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Improved CODAS Method Under Picture 2-Tuple Linguistic Environment and Its Application for a Green Supplier Selection

Siqi ZHANG[1], Guiwu WEI[1,*], Rui WANG[1], Jiang WU[2], Cun WEI[2], Yanfeng GUO[3], Yu WEI[4]

[1] *School of Business, Sichuan Normal University, Chengdu, 610101, PR China*
[2] *School of Statistics, Southwestern University of Finance and Economics, Chengdu, 611130, PR China*
[3] *School of Finance, Southwestern University of Finance and Economics, Chengdu, 610074, PR China*
[4] *School of Finance, Yunnan University of Finance and Economics, Kunming 650221, PR China*
e-mail: weiguiwu@163.com

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Abstract. In this paper, the CODAS (Combinative Distance-based Assessment) is utilized to address some MAGDM issues by using picture 2-tuple linguistic numbers (P2TLNs). At first, some essential concepts of picture 2-tuple linguistic sets (P2TLSs) are briefly reviewed. Then, the CODAS method with P2TLNs is constructed and all calculating procedures are simply depicted. Eventually, an empirical application of green supplier selection has been offered to demonstrate this novel method and some comparative analysis between the CODAS method with P2TLNs and several methods are also made to confirm the merits of the developed method.

Key words: Multiple attribute group decision making (MAGDM), picture fuzzy sets (PFSs), picture 2-tuple linguistic sets (P2TLSs), the CODAS method; green supplier selection.

1. Introduction

The idea of intuitionistic fuzzy sets (IFSs) was initially developed by Atanassov (1986) to generalize the notion of fuzzy set (Zadeh, 1965). Zhou *et al.* (2019) defined the normalized weighted Bonferroni Harmonic mean-based intuitionistic fuzzy operators for sustainable selection of search and rescue robots. There were two described variables in IFSs, including the degrees of membership and non-membership. In terms of IFSs, Atanassov and Gargov (1989) and Atanassov (1994) gave the theory of interval-valued intuitionistic fuzzy sets (IVIFSs) which can describe fuzzy numbers more exactly and reasonably. Lu and Wei (2019) designed the TODIM method for performance appraisal on social-integration-based rural reconstruction under IVIFSs. Wu *et al.* (2019a) gave the VIKOR method for financing risk assessment of rural tourism projects under IVIFSs. Wu *et al.*

*Corresponding author.

(2020) proposed some interval-valued intuitionistic fuzzy Dombi Heronian mean operators for evaluating the ecological value of forest ecological tourism demonstration areas. Wu *et al.* (2019b) designed the algorithms for competitiveness evaluation of tourist destination with some interval-valued intuitionistic fuzzy Hamy mean operators. However, in reality, there exist some particular situations when the neutral membership degree is needed to be calculated independently. Thus, to conquer this defect and obtain more rigorous information, Cuong and Kreinovich (2015) initiated the theory of picture fuzzy sets (PFSs) which took another described variable (neutral membership) into consideration. There are three described variables in PFSs which are the degrees of membership, neutral membership and non-membership. The only condition that must be fulfilled is that the three described variables' sum cannot exceed 1. As a powerful tool, PFSs deliver more comprehensive information which the application of some particular situations required more answer types of human ideas: yes, abstain, no, refusal. Cuong *et al.* (2015) found PFSs' main logic operators and developed the main operations of reasoning process in PFSs by linking the triple picture fuzzy operators of De Morgan. Garg (2017) investigated several PFSs' aggregation operators, including PFWA, PFOWA and PFMA aggregation operators. Xu *et al.* (2018) combined Muirhead mean (MM) operator with PFSs to develop PFMM operator and created a novel method which can be widely applied in attribute values to address MADM issues. Zhang *et al.* (2018a) found several novel operational rules of PFSs relying on Dombi t-conorm and t-norm (DTT) and made use of the information aggregation technology of Heronian mean (HM) to integrate PFNs. Jana *et al.* (2018) put forward a model which was related to picture fuzzy Dombi aggregation operators to address MADM issues in an updated way. Wei (2016) gave the notion about picture fuzzy cross entropy and established the entropy of the alternative attribute value of PFNs. Joshi and Kumar (2018) pointed out an approach for MADM issues derived from the Dice similarity and weighted Dice similarity measures for PFSs. Son (2017) extended the fundamental distance measure in PFSs to the generalized picture distance measures and picture association measures. Liu *et al.* (2019) explored some distance measures for PFSs and proposed Picture fuzzy ordered weighted distance measure and Picture fuzzy hybrid weighted distance measure for MAGDM method in an updated way. Singh (2015) presented the concept about the PFSs' geometrical interpretation and made a correlation coefficient of PFSs. Son (2015) found DPFCM method which was an innovative distributed picture fuzzy clustering method. Thong and Son (2015) put forward a novel hybrid model which was an application of medical diagnosis on the basis of picture fuzzy clustering and intuitionistic fuzzy recommender systems. Wang *et al.* (2018a) utilized picture fuzzy information to formulate a framework which was related to hybrid fuzzy MADM to sort the EPC projects' risk factors. Wang *et al.* (2018b) integrated the PFNP model with VIKOR method to create a method called picture fuzzy normalized projection-based VIKOR. Liang *et al.* (2018) integrated EDAS method with ELECTRE module in PFSs to infer the level of cleaner production. Ashraf *et al.* (2018) made a discussion about the weighted geometric aggregation operator's generalized form in PFSs and proposed TOPSIS method to aggregate PFNs. Ju *et al.* (2018) extended the classical GRP approach to PFSs and calculated each EVCS site's relative grey relational projection. Wei *et al.* (2019b) defined an extended bidirectional projection algorithms for picture fuzzy MAGDM issue for safety assessment of construction project. Furthermore, Wei *et al.* (2018) put forward the concept of P2TLSs on the basis of PFSs and 2-tuple linguistic term sets. Wei (2017) developed the P2TLWBM operator and the P2TLWGBM operator on the basis of Bonferroni mean. Zhang *et al.* (2018b) presented P2TLNs' novel operational laws which can conquer the limitation of existing operations relating to PFNs and P2TLNs. Zhang *et al.* (2019b) designed the MABAC method for MAGDM issue under P2TLSs.

With the continuous destruction of the human environment and the shortage of earth resources, the traditional supply chain has gradually failed to adapt to the current era and the needs of society, thus introducing the concept of green supply chain. The establishment of green supply chain has become the main challenge and trend for enterprises to provide green products and move towards a sustainable development society. Among them, the important link and core content of implementing green supply chain is the evaluation and selection of green suppliers, especially those with sustainable development who meet the requirements of green environmental protection. Because supplier selection plays an important role in green supply chain management, it directly determines the optimization of the whole chain and the core competitiveness of the enterprise. Therefore, how to efficiently determine the required suppliers from a large number of suppliers is the key problem to be solved in modern green supply chain management. The green supplier selection problem is based on multiple attributes and many experts, as it is not a single-attribute problem (He *et al.*, 2019b; Hu *et al.*, 2016; Lei *et al.*, 2019; Li *et al.*, 2020; Wang *et al.*, 2019c; Wang P. *et al.*, 2019a, 2019b). In this respect, multiple attribute group decision making (MAGDM) techniques or tools can be used to investigate this problem in a better way (Deng and Gao, 2019; Gao *et al.*, 2019; Li and Lu, 2019; Wang *et al.*, 2019a; Wang, 2019). MAGDM methods are used to rank suppliers or to choose the most appropriate and favourable supplier on the basis of multiple attributes and many experts (Mohammadi *et al.*, 2017; Paydar and Saidi-Mehrabad, 2017). Many researchers have employed different techniques to select green suppliers. Gao *et al.* (2020) developed the VIKOR method for MAGDM based on q-rung interval-valued orthopair fuzzy information for supplier selection of medical consumption products. Hashemi *et al.* (2015) defined an integrated green supplier selection approach with analytic network process and improved Grey relational analysis. Awasthi and Kannan (2016) designed the green supplier development program selection by using NGT and VIKOR under fuzzy environment. Liou *et al.* (2016) developed a new hybrid COPRAS-G MADM model for improving and selecting suppliers in green supply chain management. Wei *et al.* (2019a) proposed the supplier selection of medical consumption products with the probabilistic linguistic MABAC method. In Wang *et al.* (2019b), the q-rung orthopair hesitant fuzzy weighted power generalized Heronian mean (q-ROHFWPGHM) operator and the q-rung orthopair hesitant fuzzy weighted power generalized geometric Heronian mean (q-ROHFWPGGHM) operator are applied to deal with green supplier selection in supply chain management. Liu and Wang (2018) designed some interval-valued intuitionistic fuzzy Schweizer–Sklar power aggregation operators for supplier selection. Kannan *et al.* (2013) integrated fuzzy multi criteria decision making method and multi-objective programming approach for supplier selection and order allocation in a green supply chain.

CODAS (Combinative Distance-based Assessment) method was initially developed by Ghorabaee *et al.* (2017) to tackle the multi-criteria decision making issues. In recent years, there existed various related extensions to enrich this method. Bolturk (2018) integrated CODAS method with Pythagorean fuzzy environment. Ghorabaee *et al.* (2016) utilized linguistic variables and trapezoidal fuzzy numbers to extend the CODAS method. Badi *et al.* (2017) made use of a novel CODAS method to address MCDM issues for a steel-making company in Libya. Pamucar *et al.* (2018) presented an original MCDM Pairwise-CODAS method which was the modification of the classical CODAS method. Roy *et al.* (2019) built an assessment framework for addressing MCDM issues by extending CODAS method with interval-valued intuitionistic fuzzy numbers. So far, we have failed to find the work of the CODAS method with P2TLNs in the existing literature. Thus, investigating the CODAS method with P2TLNs is essential. The fundamental objective of our research is to develop an original method which can be more effective to address some MAGDM issues within the CODAS method and P2TLSs. Hence, the highlights of this essay are illustrated subsequently. Above all, we intend to extend the CODAS method to the picture 2-tuple linguistic environment. In addition, since the DMs are restrained by their knowledge, it is tricky to assign the criteria weights directly. Hence, CRITIC method is utilized to decide each attribute's weight. Last but not least, an empirical application is offered to demonstrate this novel approach and several comparative analysis between the CODAS method with P2TLNs and other methods are also offered to further demonstrate the merits of the novel approach.

The reminder of our essay proceeds as follows. Some fundamental knowledge of PFSs and P2TLSs is concisely reviewed in Section 2. Several aggregation operators of P2TLNs are presented in Section 3. The CODAS approach is integrated with P2TLNs and the calculating procedures are simply depicted in Section 4. An empirical application of green supplier selection is given to show the merits of this approach and some comparative analysis is also offered to further prove the merits of this method in Section 5. At last, we make an overall conclusion of our work in Section 6.

2. Preliminaries

2.1. *2-Tuple Linguistic Term Sets*

Let $S = \{s_i \mid i = 0, 1, \dots, t\}$ be a linguistic term set with odd cardinality. Any label s_i represents a possible value for a linguistic variable, and it should satisfy the following characteristics (Herrera and Martinez, 2000):

(1) The set is ordered: $s_i > s_j$, if $i > j$; (2) Max operator: $\max(s_i, s_j) = s_i$, if $s_i \geqslant s_j$; (3) Min operator: $\min(s_i, s_j) = s_i$, if $s_i \leqslant s_j$. For example, S can be defined as

$$S = \{s_0 = \textit{extremely poor}, s_1 = \textit{very poor}, s_2 = \textit{poor}, s_3 = \textit{medium}, \\ s_4 = \textit{good}, s_5 = \textit{very good}, s_6 = \textit{extremely good}\}.$$

Herrera and Martínez (2001) developed the 2-tuple fuzzy linguistic representation model on the basis of the concept of symbolic translation. It is used for representing the linguistic assessment information by means of a 2-tuple (s_i, ℓ_i), where s_i is a linguistic label from predefined linguistic term set S and ℓ_i is the value of symbolic translation, and $\ell_i \in [-0.5, 0.5)$.

2.2. *Picture Fuzzy Sets*

Definition 1 (*See* Cuong, 2014). A picture fuzzy set (PFS) on the universe X is an object of the form

$$P = \{\langle x, \mu_P(x), \eta_P(x), \nu_P(x)\rangle \mid x \in X\}, \tag{1}$$

where $\mu_P(x) \in [0, 1]$ represents the "positive membership degree of P", $\eta_P(x) \in [0, 1]$ represents the "neutral membership degree of P" and $\nu_P(x) \in [0, 1]$ represents the "negative membership degree of P", and $\mu_P(x)$, $\eta_P(x)$, $\nu_P(x)$ must meet the only condition: $0 \leqslant \mu_P(x) + \eta_P(x) + \nu_P(x) \leqslant 1$, $\forall x \in X$. Then for $x \in X$, $\pi_P(x) = 1 - (\mu_P(x) + \eta_P(x) + \nu_P(x))$ the refusal membership degree of x in P could be represented.

Definition 2 (*See* Wang *et al.*, 2017). Let $p_1 = (\mu_1, \eta_1, \nu_1)$ and $p_2 = (\mu_2, \eta_2, \nu_2)$ be any two picture fuzzy numbers (PFNs), the operation formula of them can be defined:

$$p_1 \oplus p_2 = \big(1 - (1 - \mu_1)(1 - \mu_2), \eta_1\eta_2, (\nu_1 + \eta_1)(\nu_2 + \eta_2) - \eta_1\eta_2\big), \tag{2}$$

$$p_1 \otimes p_2 = \big((\mu_1 + \eta_1)(\mu_2 + \eta_2) - \eta_1\eta_2, \eta_1\eta_2, 1 - (1 - \nu_1)(1 - \nu_2)\big), \tag{3}$$

$$\lambda p_1 = \big(1 - (1 - \mu_1)^\lambda, \eta_1^\lambda, (\nu_1 + \eta_1)^\lambda - \eta_1^\lambda\big), \quad \lambda > 0, \tag{4}$$

$$p_1^\lambda = \big((\mu_1 + \eta_1)^\lambda - \eta_1^\lambda, \eta_1^\lambda, 1 - (1 - \nu_1)^\lambda\big), \quad \lambda > 0. \tag{5}$$

Derived from the Definition 2, the properties of the operation laws are shown as follows:

(1) $p_1 \oplus p_2 = p_2 \oplus p_1$, $\quad p_1 \otimes p_2 = p_2 \otimes p_1$, $\quad \big((p_1)^{\lambda_1}\big)^{\lambda_2} = (p_1)^{\lambda_1\lambda_2}$;

(2) $\lambda(p_1 \oplus p_2) = \lambda p_1 \oplus \lambda p_2$, $\quad (p_1 \otimes p_2)^\lambda = (p_1)^\lambda \otimes (p_2)^\lambda$;

(3) $\lambda_1 p_1 \oplus \lambda_2 p_1 = (\lambda_1 + \lambda_2)p_1$, $\quad (p_1)^{\lambda_1} \otimes (p_1)^{\lambda_2} = (p_1)^{(\lambda_1+\lambda_2)}$.

2.3. *Picture 2-Tuple Linguistic Sets*

Definition 3 (*See* Wei *et al.*, 2018). A picture 2-tuple linguistic set on the universe X is an object of the form

$$P = \{(s_{\xi(x)}, \ell), \big(\mu_P(x), \eta_P(x), \nu_P(x)\big), x \in X\}, \tag{6}$$

where $(s_{\xi(x)}, \ell) \in S$, $\ell \in [-0.5, 0.5)$, $u_P(x) \in [0, 1]$, $\eta_P(x) \in [0, 1]$ and $\upsilon_P(x) \in [0, 1]$, with the condition $0 \leqslant u_P(x) + \eta_P(x) + \upsilon_P(x) \leqslant 1$, $\forall x \in X$, $s_{\xi(x)} \in S$ and

$\ell \in [-0.5, 0.5)$. $\mu_P(x)$, $\eta_P(x)$, $\nu_P(x)$ denote the degree of positive membership, neutral membership and negative membership of the element x to linguistic variable $(s_{\xi(x)}, \ell)$, respectively. Then for $x \in X$, $\pi_P(x) = 1 - (\mu_P(x) + \eta_P(x) + \nu_P(x))$ could be called the degree of refusal membership of the element x to linguistic variable $(s_{\xi(x)}, \ell)$.

For convenience, we call $\hat{p} = \langle (s_{\xi(p)}, \ell), (u(p), \eta(p), v(p)) \rangle$ a picture 2-tuple linguistic number (P2TLN), where $\mu_p \in [0, 1]$, $\eta_p \in [0, 1]$, $\nu_p \in [0, 1]$, $\mu_p + \eta_p + \nu_p \leqslant 1$, $s_{\xi(p)} \in S$ and $\ell \in [-0.5, 0.5)$.

Definition 4 (*See* Wei *et al.*, 2018). Let $\hat{p} = \langle (s_{\xi(p)}, \ell), (u(p), \eta(p), v(p)) \rangle$ be a P2TLN, a score function p of P2TLN can be represented as follows:

$$S(\hat{p}) = \Delta\left(\Delta^{-1}(s_{\xi(p)}, \ell) \cdot \frac{1 + \mu_p - \nu_p}{2}\right), \quad \Delta^{-1}\big(S(\hat{p})\big) \in [0, t]. \tag{7}$$

Definition 5 (*See* Wei *et al.*, 2018). Let $\hat{p} = \langle (s_{\xi(p)}, \ell), (u(p), \eta(p), v(p)) \rangle$ be a P2TLN, an accuracy function H of P2TLN can be represented as follows:

$$H(\hat{p}) = \Delta\left(\Delta^{-1}(s_{\xi(p)}, \ell) \cdot \frac{\mu_p + \eta_p + \nu_p}{2}\right), \quad \Delta^{-1}\big(H(\hat{p})\big) \in [0, t]. \tag{8}$$

To evaluate the degree of accuracy of P2TLN $\hat{p} = \langle (s_{\xi(p)}, \ell), (u(p), \eta(p), v(p)) \rangle$, where $\Delta^{-1}(H(\hat{p})) \in [0, t]$. The larger the value of $H(\hat{p})$, the more the degree of accuracy of the P2TLN $\hat{p}$.

In terms of the score function S and the accuracy function H, after that, an order relation between two P2TLNs should be given, which is defined as follows:

Definition 6 (*See* Wei *et al.*, 2018). Let $\hat{p}_1 = \langle (s_{\xi(p_1)}, \ell_1), (u(p_1), \eta(p_1), v(p_1)) \rangle$ and $\hat{p}_2 = \langle (s_{\xi(p_2)}, \ell_2), (u(p_2), \eta(p_2), v(p_2)) \rangle$ be two P2TLNs, $S(\hat{p}_1) = \Delta(\Delta^{-1}(s_{\xi(p_1)}, \ell_1) \cdot \frac{1+\mu_{p_1}-\nu_{p_1}}{2})$ and $S(\hat{p}_2) = \Delta(\Delta^{-1}(s_{\xi(p_2)}, \ell_2) \cdot \frac{1+\mu_{p_2}-\nu_{p_2}}{2})$ be the scores of $\hat{p}_1$ and $\hat{p}_2$, respectively, and let $H(\hat{p}_1) = \Delta(\Delta^{-1}(s_{\xi(p_1)}, \ell_1) \cdot \frac{\mu_{p_1}+\eta_{p_1}+\nu_{p_1}}{2})$ and $H(\hat{p}_2) = \Delta(\Delta^{-1}(s_{\xi(p_2)}, \ell_2) \cdot \frac{\mu_{p_2}+\eta_{p_2}+\nu_{p_2}}{2})$ be the accuracy degrees of $\hat{p}_1$ and $\hat{p}_2$, respectively. Then if $S(\hat{p}_1) < S(\hat{p}_2)$, then $\hat{p}_1$ is smaller than $\hat{p}_2$, denoted by $\hat{p}_1 < \hat{p}_2$; if $S(\hat{p}_1) = S(\hat{p}_2)$, then:

(1) if $H(\hat{p}_1) = H(\hat{p}_2)$, then $\hat{p}_1$ and $\hat{p}_2$ represent the same information, denoted by $\hat{p}_1 = \hat{p}_2$;
(2) if $H(\hat{p}_1) < H(\hat{p}_2)$, $\hat{p}_1$ is smaller than $\hat{p}_2$, denoted by $\hat{p}_1 < \hat{p}_2$.

Motivated by the operations of 2-tuple linguistic, after that, several operational laws of P2TLNs will be defined.

Definition 7 (*See* Wei *et al.*, 2018). Let $\hat{p}_1 = \langle(s_{\xi(p_1)}, \ell_1), (u(p_1), \eta(p_1), v(p_1))\rangle$ and $\hat{p}_2 = \langle(s_{\xi(p_2)}, \ell_2), (u(p_2), \eta(p_2), v(p_2))\rangle$ be two P2TLNs, then

$$\begin{aligned}
\hat{p}_1 \oplus \hat{p}_2 = &\big\langle \Delta\big(\Delta^{-1}(s_{\xi(p_1)}, \ell_1) + \Delta^{-1}(s_{\xi(p_2)}, \ell_2)\big), \\
&\big(1 - (1 - \mu_{p_1})(1 - \mu_{p_2}), \eta_{p_1}\eta_{p_2}, (\nu_{p_1} + \eta_{p_1})(\nu_{p_2} + \eta_{p_2}) - \eta_{p_1}\eta_{p_2}\big)\big\rangle; \\
\hat{p}_1 \otimes \hat{p}_2 = &\big\langle \Delta\big(\Delta^{-1}(s_{\xi(p_1)}, \ell_1) \cdot \Delta^{-1}(s_{\xi(p_2)}, \ell_2)\big), \\
&\big((\mu_{p_1} + \eta_{p_1})(\mu_{p_2} + \eta_{p_2}) - \eta_{p_1}\eta_{p_2}, \eta_{p_1}\eta_{p_2}, 1 - (1 - \nu_{p_1})(1 - \nu_{p_2})\big)\big\rangle; \\
\lambda\hat{p}_1 = &\big\langle \Delta\big(\lambda\Delta^{-1}(s_{\xi(p_1)}, \ell_1)\big), \big(1 - (1 - \mu_{p_1})^\lambda, \eta_{p_1}^\lambda, (\nu_{p_1} + \eta_{p_1})^\lambda - \eta_{p_1}^\lambda\big)\big\rangle; \\
(\hat{p}_1)^\lambda = &\big\langle \Delta\big(\big(\Delta^{-1}(s_{\xi(p_1)}, \ell_1)\big)^\lambda\big), \big((\mu_{p_1} + \eta_{p_1})^\lambda - \eta_{p_1}^\lambda, \eta_{p_1}^\lambda, 1 - (1 - \nu_{p_1})^\lambda\big)\big\rangle.
\end{aligned}$$

Derived from the Definition 7, the properties of the calculation rules are shown as follows:

(1) $\hat{p}_1 \oplus \hat{p}_2 = \hat{p}_2 \oplus \hat{p}_1$, $\hat{p}_1 \otimes \hat{p}_2 = \hat{p}_2 \otimes \hat{p}_1$, $\lambda(\hat{p}_1 \oplus \hat{p}_2) = \lambda\hat{p}_1 \oplus \lambda\hat{p}_2$, $0 \leqslant \lambda \leqslant 1$;
(2) $\lambda_1\hat{p}_1 \oplus \lambda_2\hat{p}_1 = (\lambda_1 \oplus \lambda_2)\hat{p}_1$, $\hat{p}_1^{\lambda_1} \otimes \hat{p}_1^{\lambda_2} = (\hat{p}_1)^{\lambda_1+\lambda_2}$, $0 \leqslant \lambda_1, \lambda_2, \lambda_1 + \lambda_2 \leqslant 1$;
(3) $\hat{p}_1^{\lambda_1} \otimes \hat{p}_2^{\lambda_1} = (\hat{p}_1 \otimes \hat{p}_2)^{\lambda_1}$, $\lambda_1 \geqslant 0$; $\big(\hat{p}^{\lambda_1}\big)^{\lambda_2} = \hat{p}^{\lambda_1\lambda_2}$.

Definition 8. Let $\hat{p}_1 = \langle(s_{\xi(p_1)}, \ell_1), (u(p_1), \eta(p_1), v(p_1))\rangle$ and $\hat{p}_2 = \langle(s_{\xi(p_2)}, \ell_2), (u(p_2), \eta(p_2), v(p_2))\rangle$ be two P2TLNs, then the normalized Hamming distance and the normalized Euclidean distance between $\hat{p}_1 = \langle(s_{\xi(p_1)}, \ell_1), (u(p_1), \eta(p_1), v(p_1))\rangle$ and $\hat{p}_2 = \langle(s_{\xi(p_2)}, \ell_2), (u(p_2), \eta(p_2), v(p_2))\rangle$ are defined as follows:

$$\begin{aligned}
d_H(\hat{p}_1, \hat{p}_2) = &\frac{|\mu_{p_1} - \mu_{p_2}| + |\eta_{p_1} - \eta_{p_2}| + |\nu_{p_1} - \nu_{p_2}|}{4} \\
&+ \frac{|\Delta^{-1}(s_{\xi_1}, \ell_1) - \Delta^{-1}(s_{\xi_2}, \ell_2)|}{2T},
\end{aligned} \tag{9}$$

$$d_E(\hat{p}_1, \hat{p}_2) = \sqrt{\begin{aligned}&\frac{|\mu_{p_1} - \mu_{p_2}|^2 + |\eta_{p_1} - \eta_{p_2}|^2 + |\nu_{p_1} - \nu_{p_2}|^2}{4} \\ &+ \frac{|\Delta^{-1}(s_{\xi_1}, \ell_1) - \Delta^{-1}(s_{\xi_2}, \ell_2)|^2}{2T}.\end{aligned}} \tag{10}$$

3. Picture 2-Tuple Linguistic Arithmetic Aggregation Operators

In this chapter, under the picture 2-tuple linguistic environment, some arithmetic aggregation operators are introduced, including picture 2-tuple linguistic weighted averaging (P2TLWA) operator and picture 2-tuple linguistic weighted geometric (P2TLWG) operator.

Definition 9 (*See* Wei *et al.*, 2018). Let $\hat{p}_j = \langle(s_j, \ell_j), (\mu_{p_j}, \eta_{p_j}, \nu_{p_j})\rangle$ $(j = 1, 2, \ldots, n)$ be a sort of P2TLNs. The picture 2-tuple linguistic weighted averaging (P2TLWA) operator can be defined as:

$$\text{P2TLWA}_\omega(\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_n) = \bigoplus_{j=1}^{n}(\omega_j\hat{p}_j), \tag{11}$$

where $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weight vector of $\hat{p}_j$ $(j = 1, 2, \dots, n)$ and $\omega_j > 0$, $\sum_{j=1}^{n} \omega_j = 1$.

Derived from the Definition 9, the subsequent result can be easily acquired:

Theorem 1. *The aggregated value by utilizing* P2TLWA *operator is also a* P2TLN, *where*

$$\begin{aligned} &\mathrm{P2TLWA}_\omega(\hat{p}_1, \hat{p}_2, \dots, \hat{p}_n) \\ &\quad = \bigoplus_{j=1}^{n} (\omega_j \hat{p}_j) = \Bigg\langle \Delta\Bigg(\sum_{j=1}^{n} \omega_j \Delta^{-1}(s_j, \ell_j)\Bigg), \\ &\quad \Bigg(1 - \prod_{j=1}^{n}(1-\mu_j)^{\omega_j}, \prod_{j=1}^{n}(\eta_j)^{\omega_j}, \prod_{j=1}^{n}(\nu_j + \eta_j)^{\omega_j} - \prod_{j=1}^{n}(\eta_j)^{\omega_j}\Bigg)\Bigg\rangle, \end{aligned} \tag{12}$$

where $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ *is the weight vector of* $\hat{p}_j$ $(j = 1, 2, \dots, n)$ *and* $\omega_j > 0$, $\sum_{j=1}^{n} \omega_j = 1$.

Definition 10 (*See* Wei *et al.*, 2018). Let $\hat{p}_j$ $(j = 1, 2, \dots, n)$ be a sort of P2TLNs. The picture 2-tuple linguistic weighted geometric (P2TLWG) operator can be defined as:

$$\mathrm{P2TLWG}_\omega(\hat{p}_1, \hat{p}_2, \dots, \hat{p}_n) = \bigotimes_{j=1}^{n} (\hat{p}_j)^{\omega_j}, \tag{13}$$

where $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weight vector of $\hat{p}_j$ $(j = 1, 2, \dots, n)$ and $\omega_j > 0$, $\sum_{j=1}^{n} \omega_j = 1$.

Derived from the Definition 10, the subsequent result can be easily acquired:

Theorem 2. *The aggregated value by utilizing* P2TLWG *operator is also a* P2TLN, *where*

$$\begin{aligned} &\mathrm{P2TLWG}_\omega(\hat{p}_1, \hat{p}_2, \dots, \hat{p}_n) \\ &\quad = \bigotimes_{j=1}^{n} (\hat{p}_j)^{\omega_j} = \Bigg\langle \Delta\Bigg(\prod_{j=1}^{n}\big(\Delta^{-1}(s_j, \ell_j)^{\omega_j}\big)\Bigg), \\ &\quad \Bigg(\prod_{j=1}^{n}(\mu_j + \eta_j)^{\omega_j} - \prod_{j=1}^{n}(\eta_j)^{\omega_j}, \prod_{j=1}^{n}(\eta_j)^{\omega_j}, 1 - \prod_{j=1}^{n}(1-\nu_j)^{\omega_j}\Bigg)\Bigg\rangle, \end{aligned} \tag{14}$$

where $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ *is the weight vector of* $\hat{p}_j$ $(j = 1, 2, \dots, n)$ *and* $\omega_j > 0$, $\sum_{j=1}^{n} \omega_j = 1$.

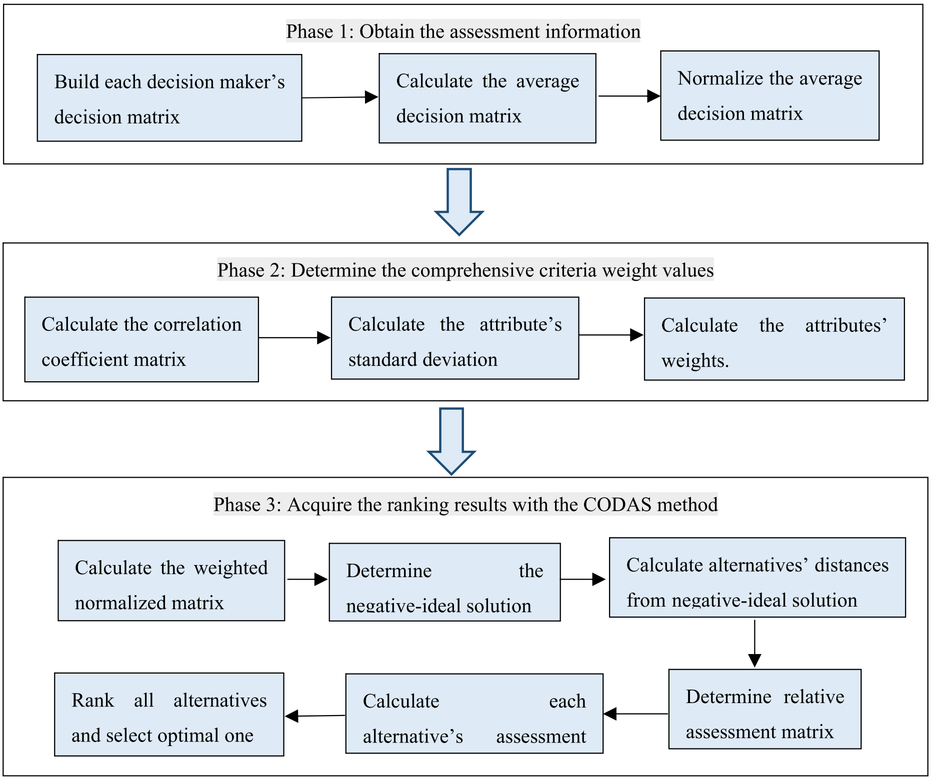

Fig. 1. The structure of the presented method.

4. The CODAS Method with P2TLNs for MAGDM Issues

In this chapter, the CODAS method will be integrated with P2TLNs, which can be utilized to conquer the limitations of the existing multi-attribute value method.

Let $C = \{C_1, C_2, \dots, C_n\}$ be the collection of attributes, $c = \{c_1, c_2, \dots, c_n\}$ be the weight vector of attributes C_j, where $c_j \in [0, 1]$, $j = 1, 2, \dots, n$, $\sum_{j=1}^{n} c_j = 1$. Assume $L = \{L_1, L_2, \dots, L_r\}$ is a collection of decision makers that have significant degree of $l = \{l_1, l_2, \dots, l_r\}$, where $l_k \in [0, 1]$, $k = 1, 2, \dots, r$. $\sum_{k=1}^{r} l_k = 1$. Let $\kappa = \{\kappa_1, \kappa_2, \dots, \kappa_m\}$ be a discrete collection of alternatives. And $G = (g_{ij})_{m\times n}$ is the comprehensive picture 2-tuple linguistic decision matrix, g_{ij} means the value of alternative κ_i regarding the attribute C_j.

After that, the specific calculation procedures will be depicted in Fig. 1.

(I) Phase 1: Obtain the assessment information

Step 1. Build each decision maker's picture 2-tuple linguistic decision matrix $G^k = (g_{ij}^k)_{m\times n}$ and then calculate the comprehensive picture 2-tuple linguistic decision matrix $G = (g_{ij})_{m\times n}$.

$$G^{(k)} = [g_{ij}^k]_{m\times n} = \begin{bmatrix} g_{11}^k & g_{12}^k & \cdots & g_{1n}^k \\ g_{21}^k & g_{22}^k & \cdots & g_{2n}^k \\ \vdots & \vdots & \vdots & \vdots \\ g_{m1}^k & g_{m2}^k & \cdots & g_{mn}^k \end{bmatrix}, \tag{15}$$

$$G = [g_{ij}]_{m\times n} = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ g_{m1} & g_{m2} & \cdots & g_{mn} \end{bmatrix}, \tag{16}$$

$$g_{ij} = \Bigg\langle \Delta\Bigg(\sum_{k=1}^{r} l_k \Delta^{-1}\big(s_{ij}^k, \ell_{ij}^k\big)\Bigg), \Bigg(1 - \prod_{k=1}^{r}\big(1-\mu_{ij}^k\big)^{l_k}, \prod_{k=1}^{r}\big(\eta_{ij}^k\big)^{l_k}, \prod_{k=1}^{r}\big(\nu_{ij}^k + \eta_{ij}^k\big)^{l_k} - \prod_{k=1}^{r}\big(\eta_{ij}^k\big)^{l_k}\Bigg)\Bigg\rangle, \tag{17}$$

where g_{ij}^k is the assessment value of the alternative κ_i $(i = 1, 2, \ldots, m)$ on the basis of the attribute C_j $(j = 1, 2, \ldots, n)$ and the decision maker L_k $(k = 1, 2, \ldots, r)$.

Step 2. Normalize the comprehensive picture 2-tuple linguistic decision matrix by utilizing the following Eq. (18).

$$G^N = \big(g_{ij}^N\big)_{m\times n} = \begin{cases} \langle (s_{ij}, \ell_{ij}), (\mu_{ij}, \eta_{ij}, \nu_{ij})\rangle, & C_j \text{is a benefit criterion,} \\ \langle \Delta(T - \Delta^{-1}(s_{ij}, \ell_{ij})), (\nu_{ij}, \eta_{ij}, \mu_{ij})\rangle, & C_j \text{is a benefit criterion.} \end{cases} \tag{18}$$

(II) Phase 2: Determine the comprehensive criteria weight values

Step 3. Determine the criterion's weighting matrix by utilizing CRITIC method.

CRITIC (CRiteria Importance Through Intercriteria Correlation) method will be proposed in this part which is utilized to decide attributes' weights. This method was initially put forward by Diakoulaki *et al.* (1995) which took the correlations between attributes into consideration. Subsequently, the calculation procedures of this method will be presented.

(1) Depending on the comprehensive picture 2-tuple linguistic decision matrix $G^N = (g_{ij}^N)_{m\times n}$, the correlation coefficient matrix $\vartheta = (\iota_{jt})_{n\times n}$ is built by calculating the correlation coefficient between attributes.

$$\iota_{jt} = \frac{\sum_{i=1}^m (S(g_{ij}^N) - S(g_j^N))(S(g_{it}^N) - S(g_t^N))}{\sqrt{\sum_{i=1}^m (S(g_{ij}^N) - S(g_j^N))^2}\sqrt{\sum_{i=1}^m (S(g_{it}^N) - S(g_t^N))^2}}, \quad j, t = 1, 2, \ldots, n, \tag{19}$$

where $S(g_j^N) = \frac{1}{m}\sum_{i=1}^m S(g_{ij}^N)$ and $S(g_t^N) = \frac{1}{m}\sum_{i=1}^m S(g_{it}^N)$.

(2) Calculate attribute's standard deviation.

$$SD_j = \sqrt{\frac{1}{m}\sum_{i=1}^{m}\big(S(g_{ij}^N) - S(g_j^N)\big)^2}, \quad j = 1, 2, \dots, n, \tag{20}$$

where $S(g_j^N) = \frac{1}{m}\sum_{i=1}^{m} S(g_{ij}^N)$.

(3) Calculate attributes' weights.

$$c_j = \frac{SD_j \sum_{t=1}^{n}(1 - \iota_{jt})}{\sum_{j=1}^{n}(SD_j \sum_{t=1}^{n}(1 - \iota_{jt}))}, \quad j = 1, 2 \dots, n, \tag{21}$$

where $c_j \in [0, 1]$ and $\sum_{j=1}^{n} c_j = 1$.

(III) Phase 3: Acquire the ranking results with the CODAS method

Step 4. Calculate the weighted normalized matrix. The weighted normalized performance values ($\tilde{g}_{ij}$) are calculated as in Eqs. (22) and (23):

$$\tilde{G} = [\tilde{g}_{ij}]_{m\times n}, \tag{22}$$

$$\tilde{g}_{ij} = c_j \otimes g_{ij}^N, \tag{23}$$

where c_j denotes the weights of the jth criterion.

Step 5. The negative-ideal solution is defined using Eqs. (24) and (25):

$$\widetilde{NS} = [\widetilde{ns}_j]_{1\times n}, \tag{24}$$

$$\widetilde{ns}_j = \min_i \tilde{g}_{ij}, \tag{25}$$

where $\min_i \tilde{g}_{ij} = \{\tilde{g}_{hj} | S(\tilde{g}_{hj}) = \min_i(S(\tilde{g}_{hj})),\ h \in \{1, 2, \dots, n\}\}$.

Step 6. Calculate alternatives' weighted Euclidean (ED_i) and weighted Hamming (HD_i) distances from the negative-ideal solution as in Eqs. (26) and (27):

$$ED_i = \sum_{j=1}^{n} d_E(\tilde{g}_{ij}, \widetilde{ns}_j), \tag{26}$$

$$HD_i = \sum_{j=1}^{n} d_H(\tilde{g}_{ij}, \widetilde{ns}_j). \tag{27}$$

Step 7. Determine relative assessment matrix (RA) as in Eqs. (28) and (29):

$$RA = [p_{ih}]_{n\times m}, \tag{28}$$

$$p_{ih} = (ED_i - ED_h) + \big(t(ED_i - ED_h) \times (HD_i - HD_h)\big), \tag{29}$$

where $h \in \{1, 2, \dots, n\}$ and t is a threshold function that is defined in Eq. (30):

$$t(x) = \begin{cases} 1 & \text{if } |x| \geqslant \theta, \\ 0 & \text{if } |x| < \theta. \end{cases} \tag{30}$$

In this function, θ is the threshold parameter of this function that can be set by decision maker. In this study, $\theta = 0.05$ is taken for the calculations.

Step 8. Calculate each alternative's assessment score (AS_i) as in Eq. (31):

$$AS_i = \sum_{h=1}^{n} p_{ih}. \tag{31}$$

Step 9. Depending on the calculation results of AS, all the alternatives can be ranked. The higher the value of AS is, the more optimal alternative will be selected.

5. An Empirical Example and Comparative Analysis

5.1. *An Empirical Example for P2TLNs MAGDM Issues*

With the rapid development of economic globalization, resources and the environment are facing enormous challenges. In this situation, green supply chain management is particularly significant, and there are a lot of challenges in evaluating green suppliers for enterprises. Green supplier selection is a classical MAGDM problem (He *et al.*, 2019a; Lei *et al.*, 2020; Lu *et al.*, 2020; Wang *et al.*, 2020; Wang P. *et al.*, 2020; Wei *et al.*, 2020). Thus, in this section, an application of selecting the optimal green supplier will be provided by making use of the CODAS method with P2TLNs, which can offer an effective solution for selecting green suppliers. Taking its own business development into consideration, a manufacturing company wants to choose a green supplier for a long-term cooperation. There are four potential green suppliers κ_i $(i = 1, 2, 3, 4, 5)$. In order to select the most appropriate supplier, the company invites three experts $L = \{L_1, L_2, L_3\}$ (expert's weight $l = (0.42, 0.35, 0.23)$ to assess these suppliers. All experts give their assessment information relying on the four subsequently attributes: ① C_1 is supply capacity; ② C_2 is product cost; ③ C_3 is the ability of external environmental management; ④ C_4 is product ecological design. Evidently, C_2 is the cost attributes, while the others are the benefit attributes. To make this evaluation, the decision-makers express their assessments using linguistic variables. The linguistics variables for rating alternatives are shown in Table 1.

(I) Phase 1: Obtain the assessment information

Step 1. Set up each decision-maker's evaluation matrix $G^{(k)} = (g_{ij}^{k})_{m\times n}$ $(i = 1, 2, \dots, m,$ $j = 1, 2, \dots, n)$ as in Tables 2–4. Derived from these tables and Eq. (15) to (17), the comprehensive picture 2-tuple linguistic decision matrix can be calculated. The results are recorded in Table 5.

Table 1
Linguistic scale for ratings of alternatives.

Linguistic term	P2TLNs
Very Low – VL	$\langle(s_0, 0), (0.05, 0.45, 0.50)\rangle$
Low – L	$\langle(s_1, 0), (0.10, 0.40, 0.45)\rangle$
Below Medium – BM	$\langle(s_2, 0), (0.15, 0.35, 0.40)\rangle$
Exactly Equal – EE	$\langle(s_3, 0), (0.30, 0.35, 0.35)\rangle$
Above Medium – AM	$\langle(s_4, 0), (0.40, 0.20, 0.15)\rangle$
High – H	$\langle(s_5, 0), (0.45, 0.15, 0.10)\rangle$
Very High – VH	$\langle(s_6, 0), (0.50, 0.10, 0.05)\rangle$

Table 2
Ratings of the alternatives on each criterion by DM_1.

	C_1	C_2	C_3	C_4
κ_1	H	EE	L	AM
κ_2	AM	EE	VH	H
κ_3	VL	BM	AM	L
κ_4	BM	L	H	EE
κ_5	EE	H	AM	BM

Table 3
Ratings of the alternatives on each criterion by DM_2.

	C_1	C_2	C_3	C_4
κ_1	VH	AM	EE	BM
κ_2	H	H	AM	VH
κ_3	AM	VL	L	EE
κ_4	L	EE	BM	AM
κ_5	AM	L	EE	EE

Table 4
Ratings of the alternatives on each criterion by DM_3.

	C_1	C_2	C_3	C_4
κ_1	VH	L	EE	H
κ_2	H	BM	VH	EE
κ_3	L	BM	AM	VL
κ_4	BM	VL	EE	AM
κ_5	AM	EE	BM	L

Step 2. Normalize the comprehensive picture 2-tuple linguistic decision matrix by utilizing Eq. (18) and the calculation results are presented in Table 6.

(II) Phase 2: Determine the comprehensive criteria weight values

Step 3. Decide the attribute weights c_j $(j = 1, 2, \ldots, n)$ by making use of CRITIC method as presented in Table 7.

(III) Phase 3: Acquire the ranking results with the CODAS method

Table 5
Comprehensive picture 2-tuple linguistic decision matrix.

	C_1	C_2
κ_1	$\langle(s_6,-0.4),(0.4796,0.1186,0.0673)\rangle$	$\langle(s_3,-0.1),(0.2973,0.2967,0.2776)\rangle$
κ_2	$\langle(s_5,-0.4),(0.4295,0.1693,0.1187)\rangle$	$\langle(s_4,-0.5),(0.3273,0.2602,0.2358)\rangle$
κ_3	$\langle(s_2,-0.4),(0.2011,0.3297,0.3231)\rangle$	$\langle(s_1,0.3),(0.1163,0.3822,0.4325)\rangle$
κ_4	$\langle(s_2,0),(0.1500,0.3500,0.4000)\rangle$	$\langle(s_2,-0.5),(0.1655,0.3922,0.4225)\rangle$
κ_5	$\langle(s_4,-0.4),(0.3599,0.2530,0.2153)\rangle$	$\langle(s_3,0.1),(0.3093,0.2569,0.2293)\rangle$
	C_3	C_4
κ_1	$\langle(s_2,0.2),(0.2221,0.3702,0.3893)\rangle$	$\langle(s_4,-0.5),(0.3356,0.2277,0.1953)\rangle$
κ_2	$\langle(s_5,0.3),(0.4671,0.1275,0.0743)\rangle$	$\langle(s_5,-0.1),(0.4377,0.1582,0.1068)\rangle$
κ_3	$\langle(s_3,0),(0.3085,0.2549,0.2226)\rangle$	$\langle(s_2,-0.5),(0.1655,0.3922,0.4225)\rangle$
κ_4	$\langle(s_4,-0.5),(0.3229,0.2452,0.2202)\rangle$	$\langle(s_4,-0.4)(0.3599,0.2530,0.2153)\rangle$
κ_5	$\langle(s_3,0.2),(0.3139,0.2767,0.2549)\rangle$	$\langle(s_2,0.1),(0.1953,0.3609,0.3926)\rangle$

Table 6
Normalized comprehensive picture fuzzy decision matrix.

	C_1	C_2
κ_1	$\langle(s_6,-0.4),(0.4796,0.1186,0.0673)\rangle$	$\langle(s_3,0.1),(0.2776,0.2967,0.2973)\rangle$
κ_2	$\langle(s_5,-0.4),(0.4295,0.1693,0.1187)\rangle$	$\langle(s_3,-0.5),(0.2358,0.2602,0.3273)\rangle$
κ_3	$\langle(s_2,-0.4),(0.2011,0.3297,0.3231)\rangle$	$\langle(s_5,-0.3),(0.4325,0.3822,0.1163)\rangle$
κ_4	$\langle(s_2,0),(0.1500,0.3500,0.4000)\rangle$	$\langle(s_5,-0.5),(0.4225,0.3922,0.1655)\rangle$
κ_5	$\langle(s_4,-0.4),(0.3599,0.2530,0.2153)\rangle$	$\langle(s_3,-0.1),(0.2293,0.2569,0.3093)\rangle$
	C_3	C_4
κ_1	$\langle(s_2,0.2),(0.2221,0.3702,0.3893)\rangle$	$\langle(s_4,-0.5),(0.3356,0.2277,0.1953)\rangle$
κ_2	$\langle(s_5,0.3),(0.4671,0.1275,0.0743)\rangle$	$\langle(s_5,-0.1),(0.4377,0.1582,0.1068)\rangle$
κ_3	$\langle(s_3,0),(0.3085,0.2549,0.2226)\rangle$	$\langle(s_2,-0.5),(0.1655,0.3922,0.4225)\rangle$
κ_4	$\langle(s_4,-0.5),(0.3229,0.2452,0.2202)\rangle$	$\langle(s_4,-0.4)(0.3599,0.2530,0.2153)\rangle$
κ_5	$\langle(s_3,0.2),(0.3139,0.2767,0.2549)\rangle$	$\langle(s_2,0.1),(0.1953,0.3609,0.3926)\rangle$

Table 7
The attributes weights c_j.

	C_1	C_2	C_3	C_4
c_j	0.3295	0.3000	0.1943	0.1762

Step 4. Calculate the weighted normalized matrix. The weighted normalized performance values $\tilde{g}_{ij}$ are calculated as in Table 8.

Step 5. Determine the negative-ideal solution depending on Table 8 and the results are presented in Table 9.

Step 6. Calculate alternatives' weighted Euclidean (ED_i) and weighted Hamming (HD_i) distances from the negative-ideal solution and the results are presented in Table 10.

Step 7. Obtain the relative assessment matrix (RA) and the results are presented in Table 11.

Table 8
The weighted normalized performance values of alternatives.

	C_1	C_2
κ_1	$\langle(s_2, -0.1612), (0.1936, 0.4953, 0.0791)\rangle$	$\langle(s_1, -0.0671), (0.0929, 0.6946, 0.1608)\rangle$
κ_2	$\langle(s_1, -0.4907), (0.1689, 0.5569, 0.1066)\rangle$	$\langle(s_1, -0.2411), (0.0775, 0.6677, 0.1848)\rangle$
κ_3	$\langle(s_1, -0.4629), (0.0713, 0.6938, 0.1751)\rangle$	$\langle(s_1, 0.4098), (0.1563, 0.7494, 0.0621)\rangle$
κ_4	$\langle(s_1, -0.3409), (0.0521, 0.7075, 0.2020)\rangle$	$\langle(s_1, 0.3588), (0.1518, 0.7552, 0.0841)\rangle$
κ_5	$\langle(s_1, 0.1797), (0.1367, 0.6358, 0.1430)\rangle$	$\langle(s_1, -0.1421), (0.0751, 0.6652, 0.1779)\rangle$
	C_3	C_4
κ_1	$\langle(s_0, 0.4197), (0.0476, 0.8244, 0.1235)\rangle$	$\langle(s_1, -0.3779), (0.0695, 0.7705, 0.0888)\rangle$
κ_2	$\langle(s_1, 0.0297), (0.1151, 0.6702, 0.0626)\rangle$	$\langle(s_1, -0.1383), (0.0965, 0.7225, 0.0688)\rangle$
κ_3	$\langle(s_1, -0.4269), (0.0692, 0.7668, 0.0994)\rangle$	$\langle(s_0, 0.2590), (0.0314, 0.8480, 0.1166)\rangle$
κ_4	$\langle(s_1, -0.3219), (0.0730, 0.7610, 0.1009)\rangle$	$\langle(s_1, -0.3691), (0.0756, 0.7849, 0.0900)\rangle$
κ_5	$\langle(s_1, -0.3802), (0.0706, 0.7791, 0.1054)\rangle$	$\langle(s_0, 0.3736), (0.0376, 0.8356, 0.1157)\rangle$

Table 9
The negative-ideal solution.

	Minimum value
C_1	$\langle(s_1, -0.4629), (0.0521, 0.7075, 0.2020)\rangle$
C_2	$\langle(s_1, -0.2411), (0.0751, 0.7552, 0.1848)\rangle$
C_3	$\langle(s_0, 0.4197), (0.0476, 0.8244, 0.1235)\rangle$
C_4	$\langle(s_0, 0.2590), (0.0314, 0.8480, 0.1166)\rangle$

Table 10
Euclidean and Hamming distances of alternatives.

	Euclidean distance	Hamming distance
κ_1	0.3338	0.9345
κ_2	0.6988	1.2442
κ_3	0.4130	0.6454
κ_4	0.4850	0.8841
κ_5	0.4611	0.8235

Table 11
Relative assessment matrix.

	p_{ik}		p_{ik}		p_{ik}		p_{ik}		p_{ik}
$\kappa_1-\kappa_2$	−0.2519	$\kappa_2-\kappa_1$	0.4469	$\kappa_3-\kappa_1$	0.3338	$\kappa_4-\kappa_1$	0.3338	$\kappa_5-\kappa_1$	0.3338
$\kappa_1-\kappa_3$	−0.0792	$\kappa_2-\kappa_3$	0.5050	$\kappa_3-\kappa_2$	0.5050	$\kappa_4-\kappa_2$	0.4108	$\kappa_5-\kappa_2$	0.4338
$\kappa_1-\kappa_4$	−0.1512	$\kappa_2-\kappa_4$	0.4108	$\kappa_3-\kappa_4$	0.3338	$\kappa_4-\kappa_3$	0.3338	$\kappa_5-\kappa_3$	0.3338
$\kappa_1-\kappa_5$	−0.1273	$\kappa_2-\kappa_5$	0.4338	$\kappa_3-\kappa_5$	0.3338	$\kappa_4-\kappa_5$	0.3338	$\kappa_5-\kappa_4$	0.3338

Step 8. Calculate each alternative's assessment score (AS_i) and the results are presented in Table 12.

Step 9. Depending on the calculating result from the **Step 8**, all alternatives can be ranked. The ranking order of these potential suppliers is $\kappa_2 > \kappa_3 > \kappa_5 > \kappa_4 > \kappa_1$. Hence, the optimal supplier is κ_2.

Table 12
Relative assessment matrix.

Alternative	Assessment score
κ_1	−0.6095
κ_2	1.7965
κ_3	1.5065
κ_4	1.4123
κ_5	1.4354

Table 13
Evaluation results of four methods.

Methods	Ranking order	The optimal alternative	The worst alternative
P2TLWA	$\kappa_2 > \kappa_1 > \kappa_4 > \kappa_5 > \kappa_3$	κ_2	κ_3
P2TLWG	$\kappa_2 > \kappa_1 > \kappa_4 > \kappa_5 > \kappa_3$	κ_2	κ_3
EDAS method	$\kappa_2 > \kappa_1 > \kappa_4 > \kappa_5 > \kappa_3$	κ_2	κ_3
The developed method	$\kappa_2 > \kappa_3 > \kappa_5 > \kappa_4 > \kappa_1$	κ_2	κ_1

5.2. *Comparative Analysis*

In this chapter, our developed CODAS method with P2TLNs is compared with several methods to illustrate its superiority.

First of all, our presented method is compared with P2TLWA and P2TLWG operators (Wei *et al.*, 2018). For the P2TLWA operator, the calculation result is $S(\kappa_1) = (s_2, 0.2311)$, $S(\kappa_2) = (s_3, -0.4229)$, $S(\kappa_3) = (s_1, 0.4485)$, $S(\kappa_4) = (s_2, -0.2457)$, $S(\kappa_5) = (s_2, -0.4714)$. Thus, the ranking order is $\kappa_2 > \kappa_1 > \kappa_4 > \kappa_5 > \kappa_3$. For the P2TLWG operator, the calculation values are $S(\kappa_1) = (s_2, 0.0365)$, $S(\kappa_2) = (s_2, 0.4075)$, $S(\kappa_3) = (s_1, 0.2467)$, $S(\kappa_4) = (s_2, -0.3801)$, $S(\kappa_5) = (s_2, 0.4881)$. So the ranking order is $\kappa_2 > \kappa_1 > \kappa_4 > \kappa_5 > \kappa_3$.

What's more, our presented method is compared with EDAS method with picture 2-tuple linguistic (Zhang *et al.*, 2019a). Then we can obtain the calculation result. The appraisal score values of each alternative are determined as: $AS_1 = 0.6225$, $AS_2 = 0.8347$, $AS_3 = 0.1803$, $AS_4 = 0.3987$, $AS_5 = 0.2284$. Hence, the ranking order of alternatives is $\kappa_2 > \kappa_1 > \kappa_4 > \kappa_5 > \kappa_3$.

Eventually, the results of these four methods are presented in Table 13.

Derived from the Table 13, it is evident that the optimal supplier is κ_2 in the mentioned methods, while the worst choice is κ_3 in most situations. That's to say, these methods' ranking results are slightly different. The EDAS method with picture 2-tuple linguistic emphasizes the positive and negative distances from the average solution respectively, while our developed method is more practical and effective, since the decision makers' bounded rationality can be fully taken into consideration relying on the Euclidean and Hamming distances in terms of the negative-ideal point and its computation procedures are relative simple.

For visibility, this optimal order and the ranking orders presented in Table 13 are all described in Fig. 2.

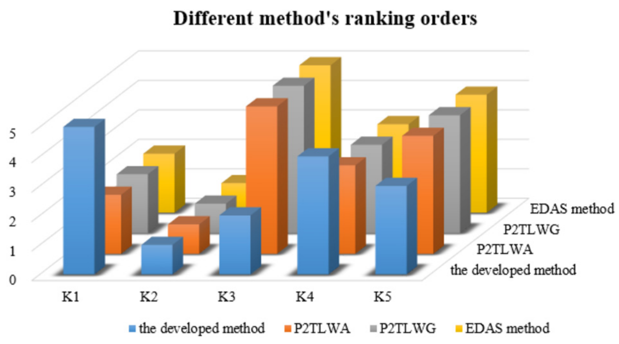

Fig. 2. Ranking orders on the basis of a same example.

6. Conclusion

In this paper, the CODAS method with P2TLNs is developed to address the MAGDM issues relying on the description of the CODAS method and some fundamental notions of P2TLSs. To begin with, the fundamental information of P2TLSs is simply reviewed. Following that, the P2TLWA and P2TLWG operators are utilized to integrate the P2TLNs. Subsequently, depending on CRITIC method, the attributes' weights are decided. What's more, applying the CODAS method to the picture 2-tuple linguistic environment, a novel method is constructed and the calculating procedures are briefly depicted. Eventually, an application of selecting the optimal green supplier has been given to confirm that this novel method is more valid and the comparative analysis between the CODAS method with P2TLNs and several methods are also made to further verify the merits of this method. The contribution of this paper can be highlighted as follows: (1) the CODAS method is modified by P2TLNs; (2) the picture 2-tuple linguistic CODAS (P2TL-CODAS) method is designed to tackle the MAGDM issues with P2TLNs; (3) the CRITIC method is utilized to decide the attributes' weight; (4) a case study for green supplier selection is designed to prove the developed method; (5) some comparative studies with existing methods are given to verify the rationality of P2TL-CODAS method. In our future research, the proposed methods and algorithm will be needful and meaningful for other real decision making problems and the developed approaches can also be extended to other fuzzy and uncertain information.

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S. Zhang is currently a master's student at School of Business, Sichuan Normal University, Chengdu, 610101, PR China.

Guiwu Wei has an MSc and a PhD degrees in applied mathematics from SouthWest Petroleum University, business administration from School of Economics and Management at SouthWest Jiaotong University, China, respectively. From May 2010 to April 2012, he was a postdoctoral researcher at the School of Economics and Management, Tsinghua University, Beijing, China. He is a professor at the School of Business, Sichuan Normal University. He has published more than 100 papers in journals, books and conference proceedings including journals such as *Omega*, *Decision Support Systems*, *Expert Systems with Applications*, *Applied Soft Computing*, *Knowledge and Information Systems*, *Computers & Industrial Engineering*, *Knowledge-Based Systems*, *International Journal of Uncertainty*, *Fuzziness and Knowledge-Based Systems*, *International Journal of Computational Intelligence Systems and Information*: *An International Interdisciplinary Journal*. He has published 1 book. He has participated in several scientific committees and serves as a reviewer of a wide range of journals including *Computers & Industrial Engineering*, *International Journal of Information Technology and Decision Making*, *Knowledge-Based Systems*, *Information Sciences*, *International Journal of Computational Intelligence Systems* and *European Journal of Operational Research*. He is currently interested in aggregation operators, decision making and computing with words.

R. Wang has a PhD degree in management from the School of Accounting, Southwestern University of Finance and Economics, Chengdu, 610074, PR China. He is an associate professor at the School of Business, Sichuan Normal University. He is currently interested in decision making in financial management.

J. Wu has a PhD degree in management science and engineering from Southwest Jiaotong University, China. He is a professor at the School of Statistics, Southwestern University of Finance and Economics, Chengdu, 611130, PR China.

C. Wei has an MSc degree in applied mathematics from SouthWest Petroleum University. Now, he is a PhD student at the School of Statistics, Southwestern University of Finance and Economics, Chengdu, 611130, PR China. He has published more than 10 papers in journals such as *International Journal of Intelligent Systems*, *Journal of Intelligent and Fuzzy Systems*, *IEEE Access*, *Mathematics*, *Information*. He is currently interested in aggregation operators, decision making and computing with words.

Y. Guo has a PhD degree in management science and engineering from Southwest Jiaotong University, China. He is an associate professor at the School of Finance, Southwestern University of Finance and Economics, Chengdu, 610074, PR China. He is currently interested in forecasting in financial and decision making in financial markets.

Y. Wei has an MSc and a PhD degree in management science and engineering from Southwest Jiaotong University, China. He is a professor at the School of Finance, Yunnan University of Finance and Economics. He has published more than 120 papers in in journals, books and conference proceedings including journals such as *Journal of Forecasting*, *Journal of Banking & Finance*, *Empirical Economics*, *Energy Economics*, *Economic Modelling*, *Applied Economics Letters*, *International Review of Economics & Finance*. He has published two books (in Chinese). He has participated in several scientific committees and serves as a reviewer of a wide range of journals including *Energy Economics*, *Journal of Forecasting*, *Emerging Market Finance and Trade*, *Physica A*. He is currently interested in volatility modelling and forecasting in financial and energy markets, density forecasting and decision making in financial markets.