

Measuring Criteria: Weights of Importance

Recent advances in decision analysis, decision support systems and multiple criteria decision making have brought forth the notion of relative importance of decision-making criteria and objectives and the measurement of importance via so-called *weights of importance*.

The issue of differential importance is not as simple as it appears on the surface. With the increasing role of descriptive decision-making models, we are naturally more interested in the actual and natural use of weights by humans, and less interested in prescribing certain mathematical formalisms connected with the old utility theory. In this process we are discovering that our understanding of criteria weights and human weighting process is limited and possibly even incorrect.

It is intuitively felt that humans do weigh decision criteria differentially: some are *very* important, some are *less* important, others can be preemptive or negligible. Very few would argue that there is only one all-important criterion, or that all criteria are somehow equally important, and thus their weights would not matter. The problem is in the actual meaning of saying that one criterion is more important than another. What is being expressed? What is the expectation of such expression? What does it mean: 'I consider the salary and the geographical location of my new job to be equally important criteria?'

Traditionally, weights of importance have been conceived as some sort of *multipliers*, usually normalized (non-negative, adding to 1), 'discounting' the contribution of a criterion performance into some sort of weighted super-function. The function:

$$U(\lambda, x) = \lambda_1 f_1(x) + \lambda_2 f_2(x) + \dots + \lambda_n f_n(x)$$

would be a typical example of such 'multiplier' concept of weights λ_i . The larger the weight of importance λ_i , the more valued is criterion performance within the aggregate of $U(\lambda, x)$.

Why should the notion of criterion importance be related to or derived from the performance of superfunction $U(\lambda, x)$ is difficult to answer.

Maximization of superfunction $U(\lambda, x)$ is in itself a criterion which can work against and overrule any a priori expressed notions of criteria importance. So, the main question seems to be emerging: Are the expressions of criteria importance related to the performance of criteria themselves, or are they related to (or derived from) the performance of the superfunction?

Let us explore a simple numerical example.

Example. Let us have five criteria judged to be (for simplicity) equally important. Consider two feasible alternatives with the following criteria performances:

$$\begin{array}{ll} \text{A: (17; 1; 1; 1; 1)} & U = 4.2 \\ \text{B: (4; 4; 4; 4; 4)} & U = 4.0 \end{array}$$

Because of the equal importance, all multipliers λ_i , would be equal to 1/5 and the weighted sums U would be 4.2 for A and 4.0 for B. By selecting the U -maximizing alternative A, the decision maker ends up with one criterion performing extremely well and others relatively poorly, although he desired their equal importance. Alternative B does not maximize U , but reflects comparatively balanced performance on all criteria. The difference can be substantial. If I am to choose an automobile according to five equally important criteria, the DSS or MCDM system could recommend car A (extremely luxurious, expansive,

unreliable, etc.) while I actually wanted car B, moderate but balanced in all criteria of choice.

It is quite possible that some of the nonacceptance, discrepancies and 'irrationality' of human decision making versus DSS models is due to the fundamentally different perceptions of the meaning of weights of criteria importance:

1. Humans maximize superfunction U and their expressions of criteria importance are simply multipliers or discount factors which bring criteria performances differentially into the weighted sum. Individual performance are less important than overall performance, collective is above individual.
2. Humans do not maximize any superfunction and their expressions of criteria importance reflect relative performance aspirations of individual criteria. Individually independent performances are more important than the overall performance, collective is secondary, individual primary.

The above two notions are as conflicting as collective socialism and competitive society of free-market individuals. Their confusion could lead to significant discrepancies between descriptive and prescriptive models.

It seems that multiple criteria decision making (MCDM) is not about maximizing a single superfunction but about maximizing all criteria individually, strictly in a vector sense, according to their expressed relative importances.

Each decision situation is characterized by a specific context of feasible alternatives. This context delimits current maximal and minimal performances achievable with respect to all criteria (ideal points). Humans expect performances of very important criteria to be close to these maxima and less important criteria to be further away from such maxima. But the main point is that if all criteria are equally important then their resulting

individual performances must be equal (within the context) in terms of percentage achievement of the individual maxima.

The problem disappears if there is only one single function: its maximum (ideal) value or performance can always be achieved, by definition.

For many decades the vector maximization problem has been stated but not treated as vector maximization but as scalar maximization (via its conversion into superfunction). So called MCDM approaches have thus never differentiated themselves from traditional OR/MS or utility-theory based economics.

Proper treatment of the vector problem would involve competition of individual criteria, not their subservience to the command of collective superfunction. The weights of importance would reflect relative power or strength of competing 'individuals'. Free market competition, within the contextual limits of the problem, decide the final performances (final choice), not the superfunction.

We now have a technology and know-how which would allow proper treatment of the vector maximization problem. It is clear that the solution cannot be analytical, but only simulational. We have experience with parallel processing, neural networks, holistic graphics, cognitive equilibrium and rule-based simulation (like autopoiesis). We also have good experiences with man-machine interaction, dialogue and exchange. It seems possible to approach MCDM de novo, from its own inner perspective, and divorce it, once and for all, from the excessive analytical baggage rooted in the utility functions of the last century.

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