $\tilde{\Im}_d = [\tilde{\Theta}_{ij}^{(d)}]_{5\times4} (d = 1(1)3)$ are aggregated (taking $q = 2$) to form an integrated decision matrix $\Im* = [\Theta_{ij}]_{5\times4}$, as shown in the following Table 2.

Step 7: The supports are calculated as $Supp(\Theta_{ij}, \Theta_{iy})(j, y = 1(1)4; (j \neq y)$ using Eq. (28). For sake of simplicity, the symbol $S^{jy} (j \neq y;\ j, y = 1(1)4)$ is used to represent

Table 2
Aggregated normalized decision matrix.

Alternative	L_1	L_2	L_3	L_4
X_1	⟨0.457546, 0.727680⟩	⟨0.377018, 0.456022⟩	⟨0.411592, 0.370298⟩	⟨0.438196, 0.600809⟩
X_2	⟨0.516059, 0.345696⟩	⟨0.578450, 0.442128⟩	⟨0.528034, 0.422266⟩	⟨0.464399, 0.226244⟩
X_3	⟨0.710191, 0.264441⟩	⟨0.438196, 0.200154⟩	⟨0.438588, 0.246261⟩	⟨0.500551, 0.375744⟩
X_4	⟨0.461582, 0.510394⟩	⟨0.443096, 0.594864⟩	⟨0.5, 0.425597⟩	⟨0.578171, 0.513749⟩
X_5	⟨0.526362, 0.432299⟩	⟨0.437910, 0.502261⟩	⟨0.562724, 0.362961⟩	⟨0.534898, 0.427268⟩

$Supp(\Theta_{ij}, \Theta_{iy})$ $(j, y = 1(1)4;\ j \neq y)$ and the following values are obtained:

$$S^{12} = S^{21} = (0.612624, 0.784901, 0.535690, 0.842471, 0.909825),$$
$$S^{13} = S^{31} = (0.639889, 0.840685, 0.556273, 0.931132, 0.815082),$$
$$S^{14} = S^{41} = (0.662497, 0.784869, 0.565558, 0.926220, 0.896442),$$
$$S^{23} = S^{32} = (0.934186, 0.843704, 0.847702, 0.842471, 0.815082),$$
$$S^{24} = S^{42} = (0.781172, 0.716582, 0.789510, 0.842471, 0.896442),$$
$$S^{34} = S^{43} = (0.808437, 0.772366, 0.810093, 0.846851, 0.815082).$$

Step 8: According to Eq. (29) the values $\psi(\Theta_{ij})$ $(i = 1(1)5;\ j = 1(1)4)$ are calculated, as presented in the following matrix:

$$\psi = \begin{bmatrix} 1.915011 & 2.327982 & 2.382513 & 2.252106 \\ 2.410456 & 2.345187 & 2.456756 & 2.273818 \\ 1.657522 & 2.172903 & 2.214069 & 2.165161 \\ 2.699824 & 2.527413 & 2.620454 & 2.615542 \\ 2.621349 & 2.621349 & 2.445245 & 2.607966 \end{bmatrix}.$$

Step 9: The power weights Ω_{ij} $(i = 1(1)5;\ j = 1(1)4)$ are calculated using Eq. (30). These values are presented in the following matrix:

$$\Omega = \begin{bmatrix} 0.1804 & 0.1030 & 0.3140 & 0.4026 \\ 0.2028 & 0.0995 & 0.3083 & 0.3894 \\ 0.1726 & 0.1030 & 0.3131 & 0.4112 \\ 0.2041 & 0.0973 & 0.2996 & 0.3990 \\ 0.2033 & 0.1016 & 0.2901 & 0.4050 \end{bmatrix}.$$

Step 10: Based on the qROFIPWAMSM operator expressed by Eq. (31), the final aggregated qROFNs are derived (taking $q = 2$, $r = 2$), as given by:

$$\Theta_1 = \langle 0.44483774, 0.564039033\rangle,$$
$$\Theta_2 = \langle 0.509610528, 0.351422694\rangle,$$
$$\Theta_3 = \langle 0.532907336, 0.302081533\rangle,$$

$$\Theta_4 = \langle 0.531043417, 0.491528853\rangle,$$
$$\Theta_5 = \langle 0.536489152, 0.414240749\rangle.$$

Step 11: The scores $V_i = V(\Theta_i)(i = 1(1)5)$ are calculated by utilizing Eq. (1), as follows:

$$V_1 = -0.1202, \quad V_2 = 0.1362, \quad V_3 = 0.1927, \quad V_4 = 0.0404, \quad V_5 = 0.1162.$$

Step 12: Since $V_3 > V_2 > V_5 > V_4 > V_1$, the priority order is $X_3 \succ X_2 > X_5 > X_4 > X_2$, hence, the most suitable alternative is X_3.

If the proposed qROFIPWG operator is applied in Step 6 and the proposed qROFIPWGMSM operator is applied in Step 10, then the following values are obtained:

$$V_1 = -0.0788, \quad V_2 = 0.1435, \quad V_3 = 0.1823, \quad V_4 = -0.0452, \quad V_5 = 0.1156.$$

Since $V_3 > V_2 > V_5 > V_4 > V_1$, the priority order of the alternatives is $X_3 \succ X_2 > X_5 > X_4 > X_2$, hence, the optimal choice is X_3.

5.3. *Effects of the Parameter ‘p’ on Ranking Orders*

Here, all possible values of p are considered in the proposed MCGDM technique to get the solution of the case study, as discussed in Section 5.1 (taking $q = 2$). To illustrate the impact of ‘p’ upon priority order, qROFIPWA operator is used in Step 6 and the proposed qROFIPWAMSM operator is used in Step 10. The related score values of alternatives and their priority position for various values of ‘p’ (taking $q = 2$) are presented in Table 3. To illustrate the effect of ‘p’ upon priority order, qROFIPWG operator is utilized in Step 6 and the proposed qROFIPWGMSM operator is used in Step 10. The related score values of alternatives and their priority position for various values of ‘p’ (taking $q = 2$) are presented in Table 4. With the increasing value of p increases, the priority order of alternatives changes in couple of cases due to the fact that the developed methodology considers interrelationships among criteria, but the best alternative (A_3) remains unaltered for any value of p when $q = 2$. For the case study presented in Section 5.1, four criteria are considered. So, maximum possible integral value of p is 4. When $p = 1$, all the criteria are independent. For $p = 2$, pairs of criteria are dependent, and for $p = 3$, any of the three criteria will be interrelated. But for $p = 4$, all the four criteria will be dependent. Depending on the given number of dependent criteria, expert/decision-maker will choose appropriate value of the parameter p.

Table 3
Effects of the parameter p when the operators qROFIPWA and qROFIPWAMSM are used.

Parameter	Score value	Ranking order
$p = 1$	$V_1 = -0.1497$, $V_2 = 0.1332$, $V_3 = 0.1849$, $V_4 = 0.0222$, $V_5 = 0.1092$	$X_3 \succ X_2 > X_5 > X_4 > X_2$
$p = 2$	$V_1 = -0.1202$, $V_2 = 0.1362$, $V_3 = 0.1927$, $V_4 = 0.0404$, $V_5 = 0.1162$	$X_3 \succ X_2 > X_5 > X_4 > X_2$
$p = 3$	$V_1 = -0.1040$, $V_2 = 0.1388$, $V_3 = 0.2002$, $V_4 = 0.0579$, $V_5 = 0.1242$	$X_3 \succ X_2 > X_5 > X_4 > X_2$
$p = 4$	$V_1 = -0.0641$, $V_2 = 0.0307$, $V_3 = 0.0484$, $V_4 = -0.0109$, $V_5 = 0.0199$	$X_3 \succ X_2 > X_5 > X_4 > X_2$

Table 4
Effects of the parameter p when the operators qROFIPWG and qROFIPWGMSM are used.

Parameter	Score value	Ranking order
$p = 1$	$V_1 = 0.0173$, $V_2 = 0.0043$, $V_3 = 0.0014$, $V_4 = 0.0020$, $V_5 = -0.0017$	$X_1 \succ X_2 > X_4 > X_3 > X_5$
$p = 2$	$V_1 = -0.0788$, $V_2 = 0.1435$, $V_3 = 0.1823$, $V_4 = -0.0452$, $V_5 = 0.1155$	$X_3 \succ X_2 > X_5 > X_4 > X_2$
$p = 3$	$V_1 = -0.1772$, $V_2 = 0.0349$, $V_3 = 0.1132$, $V_4 = -0.1631$, $V_5 = 0.0176$	$X_3 \succ X_2 > X_5 > X_4 > X_2$
$p = 4$	$V_1 = -0.1280$, $V_2 = 0.0609$, $V_3 = 0.0883$, $V_4 = -0.0983$, $V_5 = 0.0538$	$X_3 \succ X_2 > X_5 > X_4 > X_2$

5.4. *Comparative Analysis with Existing Methods*

To verify the effectiveness of our developed methodology based on the developed operators, an investigation has been conducted for the purpose of comparison between the existing methods of Jana *et al.* (2019b) and qROF Dombi weighted averaging (qROFDWA) operator; Wei *et al.* (2018) and qROF generalized weighted Heronian mean (qROFGWHM) operator and qROF generalized weighted geometric Heronian mean (qROFGWGHM) operator; Liu and Liu (2018) and qROF weighted Bonferroni mean (qROFWBM) operator; Yang and Pang (2020) and qROF weighted Bonferroni Dombi averaging (qROFWBMDA) operator; Liu and Wang (2018) and qROF weighted averaging (qROFWA) operator; Garg and Chen (2020) and qROF weighted neutrality (qROFWN) operator, and Liu *et al.* (2020) with qROF power weighted MSM (qROFPWMSM) operator. These methods are applied to the same case study presented at the beginning of Section 5. In Table 5, priority values of the considered alternatives are presented along with their ranking order. From Table 5, it is found that the ranking order obtained by our proposed method is exactly the same as obtained by other existing methods (Liu and Liu, 2018; Liu and Wang, 2018; Wei *et al.*, 2018; Jana *et al.*, 2019b; Yang and Pang, 2020; Garg and Chen, 2020; Liu *et al.*, 2020). Hence, the developed methodology based on the proposed operators is effective and feasible.

5.5. *Comparative Analysis Based on Biasness of Experts*

When evaluating alternatives in a realistic decision-making environment, experts may attempt to manipulate some initial data due to an inclination or biasness toward a particular alternative. As a result, the ranking order of alternatives may change. To reflect the actual situation, the case study presented in Section 5.1 must be modified in order to demonstrate the biased nature of experts. Assume that expert D_2 prefers alternative X_2 and has some reservations about alternative X_3 to an extent that the criteria value $\tilde{\Theta}_{31}^{(2)}$ changes to $\langle 0.1, 0.1\rangle$ from $\langle 0.7, 0.3\rangle$ and the criteria value $\tilde{\Theta}_{24}^{(2)}$ changes to $\langle 0.4, 0.2\rangle$ from $\langle 0.4, 0.3\rangle$ due to biased nature of the expert D_2. The remaining assessment values remain the same as shown in Tables 1. The outcomes from various existing methods are recorded in Table 6 (for $q = 2$).

Table 6 shows that changing the criteria values has a significant effect on the ranking order of alternatives for the related existing methods (Liu and Wang, 2018; Wei *et al.*, 2018; Jana *et al.*, 2019b; Garg and Chen, 2020). The priority order of alternatives acquired by Jana *et al.* (2019b) with qROFDWA operator is changed from $X_3 \succ X_2 > X_5 > X_4 > X_2$ to $X_2 \succ X_3 > X_5 > X_1 > X_4$ such that best alternative is transformed from the

Table 5
Comparison: existing vs. proposed (taking $q = 2$).

Method	Score value	Ranking order
Jana *et al.* (2019b) with qROFDWA operator	$V_1 = -0.0212$, $V_2 = 0.1885$, $V_3 = 0.2743$, $V_4 = 0.0522$, $V_5 = 0.1233$	$X_3 \succ X_2 \succ X_5 \succ X_4 \succ X_1$
Wei *et al.* (2018) with qROFGWHM operator	$V_1 = -0.2942$, $V_2 = -0.0703$, $V_3 = -0.0272$, $V_4 = -0.1913$, $V_5 = -0.1324$	$X_3 \succ X_2 \succ X_5 \succ X_4 \succ X_1$
Wei *et al.* (2018) with qROFGWGHM operator	$V_1 = 0.0884$, $V_2 = 0.3162$, $V_3 = 0.3317$, $V_4 = 0.2660$, $V_5 = 0.3294$	$X_3 \succ X_2 \succ X_5 \succ X_4 \succ X_1$
Liu and Liu (2018) with qROFWBM operator	$V_1 = -0.7144$, $V_2 = -0.5922$, $V_3 = -0.5780$, $V_4 = -0.6416$, $V_5 = -6163$	$X_3 \succ X_2 \succ X_5 \succ X_4 \succ X_1$
Yang and Pang (2020) with qROFWBMDA operator	$V_1 = -0.4993$, $V_2 = -0.2204$, $V_3 = -0.0846$, $V_4 = -0.4227$, $V_5 = -0.2986$	$X_3 \succ X_2 \succ X_5 \succ X_4 \succ X_1$
Liu and Wang (2018) with qROFWA operator	$V_1 = -0.0919$, $V_2 = 0.1568$, $V_3 = 0.2051$, $V_4 = 0.0309$, $V_5 = 0.1130$	$X_3 \succ X_2 \succ X_5 \succ X_4 \succ X_1$
Garg and Chen (2020) with qROFWNA operator	$V_1 = 0.5931$, $V_2 = 0.7337$, $V_3 = 0.7525$, $V_4 = 0.6783$, $V_5 = 0.7411$	$X_3 \succ X_2 \succ X_5 \succ X_4 \succ X_1$
Liu *et al.* (2020) with qROFPWMSM operator	$V_1 = -0.1154$, $V_2 = 0.1143$, $V_3 = 0.1656$, $V_4 = 0.0348$, $V_5 = 0.0934$	$X_3 \succ X_2 \succ X_5 \succ X_4 \succ X_1$
Proposed method with qROFIPWA operator and qROFIPWAMSM operator	$V_1 = -0.1202$, $V_2 = 0.1362$, $V_3 = 0.1927$, $V_4 = 0.0404$, $V_5 = 0.1162$	$X_3 \succ X_2 \succ X_5 \succ X_4 \succ X_1$
Proposed method with qROFIPWG operator and qROFIPWGMSM operator	$V_1 = -0.0788$, $V_2 = 0.1435$, $V_3 = 0.1823$, $V_4 = -0.0452$, $V_5 = 0.1156$	$X_3 \succ X_2 \succ X_5 \succ X_4 \succ X_1$

Table 6
Comparison: existing vs. proposed (taking $q = 2$).

Method	Score value	Ranking order
Jana *et al.* (2019b) with qROFDWA operator	$V_1 = -0.0343$, $V_2 = 0.2170$, $V_3 = 0.2042$, $V_4 = 0.0133$, $V_5 = 0.1445$	$X_2 \succ X_3 \succ X_5 \succ X_1 \succ X_4$
Wei *et al.* (2018) with qROFGWHM operator	$V_1 = -0.2538$, $V_2 = -0.0407$, $V_3 = -0.0642$, $V_4 = -0.2449$, $V_5 = -0.1066$	$X_2 \succ X_3 \succ X_5 \succ X_4 \succ X_1$
Wei *et al.* (2018) with qROFGWGHM operator	$V_1 = 0.1324$, $V_2 = 0.3172$, $V_3 = 0.3312$, $V_4 = 0.2084$, $V_5 = 0.3526$	$X_5 \succ X_3 \succ X_2 \succ X_4 \succ X_1$
Liu and Liu (2018) with qROFWBM operator	$V_1 = -0.7144$, $V_2 = -0.5845$, $V_3 = -0.5640$, $V_4 = -0.6416$, $V_5 = -6163$	$X_3 \succ X_2 \succ X_5 \succ X_4 \succ X_1$
Yang and Pang (2020) with qROFWBMDA operator	$V_1 = -0.4570$, $V_2 = -0.1966$, $V_3 = -0.0703$, $V_4 = -0.44442$, $V_5 = -0.2833$	$X_3 \succ X_2 \succ X_5 \succ X_4 \succ X_1$
Liu and Wang (2018) with qROFWA operator	$V_1 = -0.0405$, $V_2 = 0.1789$, $V_3 = 0.1774$, $V_4 = -0.0122$, $V_5 = 0.1350$	$X_2 \succ X_3 \succ X_5 \succ X_4 \succ X_1$
Garg and Chen (2020) with qROFWNA operator	$V_1 = 0.5930$, $V_2 = 0.7395$, $V_3 = 0.7113$, $V_4 = 0.6782$, $V_5 = 0.7410$	$X_5 \succ X_3 \succ X_2 \succ X_4 \succ X_1$
Liu *et al.* (2020) with qROFPWMSM operator	$V_1 = -0.1154$, $V_2 = 0.1161$, $V_3 = 0.1556$, $V_4 = 0.0348$, $V_5 = 0.0934$	$X_3 \succ X_2 \succ X_5 \succ X_4 \succ X_1$
Proposed method with qROFIPWA operator and qROFIPWAMSM operator	$V_1 = -0.1202$, $V_2 = 0.1422$, $V_3 = 0.1645$, $V_4 = 0.0404$, $V_5 = 0.1162$	$X_3 \succ X_2 \succ X_5 \succ X_4 \succ X_1$
Proposed method with qROFIPWG operator and qROFIPWGMSM operator	$V_1 = -0.0788$, $V_2 = 0.1523$, $V_3 = 0.1562$, $V_4 = -0.0452$, $V_5 = 0.1155$	$X_3 \succ X_2 \succ X_5 \succ X_4 \succ X_1$

Table 7
Initial assessment results of the experts.

Expert	Alternative	L_1	L_2	L_3	L_4
D_1	X$_1$	⟨0.5, 0⟩	⟨0.4, 0.4⟩	⟨0.4, 0.3⟩	⟨0.5, 0⟩
	X_2	⟨0.7, 0.2⟩	⟨0.51, 0⟩	⟨0.5, 0.1⟩	⟨0.6, 0.2⟩
	X_3	⟨0.3, 0.2⟩	⟨0.38, 0.4⟩	⟨0.6, 0⟩	⟨0.4, 0.3⟩
	X_4	⟨0.5, 0⟩	⟨0.502, 0.4⟩	⟨0.5, 0.3⟩	⟨0.5, 0⟩
	X_5	⟨0.6, 0.1⟩	⟨0.3, 0⟩	⟨0.6, 0.2⟩	⟨0.6, 0.3⟩
	X_1	⟨0.5, 0.3⟩	⟨0.5, 0⟩	⟨0.6, 0.3⟩	⟨0.6, 0.3⟩
D_2	X_2	⟨0.6, 0⟩	⟨0.6, 0.3⟩	⟨0.206, 0.2⟩	⟨0.4, 0⟩
	X_3	⟨0.4, 0.3⟩	⟨0.7, 0⟩	⟨0.35, 0.2⟩	⟨0.6, 0.4⟩
	X_4	⟨0.45, 0.4⟩	⟨0.5, 0.2⟩	⟨0.6, 0⟩	⟨0.5, 0.2⟩
	X_5	⟨0.4, 0⟩	⟨0.6, 0.3⟩	⟨0.4, 0.4⟩	⟨0.6, 0⟩
	X_1	⟨0.5, 0.3⟩	⟨0.6, 0.3⟩	⟨0.5, 0⟩	⟨0.6, 0.3⟩
D_3	X_2	⟨0.6, 0.1⟩	⟨0.49, 0.2⟩	⟨0.7, 0⟩	⟨0.2, 0.2⟩
	X_3	⟨0.7, 0⟩	⟨0.6, 0.2⟩	⟨0.71, 0.4⟩	⟨0.4, 0⟩
	X_4	⟨0.5, 0.3⟩	⟨0.7, 0⟩	⟨0.5, 0.2⟩	⟨0.6, 0.1⟩
	X_5	⟨0.62, 0.3⟩	⟨0.6, 0.3⟩	⟨0.3, 0⟩	⟨0.4, 0.2⟩

alternative X$_3$ to the alternative X$_2$. The best alternative is changed from the alternative X$_3$ to the alternative X$_2$ and the ranking order is changed from $X_3 \succ X_2 > X_5 > X_4 > X_2$ to $X_2 \succ X_3 > X_5 > X_4 > X_1$ if the approach of Wei *et al.* (2018) is used with qROFGWHM operator. In addition, the priority order generated by Wei *et al.* (2018) with qROFGWGHM operator is changed from $X_3 \succ X_2 > X_5 > X_4 > X_2$ to $X_5 \succ X_3 > X_2 > X_4 > X_1$ and the best alternative is transformed from the alternative X$_3$ to the alternative X$_5$. The best alternative changes from X$_3$ to X$_2$ and the related ranking output changes from $X_3 \succ X_2 > X_5 > X_4 > X_2$ to $X_2 \succ X_3 > X_5 > X_4 > X_1$ when the method of Liu and Wang (2018) is utilized with qROFWA operator. Moreover, the ranking order generated by Garg and Chen (2020) with qROFNWA operator changes from $X_3 \succ X_2 > X_5 > X_4 > X_2$ to $X_5 \succ X_3 > X_2 > X_4 > X_1$, and the best alternative changes from X$_3$ to X$_5$. Thus the existing methods (Wei *et al.*, 2018; Liu and Wang, 2018; Jana *et al.*, 2019b; Garg and Chen, 2020) are unreasonable for the reason that the best alternative changes due to the biased nature of the expert D_2. However, the methods of Liu and Liu (2018) with qROFWBM operator, Yang and Pang (2020) with qROFWBMDA operator and Liu *et al.* (2020) with qROFPWMSM) operator and the developed approach still have rational and unaltered ranking of the alternatives. The methods of Liu and Liu (2018), Yang and Pang (2020), Liu *et al.* (2020) and the developed approach can diminish the impact of unreasonable assessment criteria values from a biased expert.

To show the disadvantages of the methods of Liu and Liu (2018), Yang and Pang (2020), Liu *et al.* (2020), the same case study is considered with the initial assessment matrix, as given in Table 7. The ranking outcomes are presented in Table 8. From this table, it follows that the priority order of alternatives, as given by Liu and Liu (2018), are unreasonable due to the fact that they fail to distinguish the priority of alternatives. The method of Yang and Pang (2020) and the proposed approach continue to have a reasonable ranking, and the best alternative remains the same for these two approaches. This implies

Table 8
Comparison: proposed vs. existing methods (Liu and Liu, 2018; Liu *et al.*, 2020; Yang and Pang, 2020) (taking $q = 2$).

Method	Score value	Ranking order
Liu and Liu (2018) with qROFWBM operator	$V_1 = 0.019$, $V_2 = 0.019$, $V_3 = 0.019$, $V_4 = 0.019$, $V_5 = 0.019$	$X_1 = X_2 = X_3 = X_4 = X_5$
Yang and Pang (2020) with qROFWBMDA operator	$V_1 = -0.0749$, $V_2 = 0.0459$, $V_3 = -0.0633$, $V_4 = -0.0262$, $V_5 = -0.0352$	$X_2 \succ X_4 \succ X_5 \succ X_3 \succ X_1$
Liu *et al.* (2020) with qROFPWMSM operator	$V_1 = 0.264$, $V_2 = 0.264$, $V_3 = 0.264$, $V_4 = 0.264$, $V_5 = 0.264$	$X_1 = X_2 = X_3 = X_4 = X_5$
Proposed method with qROFIPWA operator and qROFIPWAMSM operator	$V_1 = 0.2204$, $V_2 = 0.2458$, $V_3 = 0.2010$, $V_4 = 0.2327$, $V_5 = 0.2278$	$X_2 \succ X_4 \succ X_5 \succ X_1 \succ X_3$
Proposed method with qROFIPWG operator and qROFIPWGMSM operator	$V_1 = 0.2165$, $V_2 = 0.2463$, $V_3 = 0.2031$, $V_4 = 0.2297$, $V_5 = 0.2257$	$X_2 \succ X_4 \succ X_5 \succ X_1 \succ X_3$

Table 9
Initial assessment results of the experts.

Expert	Alternative	L_1	L_2	L_3	L_4
D_1	X_1	$\langle 1, 0\rangle$	$\langle 0.4, 0.4\rangle$	$\langle 0.4, 0.3\rangle$	$\langle 1, 0\rangle$
	X_2	$\langle 0.7, 0.2\rangle$	$\langle 0.1, 0\rangle$	$\langle 0.5, 0.1\rangle$	$\langle 0.6, 0.2\rangle$
	X_3	$\langle 1, 0\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 1, 0\rangle$	$\langle 0.4, 0.3\rangle$
	X_4	$\langle 0.5, 0.3\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.5, 0.3\rangle$	$\langle 1, 0\rangle$
	X_5	$\langle 0.6, 0.1\rangle$	$\langle 1, 0\rangle$	$\langle 0.6, 0.2\rangle$	$\langle 0.6, 0.3\rangle$
	X_1	$\langle 0.5, 0.3\rangle$	$\langle 1, 0\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0.6, 0.3\rangle$
D_2	X_2	$\langle 1, 0\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 1, 0\rangle$	$\langle 0.4, 0.5\rangle$
	X_3	$\langle 0.4, 0.3\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.4, 0.2\rangle$	$\langle 1, 0\rangle$
	X_4	$\langle 0.5, 0.4\rangle$	$\langle 1, 0\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0.5, 0.4\rangle$
	X_5	$\langle 1, 0\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 1, 0\rangle$	$\langle 0.6, 0.4\rangle$
	X_1	$\langle 0.5, 0.3\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 1, 0\rangle$	$\langle 0.6, 0.3\rangle$
D_3	X_2	$\langle 1, 0\rangle$	$\langle 0.5, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 1, 0\rangle$
	X_3	$\langle 0.7, 0.2\rangle$	$\langle 1, 0\rangle$	$\langle 0.8, 0.2\rangle$	$\langle 0.4, 0.5\rangle$
	X_4	$\langle 1, 0\rangle$	$\langle 0.7, 0.1\rangle$	$\langle 1, 0\rangle$	$\langle 0.6, 0.1\rangle$
	X_5	$\langle 0.6, 0.3\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0.3, 0.5\rangle$	$\langle 1, 0\rangle$

that both approaches are capable of mitigating the effects of unreasonable assessment criteria values provided by a biased expert.

The same case study was examined again using the initial assessment matrix, as shown in Table 9. Table 10 exhibits the ranking results. According to Table 10, the ranking results of the alternatives obtained by Liu *et al.* (2020) are unreasonable because they fail to distinguish the priority of alternatives. The results obtained by Yang and Pang (2020) and using the proposed approach still have a reasonable ranking and the best alternative does not change for these two approaches. This suggests that both the approaches can mitigate the effects of unreasonable assessment criteria values from a biased expert.

Table 10
Comparison: proposed vs. existing methods (Liu *et al.*, 2020; Yang and Pang, 2020) (taking $q = 2$).

Method	Score value	Ranking order
Yang and Pang (2020) with qROFWBMDA operator	$V_1 = 0.6133$, $V_2 = 0.8372$, $V_3 = 0.6602$, $V_4 = 0.5890$, $V_5 = 0.6265$	$X_2 \succ X_3 \succ X_5 \succ X_1 \succ X_4$
Liu *et al.* (2020) with qROFPWMSM operator	$V_1 = V_2 = V_3 = V_4 = V_5 = 1.$	$X_1 = X_2 = X_3 = X_4 = X_5$
Proposed method with qROFIPWG operator and qROFIPWGMSM operator	$V_1 = 0.5623$, $V_2 = 0.7078$, $V_3 = 0.6156$, $V_4 = 0.5533$, $V_5 = 0.5863$	$X_2 \succ X_3 \succ X_5 \succ X_1 \succ X_4$

Table 11
Initial assessment results.

Alternative	L_1	L_2	L_3	L_4
X_1	$\langle 0.5, 0\rangle$	$\langle 0.4, 0.4\rangle$	$\langle 0.4, 0.3\rangle$	$\langle 0.5, 0.6\rangle$
X_2	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0\rangle$	$\langle 0.5, 0.1\rangle$	$\langle 0.6, 0.4\rangle$
X_3	$\langle 0.3, 0.6\rangle$	$\langle 0.3, 0.4\rangle$	$\langle 0.6, 0\rangle$	$\langle 0.4, 0.3\rangle$
X_4	$\langle 0.5, 0\rangle$	$\langle 0.7, 0.5\rangle$	$\langle 0.5, 0.3\rangle$	$\langle 0.5, 0.8\rangle$
X_5	$\langle 0.6, 0.5\rangle$	$\langle 0.3, 0\rangle$	$\langle 0.6, 0.4\rangle$	$\langle 0.6, 0.3\rangle$

Table 12
Comparison: proposed vs. Yang and Peng's method (Yang and Pang, 2020) (taking $q = 2$).

Method	Score value	Ranking order
Yang and Pang (2020) with qROFWBMDA operator	Cannot be determined due to division by zero	Cannot be generated
Proposed method with qROFIPWA operator and qROFIPWAMSM operator	$V_1 = 0.0443$, $V_2 = 0.2818$, $V_3 = 0.0709$, $V_4 = 0.0453$, $V_5 = 0.2666$	$X_2 \succ X_5 \succ X_3 \succ X_4 \succ X_1$
Proposed method with qROFIPWG operator and qROFIPWGMSM operator	$V_1 = -0.0014$, $V_2 = 0.2545$, $V_3 = 0.0589$, $V_4 = -0.0926$, $V_5 = 0.1826$	$X_2 \succ X_5 \succ X_3 \succ X_1 \succ X_4$

It is known that when all experts give the same assessment values and if all the experts have the same importance, then a MCGDM problem reduces to a MCDM problem. Suppose the initial assessment matrix for the case study is given in Table 11. Table 12 shows the scores and priorities of the alternatives. The results revealed that the method of Yang and Pang (2020) failed to generate score values and alternative preference order, as well as to solve certain decision-making problems, rendering it inefficient. However, the developed methodology is capable of producing accurate ranking of alternatives.

In real decision-making problems, the interrelationship between criteria can be seen. Methods of Wei *et al.* (2018), Liu and Liu (2018) and Yang and Pang (2020) can consider the dependency of two criteria, but they do not consider interrelationships between multiple criteria. There may be a situation in which all of the considered criteria are independent, and these methods are not appropriate for dealing with this type of decision-making problem, and may generate irrational preference of alternatives. Although the method of Garg and Chen (2020) is capable of mitigating the impact of some unreasonable assessing criteria values from some biased decision-makers and taking into account the dependency among multiple criteria, it fails to distinguish the priority orders of alternatives in some circumstances, as shown in Tables 7 and 8, respectively.

6. Conclusions

This paper presents a qROFS-based decision-making model to resolve the drawbacks of the existing methods. To develop the model, four operators, namely qROFIPWA, qROFIPWG, qROFIPWAMSM and qROFIPWGMSM, are proposed in this paper. The main advantages of the last two operators are: (1) they reduce the effects of outrageous assessing information from some biased experts, (ii) they consider the interrelationship among multiple number of criteria. A group decision-making methodology is developed based on these operators. The developed method can generate sensible ranking order of alternatives when among the qROF numbers considered, one qROF number has a (i) non-belongingness grade that equals to 0, or a (ii) belongingness grade that equals to 1. For the verification of feasibility of the proposed MCGDM method, one case study regarding personnel selection is considered. The superiority of the developed MCGDM approach is shown by comparison with existing approaches. The proposed method has two limitations: (i) it does not address the process of reaching consensus for large-scale decision-making, and (ii) it does not address the hesitancy of choosing membership and non-membership values. To address these issues, hesitant q-ROF based large scale decision-making with consensus reaching process can be developed in the future by extending the proposed operators. The proposed methodology can also be used to solve other decision-making problems and can be further extended by incorporating hesitant, probabilistic hesitant, linguistic, and probabilistic linguistic concepts.

Supplementary Material

Proof of Theorem 6, which we provided you in the main manuscript.

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