

Continuous Time Paradigms in Logic and Automata

In the last 15 years, research in Computer Science has involved many paradigms in which continuous time appears whether in a pure way or in cooperation with discrete time. In particular, this is evident in subjects concerning Automata and Logic. This issue of FI consists of four papers dedicated to such subjects. The papers will be referred to below as [HR], [S], [PRT], [T] (the initials of the authors). [HR] and [S] are about continuous-time logics, whereas in [T] and [PRT] the focus is on automata.

In [HR] the concern is about Monadic Logics of Order (MLO) and their relationship to Temporal Logics. For discrete time, decidability of Second Order Monadic Logic (SOML) follows from the connection between SOML and automata theory. It is well known that SOML covers numerous temporal logics (TL). Ultimately, these TL may be considered as syntactically sugared versions of SOML fragments. In [HR] further facts of this kind are established for continuous time. However, in this case, instead of traditional automata-theoretic techniques one needs to use properly general theorems from logic.

The logic framework developed in [S] is based on First Order Timed Logic (FOTL), that allows functions and predicates with more than one argument; moreover, it allows also some arithmetics. This makes the logic expressive enough to represent, more or less directly, continuous-time properties of distributed algorithms. But, on the other hand, it makes the logic undecidable. The fundamental observation that, nevertheless, permits the efficient use this logic for verification is as follows: the underlying theories of continuous time (e.g. the theory of real addition, Tarski algebra, etc.) are decidable or have much better complexity than the corresponding theories of discrete time. Interesting decidable classes of the verification problem are based on appropriate properties of FOTL.

The companion papers [T] and [PRT] draw their initial motivation from the literature on Hybrid Automata, Circuits and related control problems. Concrete problems about circuits (feedback reliability, completeness) and control (sample-and-hold architectures in continuous time) are also the subject of [T] and [PRT]. Yet, a more general contribution is the development of a conceptual framework that allows one to highlight the genuine distinctions and similarities between the discrete-time and continuous-time tracks.

There is a growing feeling in the community that the literature on these subjects, as well on the related logics, is plagued by a Babel of models, constructs and formalisms with an amazing discord of terminology and notation. Further models and formalisms are engendered, and it is not clear where to stop. Hence, appeals like:

1. “look back to sort out what has been accomplished and what needs to be done... by surveying logic-based and automata-based real-time formalisms and putting them into a perspective” (R. Alur and T.Henzinger).
2. “...isolate the right concepts, ... formulate the right models, and discard many others, that do not capture the reality we want to understand...”(J. Hartmanis).

The analysis of such formalisms and systems is not the goal of [S]; here the control systems under consideration are reduced to rather primitive ones by declaring the relevant properties in logic specifications. Accordingly, [S] confines itself to the following informal remark: an algorithm interacting with a continuous-time device is, usually, called a hybrid automaton.

On the other hand, [HR], [T] and [PRT] are strongly committed to the analysis in depth of the various continuous-time paradigms and to their robust conceptual integration in mainstream Automata Theory and Logic. These papers come to the following conclusions about misconceptions in the previously suggested models and logics:

- (i). The standard continuous-time model for logic was ignored as a yardstick; instead, different kinds of sequences of continuous-time bits were used ([HR]).
- (ii). Input/output behavior of automata was ignored in favor of generating devices ([T]).

More details and reflections follow:

[HR]: When it came to continuous time most of the researchers decided to abandon the natural model of the real line in favor of discrete models in which the elements of the sequence were decorated by some information concerning the real line. This may have been an attempt to pursue the connection with automata theory since automata were traditionally associated with sequences. This was the main cause in the rejection of the classical model. It complicated the subsequent research. The choice of the temporal logic became an arbitrary decision which was no longer backed up by the expressive power of classical logic. A host of different formalisms was introduced, as well as an astonishing variety of pairs “model - logic”. This work never converged to an accepted model; models and formalisms continued to proliferate.

[T]: Functions (in particular, input/output behavior of automata) are more fundamental than sets (say, languages accepted by automata). In particular, the restriction to language - theoretic formulations hides issues that are crucial for proper handling of circuits and hybrids. Surprisingly, it has been left unobserved that some flaws in the conceptual decisions concerning continuous time are identifiable already at the level of discrete-automata theory.

Editorial and historical comments. Sometimes, slightly different notation and terminology are used in the papers. This is visible especially in the companion papers [PRT] and [T] that have much in common but were written at different times. It seemed too burdensome to impose unified standards and to require the

respective revisions. Hopefully, the reader will easily overcome this lack of uniformity due to appropriate footnotes and mutual references.

The Papers

[HR] Hirshfeld Y. and Rabinovich A.

Logic for Real Time: Decidability and Complexity

[S] Slissenko A.

A Logic Framework for Verification of Timed Algorithms

[T] Trakhtenbrot B.A.

Understanding Automata Theory in the Continuous Time Setting

[PRD] Pardo D., Rabinovich A., Trakhtenbrot B.A.

Synchronous Circuits over Continuous Time: Feedback Reliability and Completeness

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