COMPUTER CHECKS ON HUMAN ANALYSES OF THE KRKB ENDGAME

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1. INTRODUCTION

The two senior authors, conform to their intention of making the ICCA Journal a repository of database results (Herschberg and Van den Herik, 1985a), are happy to report the *detailed* outcome of the undergraduate research of the two junior authors on the KRKB endgame.
As far back as 1970, Ströhlein (1970), in his Ph.D. thesis, had constructed a KRKB database. However, he did not reveal the number of won and drawn positions, nor, apparently, was there a facility to check published analyses by human authors against the database. The research to be reported was inspired, in part, by the publication, in Dutch, of Spinhoven and Bondarenko (1983). This book encompasses 24 studies in this endgame (Chapter 1) and our curiosity was challenged to determine whether some non-optimal variations were published or, more seriously, whether the studies would be correct in their assigning of games as drawn or won.
A database was constructed according to the principles published in Van den Herik and Herschberg (1985) and using the software for database construction then available at the Delft University of Technology.

As usual in this endgame, the following assumptions are made:

- White is to move;
- the black Bishop occupies a dark-coloured square;
- the black King is confined to the trapezium bordered by a1-h1-e4-d4, border squares included.

As is well-known, the second assumption reduces the positions to be investigated by a numerical factor of 2, the third assumption by 64/20, for a total of 6.4, leaving 2, 621, 440 positions to be scanned in detail.
Legitimate and illegitimate positions were conventionally (cf. Van den Herik and Herschberg, 1985, p. 70) distinguished as follows: a position is considered *illegitimate* whenever at least one of the following conditions holds:

(a) Black is in check;

(b) the WK and the BK have a distance less than 2;

(c) the WK coincides with the black Bishop;

(d) the WK coincides with the white Rook;

(e) the BK coincides with the white Rook.

All other positions are considered *legitimate.*

In Van den Herik and Herschberg (1985) condition (e) was not imposed. The difference is merely administrative because in Table 1 below the KKB WTM positions are now classified as illegimate KRKB, whereas they were counted as draws if the conditions (a) to (d) inclusive were valid only.

All legitimate positions are initialized to have a mating distance of zero. It is worth noting that whenever the WK coincides with the BB, the position is legitimate and the distance to win correctly is found to be 0, since this implies a KRK-endgame, a degenerate case of the KRKB endgame.

It follows from the above that the database is *not* organized in the *ultimate* sense, distances-to-mate being considered on a par with distances-to-conversion.
This implies that, e.g., in the position WKd6 WRh5 BKd8 BBh4 (WTM), the mating move 1. Rh8 and the converting move 1. Rxh4 are considered equivalent.

2. IMPLEMENTATION

The rest of the database construction is conventional and in Pascal requires about 1000 lines of source code. Building the database on an 8.5 MIPS IBM mainframe under CMS takes some 15 minutes of CPU time or some 8×10^9 instructions in attributed CPU time. It is instructive to compair this elapsed time to Ströhlein (1970), when it took longer by a factor of 30 for 6.5 hours, which is an implied compliment to Ströhlein.

3. RESULTS

The single most significant result to emerge from our database is the maximin of 18 moves, fully in accordance with Ströhlein (1970) and Thompson (as reported by Herschberg and Van den Herik, 1985b). Table 1 below shows the distribution of won positions (column 2) over their distances-to-win (mate or conversion), referred to the full board. Needless to say, column 2 is comparable among implementations, since each of these may adopt different symmetry stratagems which, if correct, must coincide when referred to the full board.

# of moves to win	# of the full-board WTM positions
0 (drawn)	7,441,520
1	3,016,448
2	608,436
3	57,220
4	24,064
5	16,976
6	14,024
7	11,416
8	8,376
9	6,568
10	6,148
11	6,112
12	4,228
13	2,336
14	1,568
15	1,272
16	1,256
17	416
18	232

Table 1: Distances-to-win by frequency of occurence.

It is easily read from Table 1 that

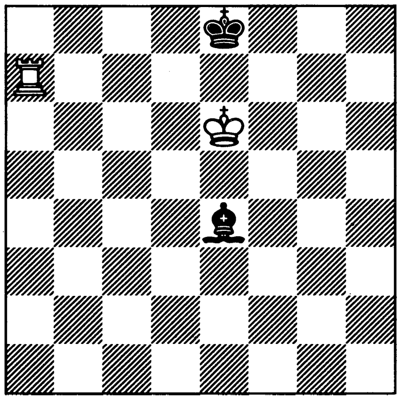
the number of won positions is	3,787,096	
the number of drawn positions is	7,441,520	
the number of illegitimate positions is	5,548,600	+
As an independent check we note that	16,777,216	$= 64^4$

It is somewhat sad to note that the KRKB endgame is essentially less interesting than other four-men endgames, such as KBNK. For one thing, the percentage of draws referred to all legitimate positions is 66.3. The complement, being *won* positions, 33.7%, is more than slightly misleading since these positions are won-in-1 or won-in-2 to an overwhelming extent. In fact, omitting these, there remain 0.4% of wins-in-more-than-2 which, we submit, are the only worthy subjects of KRKB endgame studies.

4. EXAMPLES

Exploration of our database indicates that at least 13 of the 24 studies in Chapter 1 of Spinhoven and Bondarenko (1983) can be faulted for non-optimality. Among these we choose three of the more interesting ones (nos. 6, 7 and 9).

4.1. EXAMPLE 1: Dr. H. von Gottschall (1889)



The author's solution indicates to a distance-to-win of 11; the database concurs with this conclusion. However, closer analysis shows that the solution by Dr. von Gottschall deviates four times from the optimal path. In 4.1.1 below, we present Dr. von Gottschall's solution with the database's comments (*in italics*); in 4.1.2 the optimal variation of 11 moves is shown.

White: Ke6 Ra7;
Black: Ke8 Be4;
White to move and to win.

DIAGRAM 1

4.1.1 Dr. von Gottschall's Solution

As a solution, the author presented: **1. Re7+ Kd8 2. Kd6 Bd3 3. Re3 Bc2**

Non-optimal black move, from a win-in-8 to a win-in-7. The optimal move is 3. ... Bc4.

4. Re1

Non-optimal white move, from a win-in-7 to a win-in-9. The optimal continuation is 4. Re2 Bd3 5. Rd2. This is also remarked by Spinhoven and Bondarenko. After 4. Re2 Bd1 White also continues with 5. Rd2, but 4. ... Bd1 is non-optimal, from a win-in-6 to a win-in-5.

4. ... Bd3 5. Rd1

Non-optimal white move, from a win-in-9 to a win-in-14. The author must have overlooked the optimal black countermove 5. ... Ba6. The optimal path for White here reads (with equi-optimal moves in parentheses): 5. Re3 Bc4 6. Rc3 (Re4) Be2 7. Rc2 Bd3 8. Rd2 Bg6 (Ba6) 9. Rg2 Bf7 10. Rh2 Kc8 (Bg6) 11. Rh8+ Kb7 12. Rh7 winning the Bishop.

5. ... Bc2

Non-optimal black move, from a win-in-14 to a win-in-6. May this be regarded as a real blunder in a study? The optimal defense consists in 5. ... Ba6 with the possible optimal continuation 6. Ra1 Bb7 7. Re1 Ba6 8. Re7 Kc8 9. Rc7+Kb8 10. Kc6 Bc4 (Bd3) 11. Kb6 Bb3 (Ba2) 12. Rc1 Ba2 (Ba4) 13. Ra1 Bb3 14. Ra3 Be6 15. Re3 Bf7 (Bd7) 16. Rf3 Be6 17. Rf8+ Bc8 18. Kc6 (Re8, Rd8, Rg8, Rh8) etc.

6. Rd2 Bg6 7. Rg2 Bf7 8. Rh2 Kc8 9. Rh8+ Kb7 10. Rh7 and 11. Rxf7.

4.1.2 The Optimal Variation

Below we present the optimal variation for Dr. von Gottschall's study (equi-optimal moves are in parentheses).

1. Re7+ Kd8 (Kf8) 2. Kd6 Bd3 3. Re3 Bc4 4. Rc3 (Re4) Be2 5. Rc2 Bd3 6. Rd2 Bg6 (Ba6) 7. Rg2 Bf7 8. Rh2 Kc8 (Bg6) 9. Rh8+ Kb7 10. Rh7 Ka6 (Kb6, Ka7, Ka8, Kb8, Kc8) 11. Rxf7.

4.2. EXAMPLE 2: Th. Molien (1895)

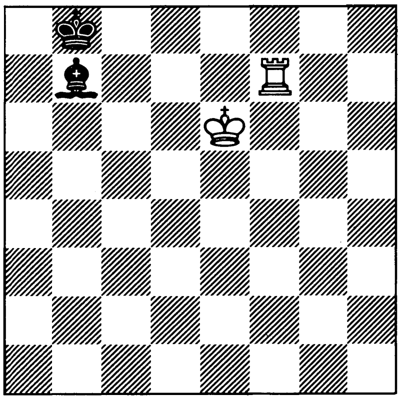


DIAGRAM 2

The published study claims a distance-to-win of 9. Since the database concludes that the distance-to-win is 14, it is safe to assume that in the human study, Black has considerably assisted White in achieving his goal. In 4.2.1 below, we republish Molien's solution with the database's comments (*in italics*) for a win-in-9, as opposed to 4.2.2, where optimal play and counterplay are presented.

White: Ke6 Rf6;
Black: Kb8 Bb7;
White to move and to win.

4.2.1 Molien's solution

As a solution, Molien proposed the following variation:

1. Kd7 Bg2

Non-optimal black move, from a win-in-13 to a win-in-7. The optimal move is 1. ... Bc8+. In their annotation, Spinhoven and Bondarenko (1983) mention this move as an equivalent to the text move. However, the 1983 authors continue after 2. Kc6 for Black with 2. ... Ka7? allowing White to have a six-move speed up (from a win-in-12 to a win-in-6); the optimal move after 2. Kc6 is 2. ... Bh3, (see the database's optimal variation in 4.2.2).

2. Rb6+ Bb7

Non-optimal black move, from a win-in-6 to a win-in-5. Optimal moves are 2. ... Ka7 and 2. ... Ka8. The 1983 authors provide an optimal analysis for 2. ... Ka7 by 3. Kc7 Be4 4. Rb4 Bc2 5. Rb2.

3. Kd8

Non-optimal white move, from a win-in-5 to a win-in-6. Optimal moves are 3. Rb5, 3. Rb4, 3. Rb3, and 3. Rb1.

3. ... Ka7 4. Kc7 Be4

Also optimal is 4. ... Bg2.

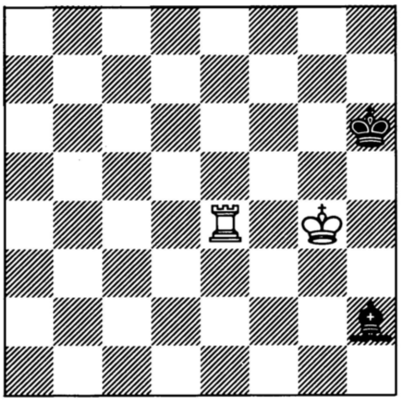
5. Rb4 Bc2 6. Rb2 Bd3 7. Ra2+ Ba6 8. Ra1 Ka8 9. Rxa6 mate.

4.2.2 The Optimal Variation

Below we present the optimal variation for Molien's study (equi-optimal moves are in parantheses).

1. Kd7 Bc8+2 Kc6 Bh3 3. Rf2 Ka7 (Bg4) 4. Rb2 Bf1 (Bg4 Bf5) 5. Rb6 Bd3 6. Kc7 Bh7 7. Re6 (Rh6) Bg8 8. Re8 Bf7 (Bh7) 9. Re7 (Rf8) Bc4 10. Re4 Bb3 (Bb5) 11. Re3 Bc4 12. Ra3+ Ba6 13. Kc6 (Ra2, Ra1, Ra4, Ra5) Ka8 (Kb8) 14. Rxa6.

4.3. EXAMPLE 3: F. Sackmann (1898)



Our third example is one of the maximin positions in the endgame with a distance-to-win of 18. Sackmann stated it to be a win-in-13. Again, we publish Sackmann's solution in 4.3.1 as adversely commented upon by the database (*in italics*), which we contrast in 4.3.2 with optimal play and counterplay as derived from the database.

White: Kg4 Re4;
Black: Kh6 Bh2;
White to move and to win.

DIAGRAM 3

4.3.1 Sackmann's Solution

As a solution to his study, Sackmann supplied a 13-move variation.

1. Kf5 Kg7

The 1983 authors present 1. ... Bb8 as an alternative; *the latter move is not optimal: from a win-in-17 to a win-in-12 (the optimal answer is 2. Rg4).*

2. Rd4 Kf8

The other moves (2. ... Kf7, 2. ... Kg8 and 2. Bb8) mentioned by the 1983 authors are not optimal .

3. Ke6 Bg3

Also optimal is 3. ... Kg7.

4. Rc4 Bh2 5. Rh4 Bg3 6. Rh3 Be1

Non-optimal black move, from a win-in-12 to a win-in-8. The optimal move was 6. ... Bc7.

7. Kf6 Kg8 8. Kg6 Kf8

Non-optimal black move, from a win-in-6 to a win-in-5. The optimal move was 8. ... Bb4.

9. Rf3+ Kg8 10. Re3 Bb4 11. Re8+ Bf8 12. Kf6 Kh7 13. Rxf8.

4.3.2 The Optimal Variation

Below we show the optimal variation for Sackmann's study (equi-optimal moves are in parentheses).

1. Kf5 Kg7 2. Rd4 Kf8 3. Ke6 Kg7 (Bg3) 4. Rd2 (Rg4) Bb8 5. Rb2 Bf4 6. Rg2+ Kh6 7. Kf5 Be3 (Bc1, Bc7, Bb8) 8. Rg6+ Kh7 9. Kf6 Bf2 (Bd4, Bc5, Bb6) 10. Kf7 Ba7 11. Rd6 (Ra6) Bb8 12. Rd8 Ba7 (Bc7) 13. Rd7 Bf2 14. Rd2 Bg1 (Bg3) 15. Rd1 (Rd3, Rg2) Be3 16. Rh1+ Bh6 17. Kf6 (Rh2, Rh3, Rh4, Rh5) Kg8 (Kh8) 18. Rxh6.

5. CONCLUSIONS

1. Ströhlein's (1970) maximin of 18 moves for KRKB (WTM) is confirmed by our database.
2. The analyses in human chess literature satisfy reasonable standards. Notably, their judgements *won* or *drawn* are correct. Yet the winning sequences regularly contain non-optimal moves. This does not, however, detract from a great appreciation of the authors having composed the chess studies.
3. Once they have been shown to a chess-player, the computer-generated paths-to-win are not difficult to follow. Without the support of a database, *some* path to win is not difficult to find for a chess-player. What *is* difficult to find for a human player without benefit of database is the *optimal* path to win. However, in the particular case envisaged, this is next to irrelevant since this endgame is so speedily decided as to need no invocation of the 50-move rule.

6. REFERENCES

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