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Pricing stocks with yardsticks and sentiments

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Abstract. Human decision making by professionals trading daily in the stock market can be a daunting task. It includes decisions on whether to keep on investing or to exit from a market subject to huge price swings, and also how to price in news or rumors attributed to a specific stock. The question then arises how professional traders, who specialize in daily buying and selling large amounts of a given stock, know how to properly price a given stock on a given day. Here we introduce the idea that people use heuristics, or "rules of thumb", in terms of "yard sticks" from the performance of the other stocks in a stock index. The under or over performance with respect to such a yard stick then signifies a general negative or positive sentiment of the market participants towards a given stock. Using empirical data of the Dow Jones Industrial Average, stocks are shown to have daily performances with a clear tendency to cluster around the measures introduced by the yard sticks. We illustrate how sentiments, most likely due to insider information, can influence the performance of a given stock over period of months, and in one case years.

Keywords: Behavioural Finance, sentiments, CAPM, dimensional analysis.

1. Introduction

One of the founders of Behavioral Finance, D. Kahneman (Shefrin, 2008), once pointed out the close resemblance in the media coverage of financial markets to a stereotypical individual. As he mentioned, the media often describe financial markets with attributes like "... thoughts, beliefs, moods and sometimes stormy emotions. The main characteristic of the market is extreme nervousness. It is full of hope one moment and full of anxiety the next day ...". One initial way to get a first quantification of the sentiment of the market is to probe the sentiments of its investors. Studying sentiments of consumers/investors and its impact on markets has become an increasingly important topic. A various number of sentiment indices of investors/consumers already exist, some of which have now been recorded in a time span over some few decades. The Michigan Consumer Sentiment index, published monthly by the University of Michigan and Thomson Reuteurs, is probably the one which has the largest direct impact on markets when published. The natural question then arises as to whether it is possible to predict market movements based on the sentiments of consumers/investors.

Fisher and Statman (2000) made a study of tactical asset allocation from data of the sentiment of a heterogeneous group (large,medium,small) of investors. The main idea in (Fisher and Statman, 2000) was to look for indicators of future stock returns based on the diversity of sentiments. The study found the sentiments of different groups do not move in unison, and further that sentiments for the groups of large and small investors, could be used as contrary indicators for future S&P 500 returns. Similar results were found by Baker and Wurgler who showed that when beginning-of-period proxies for sentiments are low, subsequent returns are relatively high for securities whose valuations are highly subjective and difficult to arbitrage (Baker and Wurgler, 2006). However other papers are reporting feedback loops

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between sentiment and market performance: past market returns determine investors sentiment who often expect a continuation of short term returns (Brown and Cliff, 2002). Recent research (Tetlock, 2007) on investors sentiment expressed in the media (as measured from the daily content of a *Wall Street Journal* column) seem to point in this direction with high media pessimism predicting downward pressure on market prices. Such results are in line with theoretical models of noise and liquidity traders (De Long et al., 1990, 1991). Other studies (Antweiler and Frank, 2004) claim very little predictability of stock returns using computational linguistics to extract sentiments on 1.5 million internet message boards posted on *Yahoo! Finance* and *Raging Bull*. However in this study it was shown that disagreement induces trading and message posting activity was shown to correlate with volatility of the market.

Common to all the aforementioned studies is the aim to predict **global** market movements from sentiments either obtained from surveys or from internet message boards. In the following we propose, instead, to obtain a sentiment related pricing of a given asset by expressing the sentiment of a given stock **relative** to the market. This is similar to the principle of the Capital Asset Price Model (Sharpe, 1964; Lintner, 1965; Mossin, 1966) (CAPM) which relates the price of a given asset to that of the price of the market, instead of trying to give the absolute price level of the asset/market. Put differently: we will in the following introduce a method that does not estimate the impact a given "absolute" level of sentiment can have on the market, but instead introduce a sentiment of an asset *relative* to the sentiment of the general market irrespective of the absolute (positive/negative) sentiment of the general market. As will be illustrated this gives rise to a pricing formula for a given stock relative to the general market, similar to the CAPM, but now with the relative sentiment of the stock to the market included in the pricing.

The sentiment indicator we introduce gives a way to actually quantify the impact of human biases and sentiments directly on the markets instead of simply postulating effects which is often done in the field of Behavioral Finance. We believe it is important to propose quantitative measures, since the validity of such measures can then directly be tested against market data. In addition such measures should shed additional light on the functioning of financial markets which go beyond the traditional rational expectation approach. Ultimately the importance of such measures will depend on whether or not they are practically applicable. In the case presented in this paper, we introduce a method that will not only enable investors to better understand market risks but also suggests new portfolio selection methods.

2. Theory

We will in the following consider how traders find the appropriate price level of a given stock on a daily basis. One could, for example, have in mind traders that specialize in a given stock actively following its price movements, maybe to consider opportune moments either to buy some amounts or, instead sell as a part of a larger order. The question is what influences the traders' decision on when to enter and when to exit the market. According to the standard economic view only expectations about future earnings/dividends and future interest rate levels should matter in the pricing of a given stock. Considering the often very large price swings during earnings or interest rate announcements, this part clearly seem to play a major role at least at some specific times. However, what about other moments when there is no news which can be said to be relevant for future earnings/interest rates? The fluctuations seen in daily stock prices simply can not be explained by new information related to these two factors, nor can risk aversion. So why do stock prices fluctuate so much and how do traders navigate the often rough seas of such fluctuations?

Here we will take a heuristics point of view, and argue that traders need some rule of thumb, or as we prefer to refer to it, "yardsticks", in order to know how to position themselves. A first rough estimate for a trader would obviously be the returns of other stocks in a given stock index. Let us denote the daily (nominal) return of stock i belonging to the given index at time t by $s_i(t)$, and the return of the remaining $N-1$ stocks in the index at time t by $R_{-i}(t)$. The contribution of stock i in R_{-i} is excluded in order to avoid any self-impact which would amount to assuming that "the price of a stock rises because it rises". Using the averaged return of the other stocks as a first crude yardstick, traders of stock i would price the stock according to

$$s_i(t) \simeq R_{-i}(t) \equiv \frac{1}{(N-1)} \sum_{j \neq i} s_j(t) \qquad (1)$$

A powerful tool to check an equation, often used in Physics, is to use dimensional analysis and ensure that the quantities of both sides have the same dimensions. By the same token, an expression should be independent of the unit used. Since Eq. (1) expresses a relationship between returns, i.e. a quantity that expresses increments in percentages, it is already dimensionless. However we argue that there is a mentally relevant "unit" present, namely the size of a typical daily fluctuation of a given stock (Vitting, 2010). Such a mental "unit of fluctuation" is created by the memory of traders who follow closely past performance of a given stock. Since the capacity of human working memory is quite small, investors are not able to analyze all available data. This causes individuals to simplify the world around them (Baddeley et al., 2009). Investors need to choose consciously or unconsciously data that seem to be the most informative. While doing this (whatever the mode they are in) investors are exposed to the recency effect according to which they will remember recent prices better than earlier ones (Miller and Campbell, 1959). The most recent prices are stored in short-term memory and therefore are easier to retrieve than earlier ones that are stored in long-term memory. Hereby, the most recent volatility of the stock becomes a "unit of fluctuations" which investors refer to while anticipating future price changes. This "unit" which enters their attention field, influences their ability to estimate probabilities in accordance with the rule that events that are easier to retrieve from the memory seem to be more likely to people than they really are (Tversky and Kahneman, 1974). Dividing Eq. (1) on each side by the size of a typical fluctuation would therefore be one way to ensure independence of such units. Taking the standard deviation as measure of a typical fluctuation, the renormalized Eq. (1) takes the form:

$$\frac{s_i(t)}{\sigma(s_i)} = \frac{R_{-i}(t)}{\sigma(R_{-i})}. \tag{2}$$

Here the standard deviation $\sigma(X)$ of the variable X is defined over a given window size T from the variance $\sigma^2 = \mathrm{Var} \equiv E_T(X^2) - E_T^2(X)$, with E denoting the expectation value. Another way of looking at Eq. (2) is to say that the dimensional analysis leads to an evaluation procedure of the investors where the price of a single stock is held against the market, with evalution determined by the Sharpe ratio.

As we will show in a moment, for most stocks, using daily returns, Eq. (2) turns out to be a good approximation. There are however strong and persistent deviations. Actually we will in the following define stocks for which Eq. (2) holds on average, as "neutral" with respect to the sentiment of the traders. Similarly we use the relation as a measure of how biased (positive or negative) a sentiment traders have on the given stock. More precisely the sentiment of a given stock i, α_i, is defined as:

$$\alpha_i(t) = \frac{s_i(t)}{\sigma(s_i)} - \frac{R_{-i}(t)}{\sigma(R_{-i})}. \tag{3}$$

We emphasize that the sentiment is defined with respect to the other stocks in the index, serving as the neutral reference. The ratio of a stock's (excess) return to its standard deviation, indicates something about its performance, or reward-to-variability ratio, also called the Sharpe ratio in Finance (Sharpe, 1966). Therefore Eq. (3) attributes a positive (respective negative) bias/sentiment to a stock, $\alpha_i > 0$ (respective $\alpha_i < 0$), when the Sharpe ratio of the stock exceeds (respective underperforms) the Sharpe ratio of the sum of the other stocks in the index (taking the risk free return used in the usual definition of the Sharpe ration equal to 0).

Rewriting Eq. (3) the pricing of stock i can now be given in terms of a renormalized performance of the other stocks in the index as well as an eventual bias:

$$s_i(t) = \sigma(s_i)\alpha_i(t) + \frac{\sigma(s_i)}{\sigma(R_{-i})} R_{-i}(t) \tag{4}$$

As a first check of Eq. (4) we take the expectation value of Eq. (4) by averaging over all stocks that an index is composed of, and average over time (daily returns). A priori, over long periods of time one would expect to find as many positive biased as negative biased stocks in an index composed of many stocks. That this is indeed the case will be shown empirically. Using this assumption the term with α_i disappears due to symmetry. One gets:

$$E(s_i) = \frac{E(R_{-i})}{\sigma(R_{-i})} \sigma(s_i) \tag{5}$$

Eq. (5) is very similar in structure to the famous Capital Asset Pricing Model in Finance (CAPM) (with the risk free return equal 0):

$$\frac{E(s_i) - R_f}{\beta_i} = E(R) - R_f; \beta_i = \frac{\mathrm{Cov}(s_i, R)}{\sigma^2(R)} \tag{6}$$

R_f in Eq. (6) is the risk free return which, since we consider daily returns, will be taken equal 0 in the following:

$$E(s_i) = \frac{\mathrm{Cov}(s_i, R)}{\sigma^2(R)} E(R). \qquad (7)$$

Apart from the exclusion of the stock itself in the expression R_{-i}, the main difference between the CAPM in the form Eq. (7) and our hypothesis Eq. (5) is that we stress the use of standard deviations in the pricing formula instead of the covariance between the stock return and the index return on the right side of Eq. (7). It should be noted that assuming correlations between R and s_i equal one in the CAPM Eq. (7), it leads to a formula similar in structure to our hypothesis Eq. (5). It is however important to remark that we do *not* assume correlations constant in the derivation of Eq. (5). Furthermore we argue that the covariance between a given stock and the index is not a very stable measure over time, whereas the variance of a given stock appears more stable. One reason for instability of the covariance could for example be sudden "shocks" in terms of specific bad or good news for a given company. After such a "shock" we postulate that the covariance between the stock and the index changes, whereas the variance remain the same but with a change in relative performance as expressed through Eq. (3). Eq. (5) is reminiscent of the so-called capital allocation line in Finance which expresses the return of a portfolio composed of a certain percentage of the market portfolio with the remaining invested in a risk free asset. The capital allocation line however only expresses the return of this specific portfolio, whereas our expression is supposed to hold for each individual asset.

3. Results

The data points in figure 1 show the CAPM hypothesis Eq. (7) on the left, and our hypothesis Eq. (5) on the right, using daily returns of the 30 stocks of the Dow Jones Industrial Average over almost a decade of data (the data used to construct figure 1 was taken from 1/1/2000 till 20/6/2008). The volatilities and co-variances were calculated in a rolling time window of a month. A perfect fit of the data to the two equations would in both cases lie on the diagonal. The data for CAPM appear tilted with respect to the diagonal, whereas the data concerning our hypothesis appear to be symmetrically distributed around the diagonal, which is what one should expect if the data on average followed Eq. (5). Figure 1b therefore gives some first evidence for the support of Eq. (5). This impression is strengthened when one

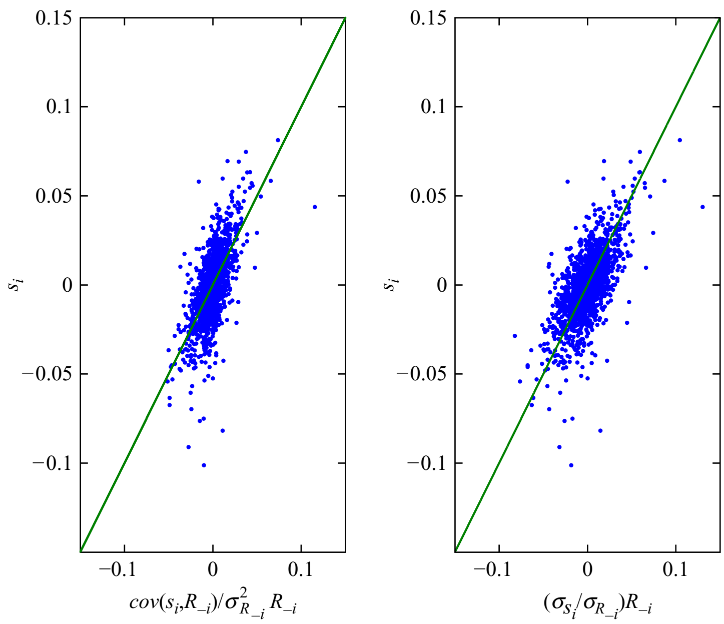

Fig. 1. The plot to the left shows the CAPM hypothesis Eq. (7) using the daily returns of the Dow Jones Industrial index over the period 03/01/2000 to 20/06/2008. Plot to the right illustrates instead Eq. (5) using same data set. Each point correspond to a daily return s_i of a given stock i.

takes a closer look at the cloud of data points in figure 1b, and considers the probability that the return of stock i takes the value s_i, *conditioned on* a given fixed value of $x \equiv \frac{R_{-i}}{\sigma(R_{-i})}\sigma(s_i)$. Figure 2 shows the probability distribution function of s_i conditioned on five different values of the variable x. From the hypothesis Eqs. (4)-(5) one would expect the most likely value of the stock return s_i to occur for the given fixed value of x. This is indeed seen to be the case with all five distributions peaking close to, if not actually at, x, giving further evidence to the assumption Eq. (5).

The sentiment α in Eq. (3) was introduced as a behavioral trait, and as such we would expect to see its effect on a long term time scale say at least of the order weeks or months. Figure 3 shows the cumulative sentiment for three different stocks, Citi Bank, and Caterpillar in the time period (03/01/2000 to 20/06/2008) and Cisco in the time period (01/06/2009 to 02/06/2011). The plots to the left show in green the return of the Dow Jones and in blue the given stock over the given time period. The case of the Citi Bank stock is particularly striking with a constant negative sentiment seen by the continuous decline in the cumulative sentiment curve of figure 3, corresponding to a constant sub-performance over two years. It should be noted that the data was chosen in order to have both a declining general market, which happens over the first half of the time period shown, as well as an increasing market which happens over the rest of the time period chosen. It is remarkable that the negative sentiment of the Citi Bank stocks stays constant independent of whether the general trend is bullish or bearish. Similarly it should be noted that the general sentiment of Caterpillar had a neutral value in the declining market, but then developed a distinguishable negative sentiment over the last three or four months of the time series where the general market is bullish. The price history for Cisco Systems tells a similar story. The only difference here is the two big jumps happening after 350 and 400 days. These two particular events took place on the 11/11/2010 and on the 10/2/2011. On the 11/11/2010 the price dropped because of a bad report for the third quarter earnings. This gave rise to a loss of confidence by investors who were expecting a sign of recovery after a couple of hard months. On the 10/2/2011 Cisco Systems announced a drop in their earnings (down 18%) together with a downward revision (-7 %) of sales for their core product. It is worth noticing that the decline of the cumulative sentiment took place **before** the two events: prior to 11/11/2010 there was a long

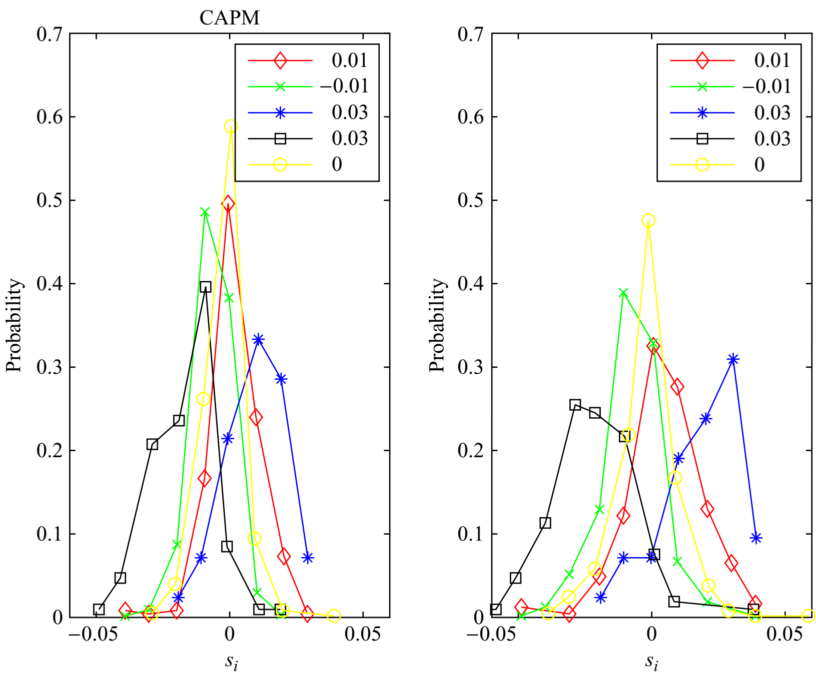

Fig. 2. Probability distribution function of s_i conditioned on 5 different values of $x \equiv \frac{cov(s_i,R)R}{\sigma_R^2}$ (CAPM result) to the left and of $x \equiv \frac{R_{-i}}{\sigma(R_{-i})}\sigma(s_{-i})$ to the right. This figure is inset to figure 1.

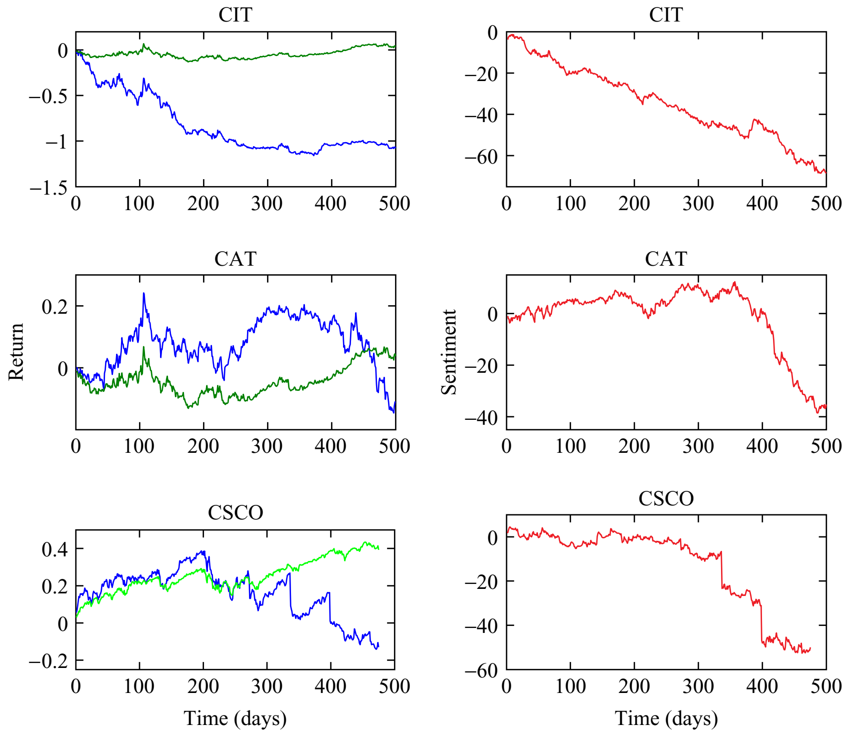

Fig. 3. The figure shows the cumulative sentiment for three different stocks, Citi Bank, Caterpilar in the time period (03/01/2000 to 20/06/2008) and Cisco in the time period (01/06/2009 to 02/06/2011). The plots to the left shows in green the return of the Dow Jones and in blue the given stock over the given time period.

slow descent of the cumulative sentiment (meaning a constant negative sentiment) and after the 11/11/2010 the descent continued. This could be taken as evidence that some investors with insider knowledge were aware of the problems of the company, which was revealed only to the public on the two aforementioned days.

Figure 4 shows the probability distribution function of the sentiment variable obtained by sampling the sentiment variable α defined in Eq. (3), using the daily return of all stocks of the Dow Jones Industrial Average in the period 03/01/2000-20/06/2008. As can be seen from the inset of figure 4 the distribution appears to follow an exponential distribution for both positive and negative sentiments. One notes that the empirical distribution appears to be symmetric with respect to the sign of the sentiment - something which was implicitely assumed in deriving Eq. (5) from Eq. (4).

Finally we would like to point out an additional difference between Eq. (5) and Eq. (4). It should be noted that Eq. (5) is a result purely related to the way traders price a given individual stock using the index to which the stock belongs as a yardstick. As such there is not an economic reasoning behind Eq. (5) whereas in Eq. (4) the return R is meant to represent a general return of the "market", and not just the specific index for which the stock belongs. In practise however practitioners use a large stock index (such as the SP500) as a proxy to represent the "market" in the CAPM case.

4. Discussion

The psychological experiments initiated by Kahneman and Tversky (Tversky and Kahneman, 1974) more than three decades ago showed that not only do people not behave rationally, but what is even more important: people do not behave randomly; people are suceptible to common judgment errors. People make these errors because they use heuristics, rather than a rational process based on logic to arrive at their decisions. Heuristics are cognitive processes that do not use all available information. Although in certain cases heuristics may produce errors, they are usually effective, and in cases when the assumption of rational models are not met (e.g. missing or highly uncertain information), they can be more useful than the rational decision process (Girgerenzer and

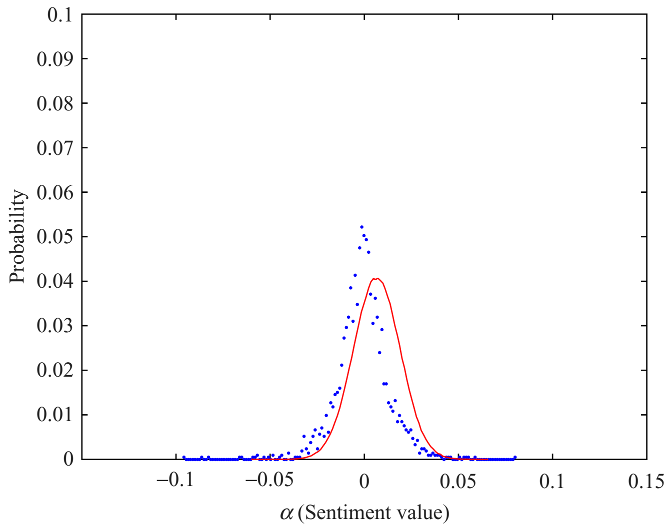

Fig. 4. Probability distribution function of the sentiment variable α_i.

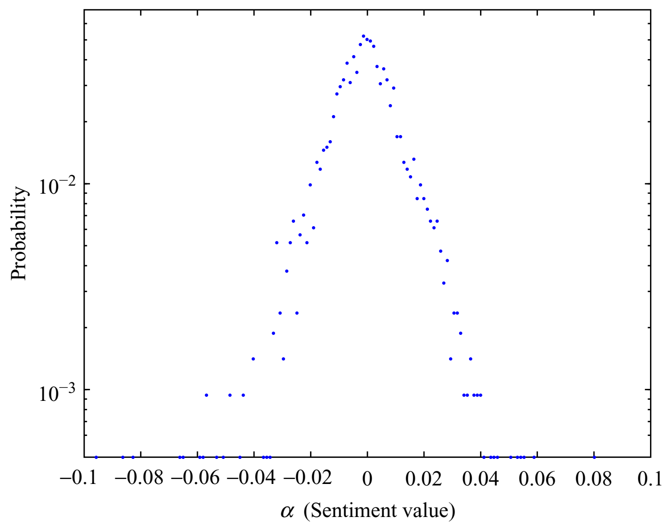

Fig. 5. This figure is inset to figure 3.

Gessmaier, 2011). Some heuristics may be extremely simple and still perform well in most situations. For example, recognition heuristics describes a process where decision makers when deciding which object has a higher value on a numerical criterion indicate the object that they better recognize (Katsikopoulos, 2011). Therefore research on sentiments is important because neither the efficient market hypothesis nor the noise trader theory explains systematic deviations of asset prices from fundamental values. So far sentiments in financial markets have been explained as either due to human overreaction or underreaction. For example Barberis et al. (2005) used psychological reasoning in explaning market deviations: overreaction with representativeness heuristics and underreaction with conservatism.

In this paper we have instead pointed out the importance of a *relative* sentiment measure of a given stock to its peers. The idea is that people use heuristics, or "rules of thumb", in terms of "yard sticks" from the performance of the other stocks in a stock index. The under-/over-performance with respect to a yard stick then signifies a general negative/positive sentiment of the market participants towards a given stock. The bias created in such cases does not necessarily have a psychological origin but could be due to insider information. Insiders having superior information about the state of a company reduce/increase their stock holding gradually causing a persisting bias over time. The introduction of a measure for the relative sentiment of a stock has allowed us to come up with a pricing formular for stocks very similar in structure to the CAPM model. Using empirical data of the Dow Jones Industrial Average, stocks are shown to have daily performances with a clear tendency to cluster around the measures introduced by the yard sticks in accordance with our pricing formular.

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