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The relationship between risk and incomplete states uncertainty: a Tsallis entropy perspective

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Abstract. This paper provides a "non-extensive" information theoretic perspective on the relationship between risk and incomplete states uncertainty. Theoretically and empirically, we demonstrate that a substitution effect between the latter two may take place. Theoretically, the "non-extensive" volatility measure is concave with respect to the standard (based on normal distribution) volatility measure. With the degree of concavity depending on an incomplete states uncertainty parameter-the Tsallis-q. Empirically, the latter negatively causes the normal measure of volatility, positively affecting the tails of the distribution of realised log-returns.

Keywords: Tsallis Entropy, Incomplete Statistics, Volatility, Uncertainty.

Introduction

Financial risk models seek to accommodate uncertainty, which is distinct of the notion of risk (Knight, 1921). In other words, they seek to account implicitly for *ex-ante* unknown future realisations (risks) and their associated probabilities (uncertainty). Classical models, however, have confused uncertainty with risk. Schinckus (2009) explains the latter confusion with the adoption of expected utility theory (EU) as a principal framework for analysing decision making under uncertainty. Its underlying principal is that "objective" probabilities can be estimated when the set of future realisations (or states) is *a-priori* complete. Hence, classical risk measures confound risk with uncertainty.

Critics of the EU framework, such as Ellsberg (1961), point to the difficulty of estimating these "objective" probabilities when the set of future states is *a-priori* incomplete. Therefore, generalisations of the EU have sought to accommodate for an added uncertainty due to incomplete *ex-ante* knowledge of future states. This is done usually by "distorting" the "objective" probabilities. For example, the prospect theory (PT) of Kahneman and Tversky (1979) introduce a value function that overweight states with small probabilities. The anticipated utility theory (AUT) of Quiggin (1982) distorts probabilities as in PT, but by distorting the probability density function as whole and not individually like in PT. Tversky and Kahneman (1992) extends the PT and AUT to the Cumulative PT by allowing probabilities to be jointly weighted without not violating the transitivity axiom (as is the case of AUT). Further examples of

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using "distorted" probabilities also include: Karmarkar (1978), Chew and MacCrimmon (1979) and Gul (1991).

In general, uncertainty is a notion that relates to what is unknown, or non quantifiable or uninsurable (see: LeRoy and Singell (1987), Langlois and Cosgel (1993) and Schinckus (2009) for further discussion). Nevertheless, statistical mechanics and information theory (Shannon, 1948; Jaynes, 1957) provide a model to circumvent uncertainty (based on Laplace principle of insufficient reason) using entropy functions. In Boltzmann (1878), entropy (or expected uncertainty) depends only on the normal standard deviation (or volatility – σ_1) which in finance is used as measure of risk. In that sense, volatility summarises both risk (future realisation is unknown) and uncertainty (unknown probabilities). However, it is unable to account for possible incompleteness of the set of future states, which renders the task of estimating probabilities even more difficult.

In this paper, two parameters determine overall uncertainty: the Tsallis parameter q and a "non-extensive" measure of risk (or standard deviation) – σ_q. The first parameter relates to the added uncertainty due to suspected incomplete knowledge of future states. The second is shown to be concave with respect to the normal standard deviation and vanishes when q tends to its limit (at $q = 3$). This suggests that a substitution effect may take place between risk (σ_q and σ_1) and incomplete states uncertainty (q).

This incomplete states uncertainty (implied from historical log-returns of the S&P-500) is shown to be exclusively caused (in Granger (1969) sense) by the S&P-500 implied volatility index (VIX) but not vice-versa. However, causes negatively the standard deviation of S&P-500 log-returns, and thus offsetts the impact of the VIX over the standard deviation of log-returns. Nevertheless, given a power-law *p.d.f.*, the standard deviation decreases in tandem with increasing distribution tail. This points out that information flowing from the VIX may carry a tail effect on the distribution of returns. Which justifies its use as a measure of "fear"[1].

An implication of our use of Tsallis (1988) entropy, is a portfolio of assets with incomplete states uncertainty that cannot be easily diversified away in the same manner risk is diversified away. For example, adding two negatively correlated assets may reduce risk (σ_1) but not the incomplete state uncertainty (q).

[1] Whaley (2000), associates the S&P 500 implied volatility index (VIX) as a measure of overall market "fear".

1. Risk and uncertainty – the non-extensive perspective

1.1. Extensive and non-extensive uncertainty

The uncertainty associated with a gaussian random variable x_t ($x_t \sim \mathcal{N}(\mu, \sigma_1)$) is summarised by Boltzmann (1878) entropy function:

$$H_B\left[f(x_t|\mu,\sigma_1)\right] = -\int_{-\infty}^{+\infty} f(x_t|\mu,\sigma_1)\ln f(x_t|\mu,\sigma_1)dx_t = \frac{1}{2}\ln(2\pi\sigma_1^2 e) \quad (1)$$

Where: $f(x_t|\mu,\sigma_1)$ is the normal *p.d.f* and σ_1^2 is the variance of x_t. Hence, in equation (1), σ_1^2 summarises the overall uncertainty associated with possible realisations and probabilities of x_t.

Tsallis (1988) generalises equation (1) to a function that accommodates long-term memory and long-range interactions within elements of x_t (Borland, 2005). In other words, Tsallis generalises Boltzmann's extensive entropy to non-extensive systems by including a parameter q that alters the entropy function to the following functional form:

$$\begin{aligned} H_q\left[f(x_t)\right] &= -\int_{-\infty}^{+\infty} f^q(x_t)\ln_q f(x_t)dx_t \\ &= \frac{1-\int_{-\infty}^{+\infty} f^q(x_t)dx_t}{q-1} \\ \lim_{q\to 1} H_q\left[f(x_t)\right] &= -\int_{-\infty}^{+\infty} f(x_t)\ln f(x_t)dx_t \end{aligned} \quad (2)$$

This does not only alter the entropy functional form, but also modifies the computation of statistical moments. A notable modification of the latter is the use of escort probabilities to compute volatility (and other statistical moments):

$$\sigma_q^2 = \frac{\int_{-\infty}^{+\infty} x_t^2 f^q(x_t)dx_t}{\int_{-\infty}^{+\infty} f^q(x_t)dx_t} \quad (3)$$

The use of escort probabilities to compute statistical moments is justified by their generating power law distributions (by maximising equation (2) subject to equation (3)) that cannot be generated otherwise (Tsallis, 1991; Tsallis et al., 1998). It is also justified from an incomplete statistics point of view (Wang, 2002; Darooneh et al., 2010) where q acts as a non-linear normalising parameter that accounts for possible incomplete set of future states. It does so by "distorting" the *p.d.f.* Gell-Mann and Tsallis (2004) review the connection that this parameter has with the dynamics associated with a given system. For example, maximising equation (2) subject to equation (3) yields a distribution that describes a Lévy-like anomalous diffusion or a distribution associated with a stochastic differential equation with an additive and multiplicative noise (Nakao, 1998).

An interpretation of q depends on its context. For example, Ludescher et al. (2011) show that it is logarithmically increasing with the average interoccurrence time between losses associated with holding a stock. In Borland (2002) it is associated with the option implied volatility smile. Borges and Roditi (1998) relate it to the Jackson q-derivative, and so on. The present paper adopts the incomplete statistic narrative, where it is a measure of the uncertainty over the completeness of the set of future states. Furthermore, it is also related to the generalisation of the expected utility theory. An analogy that has been documented in Tsallis et al. (2003).

Maximising equation (2) subject to equation (3) and to $\int f(x_t)dx_t = 1$ yields a power-law *p.d.f*:

$$
\begin{aligned}
f(x_t) &= \frac{\left(1-(1-q)\beta^* x_t^2\right)^{\frac{1}{1-q}}}{Z(q,\sigma_q)} \\
Z(q,\sigma_q) &= \frac{\sqrt{\pi(\frac{2}{q-1}-1)}\Gamma\left(\frac{1}{q-1}-\frac{1}{2}\right)\sigma_q}{\Gamma\left(\frac{1}{q-1}\right)} \\
\beta^* &= \frac{\beta}{\int_{-\infty}^{+\infty} f^q(x_t)dx_t} = \frac{1}{(3-q)\sigma_q^2} \\
\int_{-\infty}^{+\infty} f^q(x_t)dx_t &= \frac{\pi^{\frac{1}{2}-\frac{q}{2}}(q-1)\Gamma(\frac{1}{2}+\frac{1}{q-1})\sigma_q^{1-q}\left(\frac{\sqrt{(\frac{2}{q-1}-1)}\Gamma(-\frac{1}{2}+\frac{1}{q-1})}{\Gamma(\frac{1}{q-1})}\right)^{-q}}{\sqrt{(3-q)(q-1)}\Gamma\left(\frac{1}{q-1}\right)} \\
q &\in [1;3]
\end{aligned}
\tag{4}
$$

Figure 1 plots equation (4) for several values of q assuming x_t is standardised as well as the escort *p.d.f* used in equation (3).

Using equation (4) and equation (2), a functional form of uncertainty ($H_q\left[f(x_t)\right]$) is obtained. This uncertainty is now determined by a "non-extensive" volatility parameter (σ_q) and a parameter q. Inserting $\int_{-\infty}^{+\infty} f^q(x_t)dx_t$ in equation (2) and taking the limits $q \to 1$ and $q \to 3$, yields the followings:

$$
\begin{aligned}
&\lim_{q\to 1} H_q\left[f(x_t)\right] = 2\ln(2\pi\sigma_1^2 e) \quad and \\
&\lim_{q\to 3} H_q\left[f(x_t)\right] = \frac{1}{2}
\end{aligned}
\tag{5}
$$

Interestingly, "non-extensive" uncertainty ($H_q[f(x_t)]$) converges to $\frac{1}{2}$ as $q \to 3$ regardless of σ_q. To explain it, a closer analysis of σ_q and its relation to the classical measure of standard deviation (σ) is required.

1.2. Risk and state incompleteness

Using equation (2) and equation (4) allows also to relate volatility to the associated Lagrange multiplier (β) and to the Tsallis entropy:

$$
\sigma_q^2 = \frac{1-(q-1)H_q\left[f(x_t)\right]}{\beta(3-q)} \tag{6}
$$

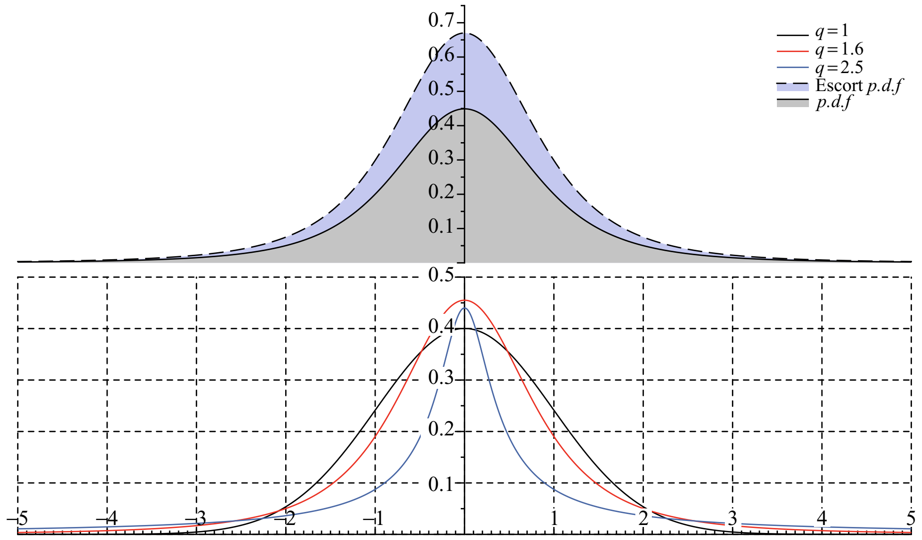

Fig. 1. The Tsallis *p.d.f.* for different values of q (bottom panel) and a comparison between the Tsallis *p.d.f.* and its escort *p.d.f.* used in equation (3)

A direct implication of this result is that for $q=1$, equation (6) becomes the solution of Boltzmann's entropy maximisation (i.e: $\beta=\frac{1}{\sigma_1^2}$). Therefore, it is possible to assume that β is the same regardless of whether Tsallis or Boltzmann entropy are being maximised. However, for the volatility it is not quite the case.

Using equation (4) and writing σ_1 and σ_q as "volatilities" corresponding respectively to the solutions of maximising Boltzmann's and Tsallis entropies, a functional form of σ_q is deduced:

$$\sigma_q = \sigma_1^{\frac{2}{1+q}} \mathcal{B}(q) \tag{7}$$

Where:

$$\mathcal{B}(q) = \left[\frac{2\pi^{\frac{1}{2}-\frac{q}{2}}(q-1)\Gamma\left(\frac{1}{2}+\frac{1}{q-1}\right)\left(\frac{\Gamma\left(\frac{1}{q-1}-\frac{1}{2}\right)\sqrt{\frac{2}{q-1}-1}}{\Gamma\left(\frac{1}{q-1}\right)}\right)^{-q}}{\sqrt{(3-q)(q-1)}\Gamma\left(\frac{1}{q-1}\right)} \right]^{\frac{1}{1+q}} \tag{8}$$

Equation (7), thus, relates the "non-extensive" volatility (σ_q) to the q parameter and the traditional normal measure of volatility (i.e.: standard deviation – σ_1). Naturally, σ_q converges to σ_1 as $q\to 1$. However, taking the limits of σ_q when $q\to 3$ yields the following:

$$\lim_{q\to 3} \sigma_q = 0 \tag{9}$$

Considering the narrative of incomplete statistics, the above result is perhaps surprising as one would expect σ_q to be absolutely increasing with q. Nevertheless, equation (8) may suggest that from a "non-extensive" perspective, there is a substitution effect that takes place between σ_q (measuring risk) and q that relates to incomplete state uncertainty. This effect is more pronounced when the normal measure of volatility and the q parameter are increasing. Figure 2 plots equation (7) for given values of q and σ_1. It indicates the concavity of σ_q with respect to σ at a predetermined value of $q>1$. Some of the empirical implications of this relationship are discussed in the next section.

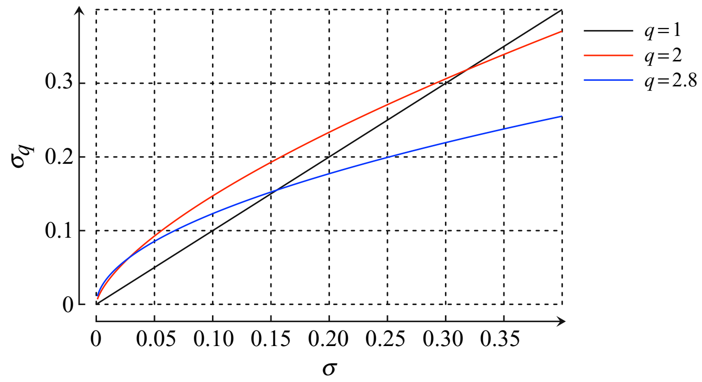

Fig. 2. This figure plots equation (7) for predetermined values of q (black – q=*1*, red – q=*2* and blue – q=*3*). For a given $q > 1$, σ_q is concave with respect to the normal measure of volatility σ.

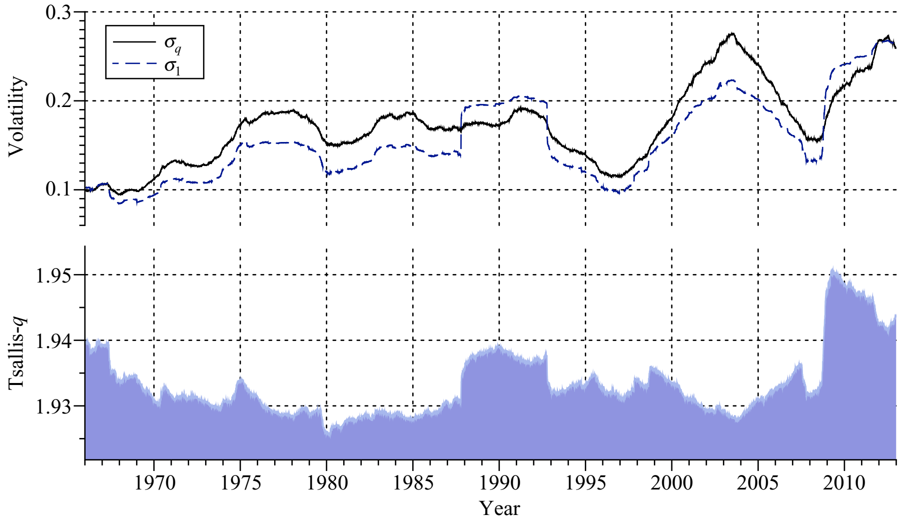

Fig. 3. Top panel indicates the historical (normal) and non-extensive volatility measures (σ_1 and σ_q, respectively). The bottom panel indicates the estimated q-parameter.

2. Implications of "non-extensivness" on and risk and uncertainty

Daily closing prices of the S&P-500 and S&P-500 implied volatility index (VIX – σ_{VIX}) are downloaded from the Yahoo finance[2] website. The causality relationship between σ_1, σ_{VIX} and q are examined. To estimate q, the log-likelihood function of $f(x_t|q, \sigma_q)$ (x_t being standardised log-returns) is maximised over a select moving time window. Figure 3 plots σ_1, σ_q and q, estimated from a moving time window of 5 years. The covered period for figure 3 ranges from 1966 to 2012.

Figure 3 confirms the intuition presented in the previous section. If the normal volatility measure (σ_1) is a measure of risk, relative to the "non-extensive" risk measure (σ_1), it underestimate it. Nevertheless, in time frames where tail events occurred (such as October 1987 and the 2008 crisis), σ_1 relatively overstate risk. In such periods, a jump in the value of q is observed. Figure 3, therefore, may describes well a substitution effect that takes place between (non-extensive) risk and incomplete state uncertainty (q).

The next subsection discusses the possible lead-lag relationship q may have with σ_1 and σ_{VIX}. The subsection that follows, discuss some of the implication on portfolio diversification.

[2] `www.finance.yahoo.com`

2.1. Possible lead-lag relationship with the S&P-500 implied volatility index (VIX)

The S&P-500 implied volatility (VIX) is a well known measure of market fears. It is calculated from a cross-section of traded options on the S&P-500 index with 30 to 60 days to expiration. Its appeal is that it provides a 30 days ahead "model-free" forecast of S&P-500 volatility. Figure 4 plots the VIX, σ_q, and q (estimated using 252 trading days moving time window) for the period of 1990–2012.

Equation (10) represents a VAR(1) equation that involves the state incompleteness parameter (q), the "normal" volatility measure (σ_1) and the VIX (σ_{VIX}).

$$\begin{bmatrix} \triangle \ln q_t \\ \triangle \ln \sigma_{1,t} \\ \triangle \ln \sigma_{VIX,t} \end{bmatrix} = \mathbf{C} + \gamma \begin{bmatrix} \triangle \ln q_{t-1} \\ \triangle \ln \sigma_{1,t-1} \\ \triangle \ln \sigma_{VIX,t-1} \end{bmatrix} + \mathbf{E_t} \tag{10}$$

Where: $\mathbf{C}$ is a 3×1 vector of constants, γ is 3×3 matrix of regression coefficients and $\mathbf{E_t}$ is a vector of stochastic error terms. The purpose of estimating the VAR(1) in equation (10) is to further clarify the effects that state incompleteness uncertainty (q) might have in determining the aggregate measure of implied risk (VIX).

The "non-extensive" volatility parameter (σ_q) is omitted from this analysis because it explicitly depends on q and σ_1 (equation (7)). The VAR(1) in equation (10) is estimated for the sample period of 1990–2012 and the two sub-periods of 1997–2003 and 2007–2012. These two periods are marked by a volatile index. The first sub-period covers such events as the Asian crisis, LTCM and dot-com bubble burst. The second sub-period covers 2008 crisis, the European sovereign bond crisis and downgrade of U.S. debt. The estimated parameters are reported in table 1.

The results of estimating equation (10) reveal that σ_{VIX} exclusively causes q (in Granger (1969) sense), but not vice-versa. While, the latter variables are causal to σ that feeds back to σ_{VIX}. This relationship is stable across selected sub-periods. The insignificance of $\triangle \ln q_{t-1}$ in the equation that that forecasts $\triangle \ln \sigma_{VIX,t}$ may also be attributed to the possibility that $\triangle \ln \sigma_{1,t-1}$ also embeds information on $\triangle \ln q_{t-1}$ in the equation that determines $\triangle \ln \sigma_{VIX,t}$. Figure 5 summarises the flow of information that is suggested by the estimated coefficients in equation 10.

Table 1 and figure 5, suggests a flow of information where the VIX affects S&P-500 log-returns both through σ_1 and q. Furthermore, q also negatively causes σ_1 which increases the tails of the distribution of log-returns. Given the indicated results, σ_{VIX} affects the both the tail and scale of log-returns

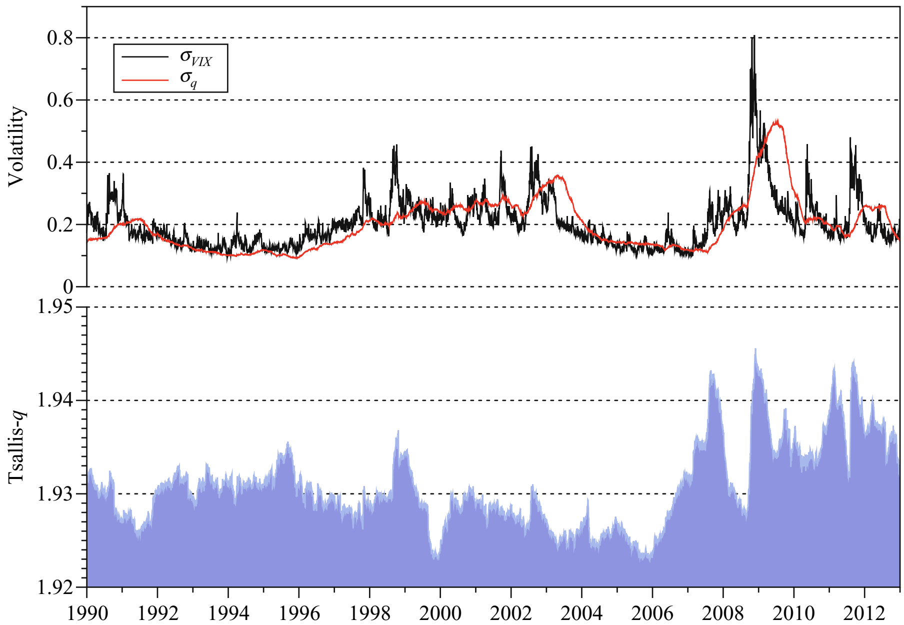

Fig. 4. Top panel indicates the implied and non-extensive volatility measures (σ_{VIX} and σ_q, respectively). The bottom panel indicates the estimated q-parameter.

Table 1

Estimation results for equation 10

Sub-period	**1997–2003**			**2007–2012**			**1990–2012**		
Dependent Variable				$\triangle q_t$					
	$Est.$	$t-value$	$Pr(\|t\|>0)$	$Est.$	$t-value$	$Pr(\|t\|>0)$	$Est.$	$t-value$	$Pr(\|t\|>0)$
$\triangle \ln q_{t-1}$	0.97	167.2	0.00^{***}	0.98	208.5	0.00^{***}	0.98	350.5	0.00^{***}
$\triangle \ln \sigma_{1,t-1}$	-6.2×10^{-6}	-0.2	0.87	-2.0×10^{-5}	-0.6	0.55	-1.5×10^{-5}	-0.7	0.47
$\triangle \ln \sigma_{VIX,t-1}$	6.1×10^{-5}	3.3	$9\times10^{-4***}$	1.4×10^{-4}	6.7	0.00^{***}	6.9×10^{-5}	6.4	0.00^{***}
const.	-6.1×10^{-7}	-0.03	0.97	-7.9×10^{-6}	0.3	0.75	2.3×10^{-7}	0.02	0.98
R^2	0.92			0.97			0.96		
Dependent Variable				$\triangle\sigma_{1,t-1}$					
	$Est.$	$t-value$	$Pr(\|t\|>0)$	$Est.$	$t-value$	$Pr(\|t\|>0)$	$Est.$	$t-value$	$Pr(\|t\|>0)$
$\triangle \ln q_{t-1}$	-1.37	-2.4	0.015^{**}	-1.48	-3.8	0.00^{***}	-0.74	-3.0	0.003^{***}
$\triangle \ln \sigma_{1,t-1}$	0.98	259.0	0.00^{***}	0.98	343.6	0.00^{***}	0.98	550.0	0.00^{***}
$\triangle \ln \sigma_{VIX,t-1}$	0.02	9.3	0.00^{***}	2.8×10^{-2}	15.6	0.00^{***}	1.8×10^{-2}	19.4	0.00^{***}
const.	1.4×10^{-4}	0.09	0.92	-8×10^{-4}	-0.4	0.67	-6.6×10^{-5}	-0.07	0.94
R^2	0.98			0.98			0.98		
				$\triangle\sigma_{VIX,t-1}$					
	$Est.$	$t-value$	$Pr(\|t\|>0)$	$Est.$	$t-value$	$Pr(\|t\|>0)$	$Est.$	$t-value$	$Pr(\|t\|>0)$
$\triangle \ln q_{t-1}$	0.81	0.4	0.71	0.35	0.2	0.82	1.63	1.55	0.12
$\triangle \ln \sigma_{1,t-1}$	-0.04	-3.02	$2.5\times10^{-3***}$	-4.6×10^{-2}	-3.8	0.00^{***}	$-4.\times10^{-2}$	-6.1	0.00^{***}
$\triangle \ln \sigma_{VIX,t-1}$	0.96	137.9	0.00^{***}	0.97	130.0	0.00^{***}	0.96	239.1	0.00^{***}
const.	1.5×10^{-3}	0.2	0.80	4.8×10^{-3}	0.6	0.58	-2.2×10^{-4}	-0.06	0.95
R^2	0.92			0.92			0.91		

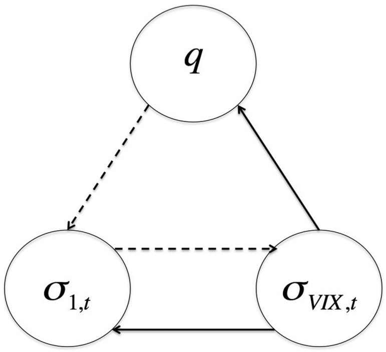

Fig. 5. Information flows from σ_{VIX} to q and to $\sigma_{1,t}$. While, information also flows from q to $\sigma_{1,t}$ but not vice-versa.

distribution. This tail effect, validates the VIX as an index of aggregate market "fears".

2.2. *Portfolio diversification*

Diversifying away risk by including more assets in a portfolio is at the core of asset and risk management. Common wisdom suggests that the larger the number of statistically independent assets in a portfolio the smaller would the risk associated with it. In an equally weighted portfolio, the risk associated with individual assets is completely diversified away as the number of asset is increasing, while overall portfolio risk converges to an average returns covariance. However, can incomplete state uncertainty be diversified away?

To examine this, 30 U.S. stocks closing prices are downloaded[3] from Yahoo, covering the period of December 2007 to December 2012. From these closings prices, 30 portfolios are created. The first portfolio is composed of one stock, the second of two stocks and so on. Thus, in the experiment there are 30 portfolios that are equally weighted, each with more asset then the previous one. Figure 6 plots the risk (σ_1) and incomplete state uncertainty of each portfolio.

Figure 6 implies that while risk can be easily diversified away, it is not the case for incomplete state uncertainty. As a matter of fact, in this experiment, this uncertainty increases with the number of included assts. Thus, the first implication of incomplete state uncertainty is that it may be difficult to diversify away this uncertainty.

In a second set of experiments, the mean-variance frontier associated with the 30 stocks is calculated. Figure 7, plots mean-variance frontier, as well as the relationship between q, optimal portfolio returns and non-extensive risk (denoted $\sigma_{p,q}$). This figure suggests that portfolio expected returns are decreasing with respect to q. While, both risk measures are also decreasing with respect to to q.

The result in figure 7, might be seen as a direct consequence of what is suggested in figure 6. That is: it is possible that a trade-off exists between risk and incomplete state uncertainty. The latter, erodes the portfolio expected returns as risk is being diversified away. If reality supports this claim, then it is possible that the incomplete state uncertainty implies a hidden cost of risk diversification.

The implications of incomplete state uncertainty on portfolio management might suggest that the associated portfolio incomplete state uncertainty increases as risk is being diversified away from this portfolio. In other words, the tails of the portfolio returns distribution increase while it decrease in scale.

3. Conclusion

The "non-extensive" Tsallis entropy perspectives that accounts for uncertainty were presented in this paper. Given the volatility of financial assets returns, it depends on a "non-extensive" measure of volatility (that is concave with respect to the normal volatility) and an incomplete states uncertainty parameter (q). In essence, this paper draws a link between ("normal" and "non-extensive") risk and incomplete states uncertainty. One conclusion that emerges from this analysis is that there is a possible substitution effect that takes place between risk and incomplete states uncertainty. That is, while volatility might be decreasing (decreasing the scale of returns distribution) the distribution tails might be increasing. This relationship has been investigated both theoretically in the first section of this paper and empirically in the second section.

This papers also characterised the flow of information between the S&P-500 options implied volatility index and the implied from S&P-500 log-returns normal volatility and the incomplete states uncertainty parameter (q). We found that the q implied from realised log-returns is exclusively caused by the VIX. While, this parameter causes normal volatility. Given that the VIX is regarded as a measure of "fear", the results obtained from this causality test shed some

[3]The Quantmod R-Package (Ryan, 2011) is used to download data from the Yahoo-Finance website

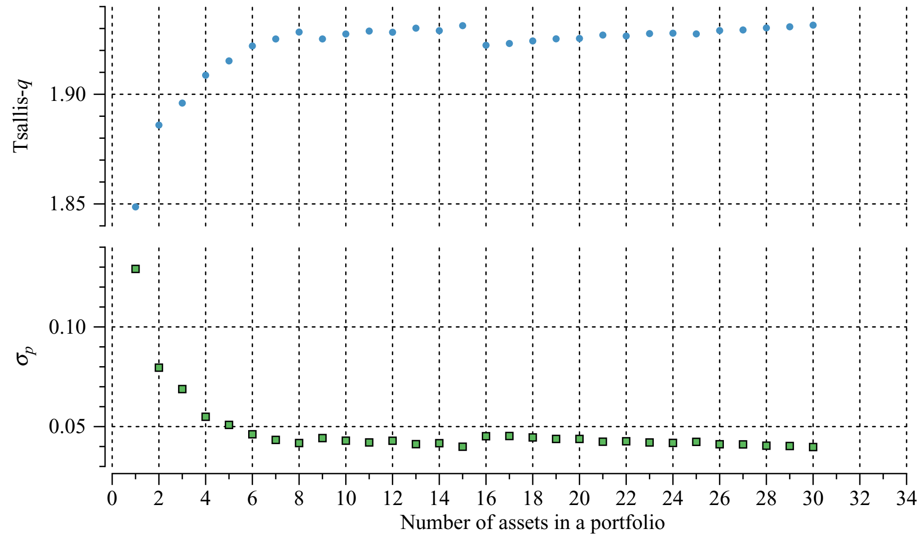

Fig. 6. Top panel indicates the Tsallis-q associated with the ith portfolio. Bottom panel indicates σ_p – the risk associated with the ith portfolio.

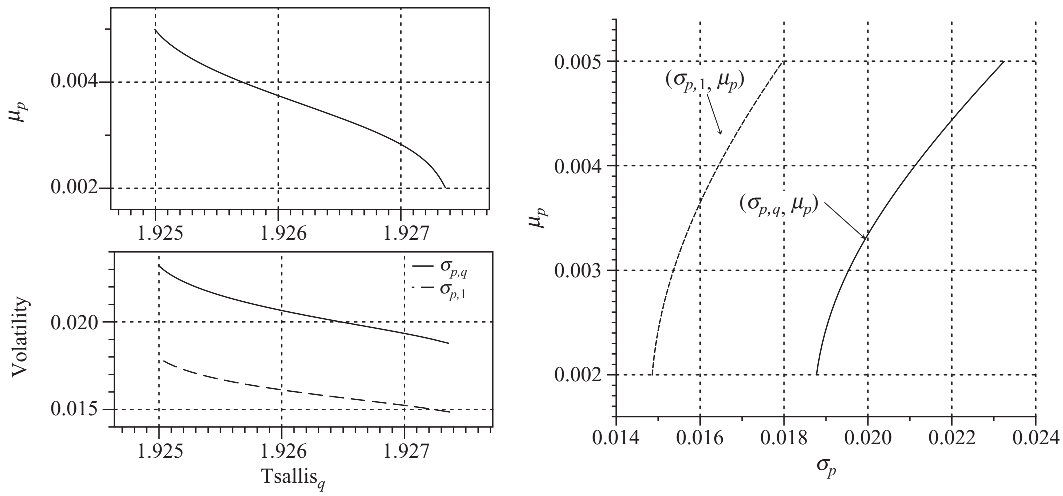

Fig. 7. Top left corner of this figure plots expected portfolio return μ_p vs. the Tsallis-q. Bottom left corner plots the portfolio $\sigma_{p,q}$ and $\sigma_{p,1}$ vs. the Tsallis-q. The right side plot the mean-variance frontier with respect to $\sigma_{p,q}$ and $\sigma_{p,1}$.

light over how an aggregate market "fears" affect the distribution of realised log-returns.

At last, this paper discussed some of the possible implications of applying the "non-extensive" information theory to asset management. Importantly, incomplete states uncertainty might be difficult to diversified away, even if two assets are negatively correlated with each other. Nevertheless, these implications should be further analysed. For example, analysing the effects of this uncertainty over calculated VaR or the likelihood for left tail events. Or, to expand the analysis to more sophisticated assets such as collateral debt obligations. Such an analysis will require to model the joint uncertainty ("non-extensive" entropy) between several random variables. These are topics for further research.

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