Algorithmic Finance 1 (2011) 123–139
DOI 10.3233/AF-2011-010
IOS Press

News Reaction in Financial Markets within a Behavioral Finance Model with Heterogeneous Agents

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Abstract. This paper presents a Heterogeneous Agent Model of a financial market with chartist and fundamentalist traders that exhibit bounded rationality and short-term thinking to explain the effect of under and overreaction to news. The existence of the Market Maker's finite price adjustment speed and the presence of risk aversion lead to the fact that prices do not adjust instantaneously to new information. Chartists use moving average rules to make their investment decisions. They can transform an underreaction-only scenario into a market with overreaction. The use of long moving average rules might even make the market unstable. Higher market efficiency (low deviations from fundamental value), on the other hand, is achieved if high rationality and long-term thinking for the agents is assumed.

JEL classification: G14, D84, C62, C15.

Keywords: Heterogeneous Agent Model, stock market, under and overreaction to news, moving average rules, financial stability.

1. Introduction

This paper shows that the phenomenon of under and overreaction to news can be explained by a Heterogeneous Agent Model (HAM) of a financial market. This effect is only considered scantly in the literature on HAMs[1]. First, an analytical discussion of a simplified linearized version of the model without noise is presented. Instead of using Bifurcation Theory, the analytical framework of classical linear control theory is applied. We show that the emergence of overreaction and instability depends on the chartists' strategy. Underreaction occurs due to finite price adjustment speed and risk aversion (especially) by fundamental traders. It can be dampened by chartist behavior. In the case of a combined under and overreaction scenario high aggressiveness of chartists and high price adjustment speed can lead to instability. Secondly, a simulation-based approach of the complex model shows that a low degree of agent rationality as well as short-term thinking increases the effect of both under and overreaction and therefore decreases market efficiency.

HAMs dating back to Day and Huang (1990) have recently become very popular for discussing the behavior of stock markets. These models rely on two basic assumption: agents (i) exhibit bounded rationality and (ii) form heterogeneous beliefs. The HAMs in finance normally distinguish between fundamentalists, technical, and noise traders. The models have been applied to different markets such as commodities (Reitz and Westerhoff, 2007), foreign exchange (De Grauwe and Grimaldi, 2006), options (Frijns et al., 2010), and stocks (Westerhoff, 2008). The models are

*I am grateful to Jesper Riedler, Ingo Barens, Andreas Röthig, Philip Maymin and one anonymous referee for numerous helpful comments. Philip Savage contributed stylistic advice. Of course, all remaining errors are mine.

[1]Boswijk et al. (2007) present a HAM of the S&P500 explaining the DotCom-bubble by the overreaction to good fundamental news.

able to replicate several stylized facts found in actual financial markets such as excess volatility, random walk behavior (indicated by insignificant autocorrelations in returns), volatility clustering (as indicated by significant slowly decreasing autocorrelations in absolute returns), skewness as well as excess kurtosis of return distribution (Lux, 2009). In a mathematical sense, these models are represented by non-linear difference equations. Current research expands these models to incorporate realistic trading strategies (e.g. Westerhoff (2006)), whilst the mathematical analysis, mostly relying on the tools of Bifurcation Theory, is brought to a more sophisticated level (e.g. Hommes and Wagener (2009)). This analysis helps to understand which parameters or model features drive the stylized facts (e.g. He and Li (2007)) and correspond to the stability of the market (e.g. Chiarella et al. (2009)). Major factors seem to be the rules used by chartist traders (Chiarella et al., 2006) and the noise in financial markets (Chiarella et al., 2011).

In this paper we use the HAM framework to examine the effect of under and/or overreaction. This effect is inconsistent with the Efficient Market Hypothesis assuming instant price reaction to news fundamentals. Nevertheless, several empirical studies seem to confirm these effects in real markets. Underreaction describes the idea that prices only sluggishly react to new information and is therefore also often referred to as the Momentum Effect. This effect implies that past price movements have predictive power for future prices, since they are followed by returns of the same sign. Overreaction on the other side states that markets overreact to good or bad news, but returns adjust to a mean in the long run. Therefore, this effect is also known as Mean Reversion. These effects seem to be contradictory. Note that underreaction is mostly measured in the short run, whilst overreaction is found in longer horizons of roughly three to five years (Beechey et al., 2000).

Several models explain the effects of under and overreaction based on findings of Behavioral Finance. Daniel et al. (1998) attribute these effects to overconfidence and biased self-attribution. Individuals overestimate the precision of private signals (overconfidence). By contrast, reaction to public events is asymmetrical: events that confirm the validity of private information are attributed to high forecast ability, while public information that disconfirms private information is blamed on noise or sabotage (biased self-attribution). Daniel et al. (1998) provide simulations that show short-run Momentum followed by long-run reversals. This is also measured by short-run positive and long-run negative autocorrelations in returns. Note that the model predicts initial overreaction followed by even more overreaction. Another approach for explaining both effects in a unified theoretical framework is presented by Barberis et al. (1998). They assume the two psychological effects of representativeness and conservatism. The former refers to the effect that market participants tend to see patterns based on few observations, while the latter refers to the slow updating of beliefs. The combination of these two effects is able to replicate the effect of short-term Momentum and long-run Mean Reversion. While these models rely on the idea of a single representative agent, the approach of Hong and Stein (1999) introduces the interaction of different trader types as a key to understand both effects. Due to slow diffusion of private information among so-called Information Traders, there is underreaction and Momentum in the prices, which evokes the action of Momentum Traders with positive feedback behavior creating the effect of overreaction. The authors present a hump-shaped price reaction function and are also able to measure the short-run positive and long-run negative autocorrelations. Under and overreaction are both stronger when low information diffusion is considered. Both Hong and Stein (1999) and Barberis et al. (1998) present a model with initial underreaction followed by subsequent overreaction.

In the remainder of this paper, we follow the rationale of Hong and Stein (1999) that combined under and overreaction can be explained by the interaction of heterogeneous agents with bounded rationality. Therefore, a very common representation of a HAM is presented in section 2. Based on a linearized version of the model, the conditions for under and overreaction are examined analytically in section 3. In line with Chiarella et al. (2006) it is assumed that technical traders use moving average rules. The window length of this rule proves to be crucial for systemic stability. Longer moving average rules might even lead to instability. Furthermore, we discuss the interaction of the parameters of chartists and fundamentalists aggressiveness as well as price reaction speed of the Market Maker. One key finding is that due to the fact that markets have a finite price adjustment speed and are therefore not cleared at any time as assumed by Walrasian aucutioneer, trend-following chartist traders emerge and eventually lead to overreaction or even instability. In section 4 the complex model is discussed on a simulation based approach. Both analytical and simulation-based approaches confirm that noise trading in combination with Momentum trading is a crucial factor that drives real markets and affects market stability. Section 5 concludes and gives directions for further research.

2. Basic model

This section presents the basic model. The model presented is closely related to well-known HAMs of financial markets as presented in recent surveys by Hommes and Wagener (2009) and Chiarella et al. (2009). We assume a world with two assets: a risky asset with expected return $E_t^i(r_{t+1})$ and a risk-free asset with safe return of ${r_f}^2$. The demand for risky asset is derived with mean-variance portfolio optimization (Hommes and Wagener, 2009):

$$D_t^i = \frac{E_t^i(r_{t+1}) - r_f}{RA \cdot \sigma_{t,r}^2}. \tag{1}$$

The demand of a certain group of agents i at time t therefore crucially depends on the group's individual expectation of future returns. Demand for risky assets increases with high expected excess returns (relative to risk-free rate). Inversely, demand is low in the case of high risk aversion RA and high volatility of returns $\sigma_{t,r}^2$.

The market-clearing in classic economic models is modeled as a Walrasian auctioneer. The key idea is that after determining the excess market demand, the auctioneer keeps announcing prices and interacts with the market feedback until the excess demand equals zero. This yields the classic *demand equals supply* equation:

$$\sum_{i=1}^{n} W_t^i D_t^i = N_t. \tag{2}$$

In this case, $0 < W_t < 1$ represents the market weight of a specific group of agents. The aggregate demand should equal the supply N_t. Since agents can go short in stocks in the case that they expect prices to fall, they can also supply stocks ($D_t^i < 0$). Thus, no external supply N_t is necessary. This case shall be referred to as Zero Net Supply.

As presented in Chiarella et al. (2009), this modeling approach, even though widely used in economic analysis, only plays a part in one real market (the market for silver in London). Therefore, it is convenient to model a so-called Market Maker mechanism for market-clearing (e.g., Chiarella et al., 2006; Westerhoff, 2008). Even though this approach is still very simplified, it comes closer to price determination in actual markets. The key idea here is that an institution named Market Maker takes an offsetting long or short position to assure that excess demand in period t equals zero. In the next period, the Market Maker announces a new log-price p_{t+1} to reduce excess demand[3]:

$$p_{t+1} = p_t + \mu \left(\sum_{i=1}^{n} W_t^i D_t^i - N_t \right). \tag{3}$$

In this case, $\mu > 0$ represents the price reaction speed of the Market Maker. If we assume infinite reaction speed, this approach reduces to a Walrasian auctioneer:

$$\lim_{\mu \to \infty} \left(\frac{p_t - p_{t+1}}{\mu} + \sum_{i=1}^{n} W_t^i D_t^i - N_t \right) = \sum_{i=1}^{n} W_t^i D_t^i - N_t = 0. \tag{4}$$

This result will be of interest when the dynamic properties of the system are analyzed in the following section. Furthermore, the parameter μ can be interpreted as the liquidity of the market. In time of illiquid markets μ is high and prices react severely to excess demand.

In the basic model, the weights of the different agents vary in time. This represents the empirical fact pointed out by Menkhoff and Taylor (2007) that traders do not stick to a certain rule, but instead use a combination of both technical and fundamental analysis. The weights of the groups are derived using a Multinominal Logit Model as presented in Manski and McFadden (1981):

$$W_t^i = \frac{e^{\gamma A_t^i}}{\sum_{i=1}^{n} e^{\gamma A_t^i}}. \tag{5}$$

The application of the Multinominal Logit Model as a strategy switching model was introduced in Brock and

[2]The effect of dividends is neglected since we assume day-trading behavior. Since dividends are normally only paid out once a year only, they do not matter for all but one trading period a year.

[3]The model uses log-prices p_t instead of real prices P_t. This is briefly discussed in appendix A.

Hommes (1997), whilst its application in the financial market context dates back to Brock and Hommes (1998). Due to the construction of the equation, the individual weights sum up to one. The parameter γ presents a degree of rationality in choosing a strategy. In case γ equals zero, the weights of the groups are constant and amount to $1/n$. The other extreme case with γ converging to infinity represents the case in which all individuals choose the optimal forecast. De Grauwe and Grimaldi (2006) therefore interpret this parameter as a model of the behavioral effect of Status Quos Bias as presented in Kahneman et al. (1991). This effect implies that individuals find it is difficult to change a decision rule they used in the past. In a more general way, this parameter can also be considered as a value for bounded rationality in the sense of Simon (1955). Due to the limited resources of time and money, individuals use suboptimal rules.

The weight of a strategy W_t^i in the market is evaluated by its attractiveness A_t^i in a period t. This parameter is modeled in the following way[4]:

$$\begin{aligned} A_t^i &= D_{t-1}^i \cdot (r_t - r_f) + \eta A_{t-1}^i \\ &\approx D_{t-1}^i \cdot (\ln(1+r_t) - \ln(1+r_f)) + \eta A_{t-1}^i \\ &= D_{t-1}^i \cdot (p_t - p_{t-1} - \ln(1+r_f)) + \eta A_{t-1}^i. \end{aligned} \quad (6)$$

It considers the profits a strategy yielded between period $(t-1)$ and t. Note that a profit is made in the case where risky assets are bought when returns are higher than risk-free returns, or risky assets are sold when their return is lower than the return of the risk-free asset. The parameter $0 < \eta < 1$ represents the memory of the agent. If it is set to zero, myopic traders that only value the very last success of the strategy are considered. In the case of $\eta = 1$, instead of profits the accumulated wealth of a group is taken into account. This modeling approach enables us to investigate the effect of short-term focusing in financial markets. The parameters γ and η are therefore the key to measuring the degree of irrationality in markets.

The model investigates three different strategies: (i) fundamentalism, (ii) chartism using moving average rules, and (iii) noise trading. *Fundamental* traders know the true fundamental log-value of an asset f_t and expect the prices to converge to it. Their expectations can therefore be modeled in the following way:

$$E_t^F(p_{t+1} - p_t) = \alpha(f_t - p_t). \quad (7)$$

[4]This equation builds on the results presented in appendix A.

The parameter $\alpha > 0$ measures the speed at which fundamentalist traders expect prices of stock to converge to their true underlying value. This strategy can be interpreted as the the Hedge Fund strategy of so-called Alpha Seeking trying to buy undervalued and to sell overvalued securities in the market (securities whose α, representing the deviation from the Security Market Line, are positive, respectively negative). Their action contributes to higher market efficiency.

Chartists on the other hand do not consider fundamental prices, but derive order signals from past prices. There are several studies indicating widespread use of technical analysis (even) among professional traders in particular in foreign exchange markets. Chartism is especially important for short-term forecast horizons[5]. Hong and Stein (1999) show that chartism can be useful in exploiting the general underreaction of markets. Chartism is often also referred to as Technical Trading, since it derives its trading signals from clear rules that can be automated. For this reason it is also very easy to implement these rules in a HAM. One of the easiest rules to implement is the moving average rule:

$$E_t^C(p_{t+1} - p_t) = \beta \left[\frac{1}{N_s} \sum_{i=0}^{N_s-1} p_{t-i} - \frac{1}{N_l} \sum_{i=0}^{N_l-1} p_{t-i} \right]. \quad (8)$$

This strategy compares a long to a short-moving average ($N_s < N_l$). The use of the moving averages can be explained by market noise: it filters fluctuations around a long-run trend (Menkhoff and Taylor, 2007)[6]. Normally, an intersection of the two moving averages is required to generate a trading signal. If we neglect this condition, this rule can generate a trading signal in each trading period implying that traders are always in the market (Brock et al., 1992). Another important feature of this rule is that it shows Momentum behavior by generating buying signals in case of increasing prices and selling signals in case of decreasing prices (Menkhoff and Taylor, 2007)[7].

[5]For a survey the reader is referred to Menkhoff and Taylor (2007).

[6]In a control theory sense, a moving average acts as a low-pass filter, which filters away high-frequency noise.

[7]The opposite is the case for a Mean Reversion strategy, which is heavily used by Hedge Funds. If a short moving average is below a long moving average, a buying signal is perceived. The long-moving average in the Mean Reversion strategy therefore can therefore be considered a proxy for the fundamental value derived upon historic data.

The parameter $\beta > 0$ measures the aggressiveness with which the chartist traders take positions in the market. Comparable to the parameter α for the chartist traders this can also be interpreted as an individual inverse risk aversion of each strategy meaning that for high values these traders exhibit low risk aversion and vice versa.

A crucial factor in market trading is *noise trading*. According to Black (1986), noise traders trade on noise as if it were information. Noise is modeled as an i.i.d. process with mean zero and variance σ_i^2. This is consistent with the consideration of Shleifer (2000) that noise should, on mean, cancel itself out. Noise trading can also be explained by the need for liquidity (here the need to raise capital for other reasons (Bouchaud et al., 2009)). In line with Westerhoff (2008), noise is considered in three parts of the model. First, there is a demand of pure noise traders a_t which is included in the Market Maker equation:

$$p_{t+1} = p_t + \mu \left(\sum_{i=1}^{n} W_t^i D_t^i - N_t \right) + a_t. \qquad (9)$$

On the other side, both fundamentalist and chartist traders have features of noise traders. Therefore their expectations formation is also superimposed by noisy processes b_t and c_t:

$$E_t^F(p_{t+1} - p_t) = \alpha(f_t - p_t) + b_t. \qquad (10)$$

$$E_t^C(p_{t+1} - p_t) = \beta \left[\frac{1}{N_s} \sum_{i=0}^{N_s-1} p_{t-i} - \frac{1}{N_l} \sum_{i=0}^{N_l-1} p_{t-i} \right] + c_t. \qquad (11)$$

Since chartists exhibit more irrational noisy behavior, it is assumed that $\sigma_c > \sigma_b$.

3. Analytical approach in a linearized version of the model

The analytical approach applies the techniques of control theory in the frequency domain. These rules have been developed for linear differential equations. The use of linear differential equations for the modeling of stock market behavior dates back to Beja and Goldman (1980) and is still widely used in models such as Chiarella et al. (2011). One key advantage of the method of classic linear control theory is that we can derive closed form solutions between inputs (in our case: news shocks) and outputs (here: prices) and discuss them (Davis et al., 2012)[8]. Since the model consists of non-linear difference equations, several simplifications have to be made. First, we assume that prices are described by a continuous time function $p(t)$ instead of a discrete function p_t with the following property:

$$p_{t+1} - p_t \approx \frac{dp(t)}{dt} = \dot{p}. \qquad (12)$$

Furthermore, the simplified model assumes risk-neutral investors and a risk-free rate of zero, which leads to the fact that demand equals the expected change of log-prices of each group scaled down by a constant risk aversion ρ[9]:

$$D_t^i = \frac{E_t^i(p_{t+1} - p_t)}{\rho}. \qquad (13)$$

In the two-trader case the model we can follow the well-established approach of Brock and Hommes (1998) modeling the non-linear behavior of the model using a $\tanh$-function. This model observes the relative difference of weights and attractiveness of chartist and fundamentalists strategy. The difference in weights m_t is defined as follows:

$$m_t = W_t^F - W_t^C \longleftrightarrow W_t^F = \frac{1}{2} + \frac{m_t}{2} \quad \text{and}$$

$$W_t^C = \frac{1}{2} - \frac{m_t}{2}. \qquad (14)$$

By introducing the difference between attractiveness $U_t = A_t^F - A_t^C$ the difference in weights can be represented by a $\tanh$-function:

$$m_t = \frac{e^{\gamma A_t^F} - e^{\gamma A_t^C}}{e^{\gamma A_t^C} + e^{\gamma A_t^F}} = \frac{e^{\gamma (A_t^F - A_t^C)} - 1}{e^{\gamma (A_t^F - A_t^C)} + 1}$$

$$= \frac{e^{\gamma U_t} - 1}{e^{\gamma U_t} + 1} = \tanh\left(\frac{\gamma}{2} U_t\right) \qquad (15)$$

[8]A short general introduction in the methodology is given in the appendix in section B.

[9]This formation of demand is based on the results presented in appendix A.

This makes the following equation for prices[10]:

$$\begin{aligned}\dot{p} &= \mu\left[\left(\frac{1}{2}-\frac{m}{2}\right)D^C+\left(\frac{1}{2}+\frac{m}{2}\right)D^F\right]\\ &= \frac{\mu}{2}\left[D^C+D^F+m(D^F-D^C)\right]\\ &= \frac{\mu}{2}\left[D^C+D^F+\tanh\left(\frac{\gamma}{2}U\right)(D^F-D^C)\right].\end{aligned} \tag{16}$$

This formulation is very useful, since it disentangles the linear (first two terms) and the non-linear part (third term) of the system's behavior. If we consider the case of $\gamma=0$, we model totally irrational individuals who stick to a rule, even though it is not profitable. With this strong assumption chartists and fundamentalists always have the same market share. Moreover, the system loses its nonlinearity and therefore can be analyzed with the tools of classical linear control theory. It takes the following form:

$$\dot{p} = \frac{\mu}{2}(D^C+D^F). \tag{17}$$

This result is identical to the one of Chiarella et al. (2011). The noise terms in the models are set to their expected value of zero (De Grauwe and Grimaldi, 2006). The nonlinear term will be of importance, when we compare the linearized version with non-linear simulation results.

First, we want to examine the fundamentalist-only case ($\beta=0$). This results in the following law of motion for log-prices p:

$$\dot{p} = \frac{\mu}{2\rho}(\alpha(f-p)). \tag{18}$$

If we transfer this equation into the frequency domain, the following response function $F(s)$ to a step-shock in fundamental value can be derived[11]:

$$F(s) = \frac{p(s)}{f(s)} = \frac{1}{1+\frac{2s\rho}{\mu\alpha}}. \tag{19}$$

By assuming a step function, we examine the effect of prices in the case where the log-fundamental value f suddenly changes from zero to one. The result resembles the classic PT_1-behavior of control theory (Franklin et al., 2005):

$$F(s) = \frac{K}{1+Ts}. \tag{20}$$

In the time domain the system converges to a final value of K with a speed of T (see figure 1)[12]. Since in this case $K=1$, the model converges to its fundamental value. In this case an underreaction-only

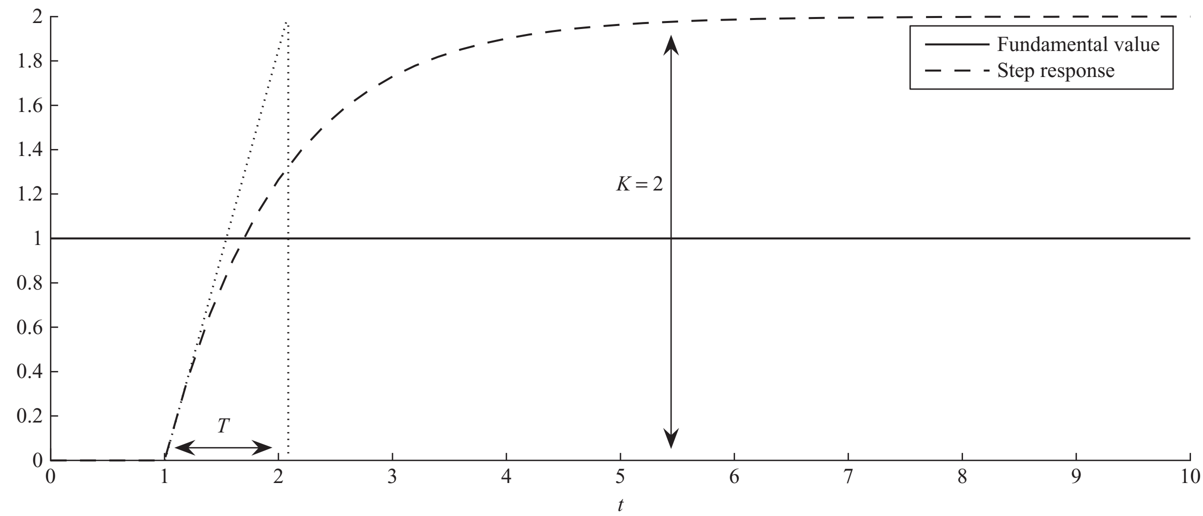

Fig. 1. Response of a PT_1 system to a step function

[10]Note that in this case we skip the index t. This is to show that we transformed from a discrete function to a continuous-time function and thereby from a difference equation to a differential equation.

[11]This result is derived in section C of the appendix

[12]If we transform equation 20 back in the time domain this results in $F(t) = K\left(1-e^{-\frac{t}{T}}\right)$, which converges to K for $t\to\infty$.

scenario is produced. The effect of underreaction is stronger for high values of T:

$$T = \frac{2\rho}{\mu\alpha}. \tag{21}$$

The effect of underreaction is therefore stronger in case the case of low price adjustment speed μ (i.e. high market liquidity) as well as the low aggressiveness of fundamental agents α (i.e. high risk aversion of fundamentalists) and high overall risk aversion ρ. The eigenvalue is given as follows[13]:

$$s = -\frac{1}{T} = -\frac{2\rho}{\mu\alpha} < 0. \tag{22}$$

The system is stable since the eigenvalue s is always negative. If we furthermore take into account a Walrasian auctioneer as a special case of Market Maker with infinite conversion speed, there is no underreaction ($T = 0$). The same result can be derived for the case of risk-neutral fundamental trader (α converging to infinity) or general risk neutrality ($\rho = 0$). This is consistent with the idea of the EMH that prices adjust instantaneously to news (Menkhoff and Taylor, 2007).

Now, the effect of different technical rules on the behavior of prices is investigated. We start by assuming the very simple case of $N_s = 1$ and $N_l = 2$. This yields the following demand for chartists:

$$D^C = \frac{\beta}{\rho}\left(p_t - \frac{1}{2}(p_t - p_{t-1})\right) = \frac{\beta}{2\rho}(p_t - p_{t-1}). \tag{23}$$

This modeling for the demand of chartists is frequently used in HAMs (e.g. Westerhoff, 2008). The main idea is that chartists expect the most recent trend to continue at a speed of $\frac{\beta}{2}$. Considering differential instead of difference equation chartist demand can be presented as follows[14]:

$$D^C = \frac{\beta}{2\rho}(p_t - p_{t-1}) = \frac{\beta}{2\rho}(\dot{p} - \ddot{p}). \tag{24}$$

If we insert this into the Market Maker equation and transfer it into the frequency domain, the following response function $F(s)$ to a step in fundamental value can be derived[15]:

$$F(s) = \frac{p(s)}{f(s)} = \frac{1}{\frac{\beta}{2\alpha}s^2 + \frac{4\rho - \mu\beta}{2\mu\alpha}s + 1}. \tag{25}$$

This behavior represents the so-called PT_2 function of control theory (Franklin et al., 2005):

$$F(s) = \frac{K}{\frac{1}{\omega_0^2}s^2 + \frac{2D}{\omega_0}s + 1}. \tag{26}$$

The eigenvalues of the system are defined by the following equation:

$$s_{1/2} = \omega_0(-D \pm \sqrt{D^2 - 1}). \tag{27}$$

In this case, the variables D and ω_0 are given as follows:

$$\omega_0 = \sqrt{\frac{2\alpha}{\beta}}. \tag{28}$$

$$D = \frac{4\rho - \mu\beta}{2\sqrt{2}\mu\sqrt{\alpha\beta}}. \tag{29}$$

Depending on the value of D three cases can be distinguished (see figure 2). In technical application the parameter D is normally considered as the damping of a system. Negative values for D indicate instable systems, while the degree of D determines the dynamic behavior of a stable system. In an economic sense, this can be interpreted as the behavior of under ($D > 1$) and overreaction ($0 < D < 1$). In the first case $D > 1$, the system converges in a slow process of underreaction to its fundamental value like the PT_1 transfer function. The condition for underreaction-only therefore is as follows:

$$4\rho > \mu\sqrt{\beta}(\sqrt{\beta} + 2\sqrt{2}\sqrt{\alpha}). \tag{30}$$

Low values of price adjustment speed μ as well as low aggressiveness of agents α and β therefore lead to the underreaction-only scenario. Keeping in mind that low aggressiveness can also be interpreted as high risk aversion by agents this leads to the result that underreaction is promoted in a scenario with high trader risk aversion (low values for α and β) or high

[13] As presented in section B of the appendix the eigenvalue s can be calculated by setting the denominator of the transfer function in the frequency domain to zero.

[14] A derivation of this result is presented in appendix C.

[15] The derivation and the discussion of the transfer function is presented in section C of the appendix.

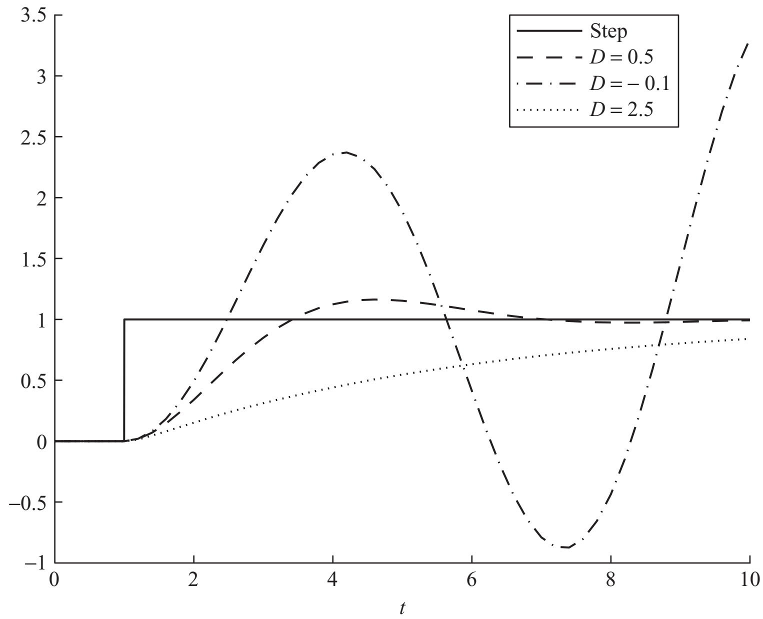

Fig. 2. Response of a PT_2 system to a step function with exemplary values for D

overall risk aversion ρ. If we switch the $>$ with a $<$ sign we get the conditions for overreaction. Since low values of μ can be interpreted as high liquidity, this implies that overreaction tends to occur more frequently in illiquid markets. Note that in the presence of chartists high aggressiveness by both chartist and fundamental traders lead to overreaction.

Overreaction on the other side occurs in second case of $0 < D < 1$. As presented in Hommes (2011), the effect of overreaction can only be produced in the case where chartist traders with autoregressive behavior of at least second order (AR(2) behavior) are assumed. The simulation shows the well-known hump-shaped price pattern as presented in Daniel et al. (1998) and Hong and Stein (1999) of underreaction in the first instance followed by subsequent overreaction (see figure 2). In the long-run, the system converges to its underlying fundamental value.

This does not hold in the third case ($D < 0$). For $\mu\beta > 4\rho$ we have an unstable system. High price adjustment speed μ and the high aggressiveness of chartist traders in the presence of low overall risk aversion ρ therefore lead to instability.

The parameter ω_0 (see equation 28) represents the frequency of price behavior. In the case of underreaction-only, high values of ω_0 therefore indicate fast conversion to fundamental value, whilst in the case of combined under and overreaction they lead to faster swings between under and overreaction. High values for fundamentalist aggressiveness α relative to the aggressiveness of chartists β therefore at first sight might lead to less underreaction. On the other side, as presented in equation 30, higher values of α lead to overreaction. In other words, high aggressiveness of fundamentalists in order to reduce underreaction leads to the effect of overreaction of market prices to news.

If we now assume $N_s = 1$ and $N_l = 3$, the following chartist demand can be derived:

$$D^C = \frac{\beta}{\rho}\left[p_t - \frac{1}{3}\sum_{i=0}^{2} p_{t-i}\right] = \frac{\beta}{\rho}\left[\frac{2}{3}p_t - \frac{1}{3}p_{t-1} - \frac{1}{3}p_{t-2}\right]. \tag{31}$$

The price reaction function is described by the following equation[16]:

$$F(s) = \frac{1}{-\frac{\beta}{\alpha 3}s^3 + \frac{4\beta}{3\alpha}s^2 + (\frac{2\rho-\mu\beta}{\mu\alpha})s + 1}. \tag{32}$$

This system is always unstable. This also holds true for all other $N_l > 3$ as proofed in the appendix in section D. Therefore, the theoretical results of Chiarella et al. (2009) which show that longer moving average rules destabilize the market are confirmed.

[16]The determination of this equation is presented in appendix C.

4. Simulation of the complex model

As discussed in section 3, the analytical approach required some simplifications. Therefore, the simulation results of the complex model are compared with the linearized model.

Applying a shock of $\ln(2) \approx 0.69$ in log-fundamental value f_t is identical to a doubling of real fundamental value F_t[17]. Figure 3 shows simulation results for the case with $L_s = 1$ and $L_l = 2$ in a zero-noise-framework. The parameters are set to $\mu = 1$ and $\alpha = \beta = 0.8$ implying underreaction for the linearized model. Furthermore, the values of $\eta = 0.985$ for memory and $\gamma = 20$ for rationality are assumed[18]. Therefore, there are the same assumption for the linearized and complex case. In fact, the complex and the linearized model show very similar behavior, whilst the effect of underreaction is less strong in the complex case. The rationale for this is that news makes fundamental trading more attractive ($U_t > 0$) and therefore the market weight of fundamental traders increases ($m_t > 0$)[19]. Therefore, the model predicts that periods with news in markets are characterized by strong fundamental trading activity. It is also interesting to observe that the fundamental traders start taking long positions in the market ($D_t^F - D_t^C > 0$). When later on the chartists jump on the bandwagon, they take stronger buying positions than the fundamental traders ($D_t^F - D_t^C < 0$). Chartist traders react in a lagged manner because they have to observe market price movements to start trading. Since they trade in the same direction as fundamentalists, they contribute to market efficiency. In the long run of the scenario without noise, trading activity converges to zero, prices to fundamentals and the attractiveness as well as the weight of both strategies reach identical levels.

If we take the assumptions as before but increase the values of aggressiveness α and β to 1.2, *ceteris paribus*, in both linearized and complex case overreaction can be observed (see figure 4). The complex model shows stronger dynamics with prices

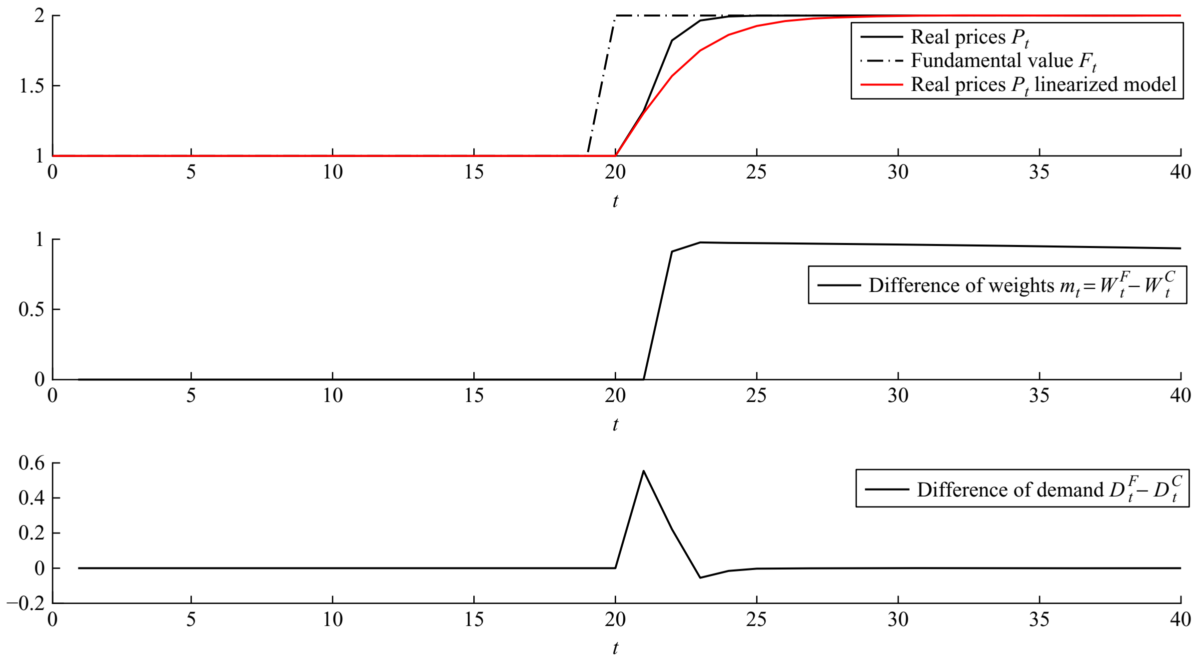

Fig. 3. Reaction of real prices P_t, difference of attractiveness U_t, difference of weights m_t, and difference of demand $D_t^F - D_t^C$ to a step shock in news in both linearized and complex model for the underreaction case

[17]We only show results for positive news shocks. Since there is no asymmetrical component in the model these results also apply to negative news shocks.

[18]The risk aversion is assumed as $\rho = 1 = const$ and the risk-free rate as $r_f = 0.0\%$.

[19]Recall from the preceding section that $m_t = \tanh\left(\frac{\gamma}{2}U_t\right)$.

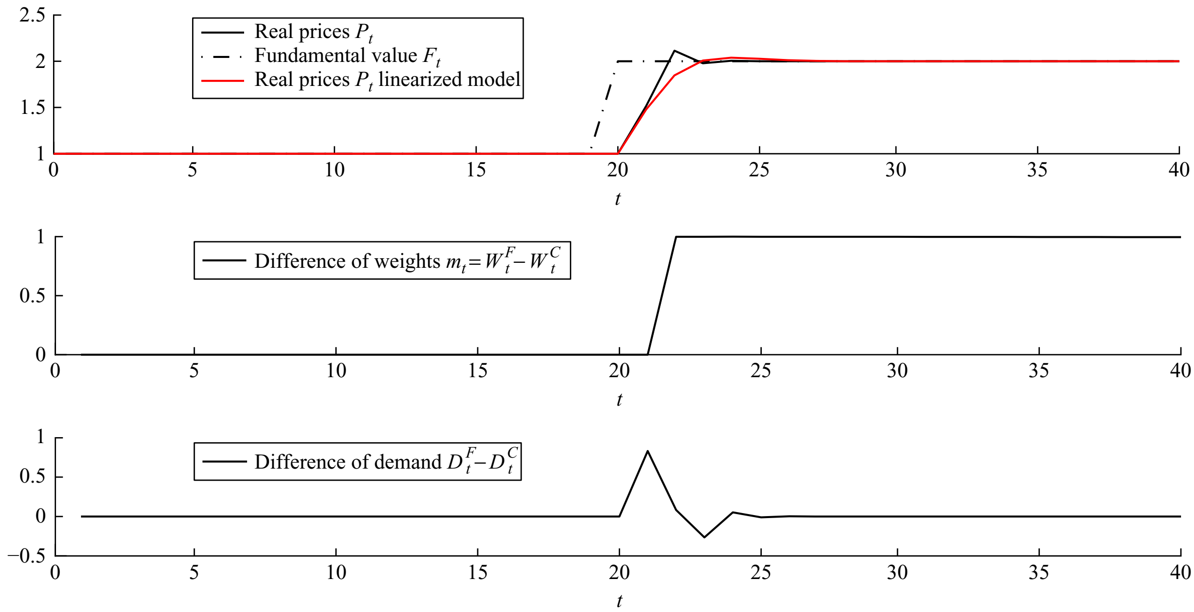

Fig. 4. Reaction of real prices P_t, difference of attractiveness U_t, difference of weights m_t, and difference of demand $D_t^F - D_t^C$ to a step shock in news in both linearized and complex model for the overreaction case

converging faster to fundamentals but also a higher degree of overreaction. This can be attributed to the weight shift to fundamental traders and their stronger demand reaction. Since we increase the aggressiveness of traders with 50 %, not surprisingly, the difference in demand ($D_t^F - D_t^C$) also increases with the same amount. This also contributes to the attractiveness of the fundamental strategy and its higher market share over time compared to the underreaction case. Once again we can also observe the lagged behavior of chartist traders. In period 23, where fundamental traders already started selling the asset due to the effect of overreaction, chartists still take long positions ($D_t^F - D_t^C > 0$).

In the complex case, we can also investigate the effect of the important variables rationality γ and memory η. Figure 5 presenting the underreaction case ($\alpha = \beta = 0.8$) shows that higher rationality leads to higher weight of fundamental traders ($m_t > 0$) and therefore to less underreaction and faster convergence to fundamentals. As we already seen, in times of fundamental news fundamental trading is more attractive than the chartist strategy. Therefore, high values of rationality contribute to a stronger adoption of this strategy. High values of memory η implying long term thinking also result in a higher attractiveness and thereby a more enduring weight of fundamental traders (see figure 6). The technical rationale for this is that high values of η induce stronger autoregressive behavior in the difference of attractiveness U_t. Furthermore, it is interesting to notice that zero memory for a short period of time makes chartism more attractive ($U_t < 0$) and thereby eventually even leads to a slight overreaction.

Similar result for the variation of rationality γ and memory η can be found for the overreaction case ($\alpha = \beta = 1.2$). The stronger switching to the fundamental strategy due to higher rationality γ as well as the more stable weight of this strategy due to higher memories η contribute to higher market efficiency in the sense of lower overreaction.

Up to this point, simulation assumed zero noise. Now, different forms of noise are applied for the underreaction case (see figure 7). The figure shows the mean as well as the 5% and the 95% quantile of the different of weights m_t after $n = 100$ simulation results with varying seeds. First of all and not surprisingly, higher degrees of noise also lead to higher variance in the model outcome. The most interesting effect is visible in the time before the news shock. Overall market noise σ_a has little effect on the weight composition of different strategies. If, on the other hand, noise is introduced into the fundamentalist

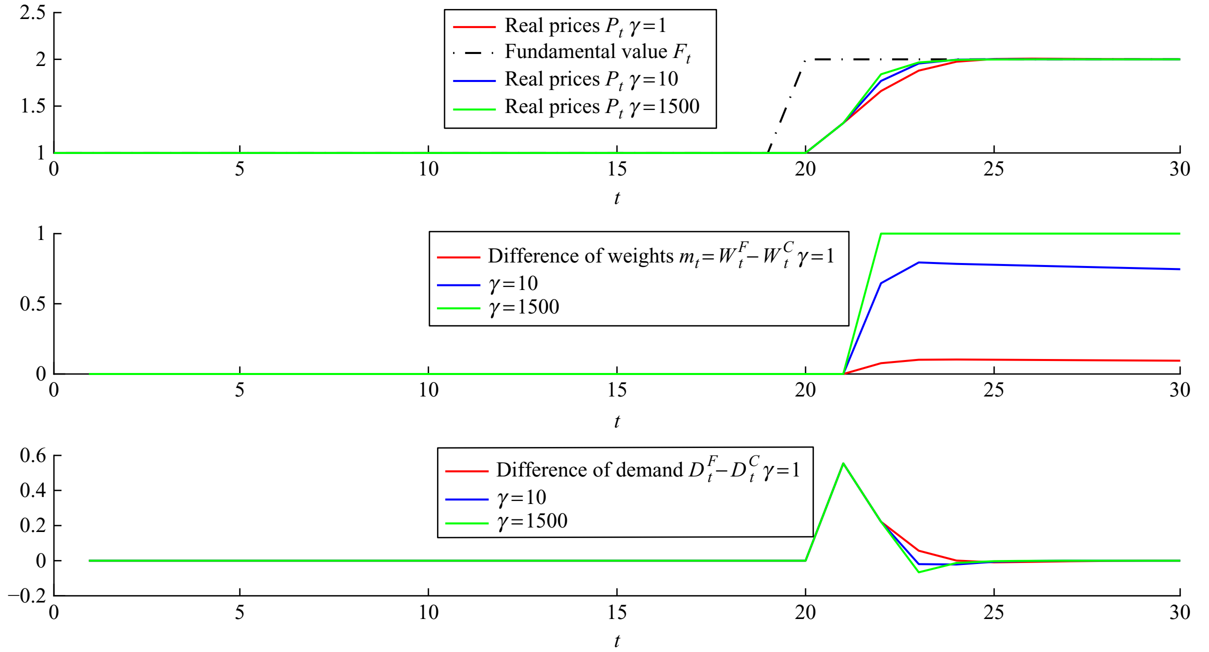

Fig. 5. Reaction of real prices P_t, difference of weights m_t, and difference of demand $D_t^F - D_t^C$ to a step shock in news for the complex model for the underreaction case with variation of rationality γ

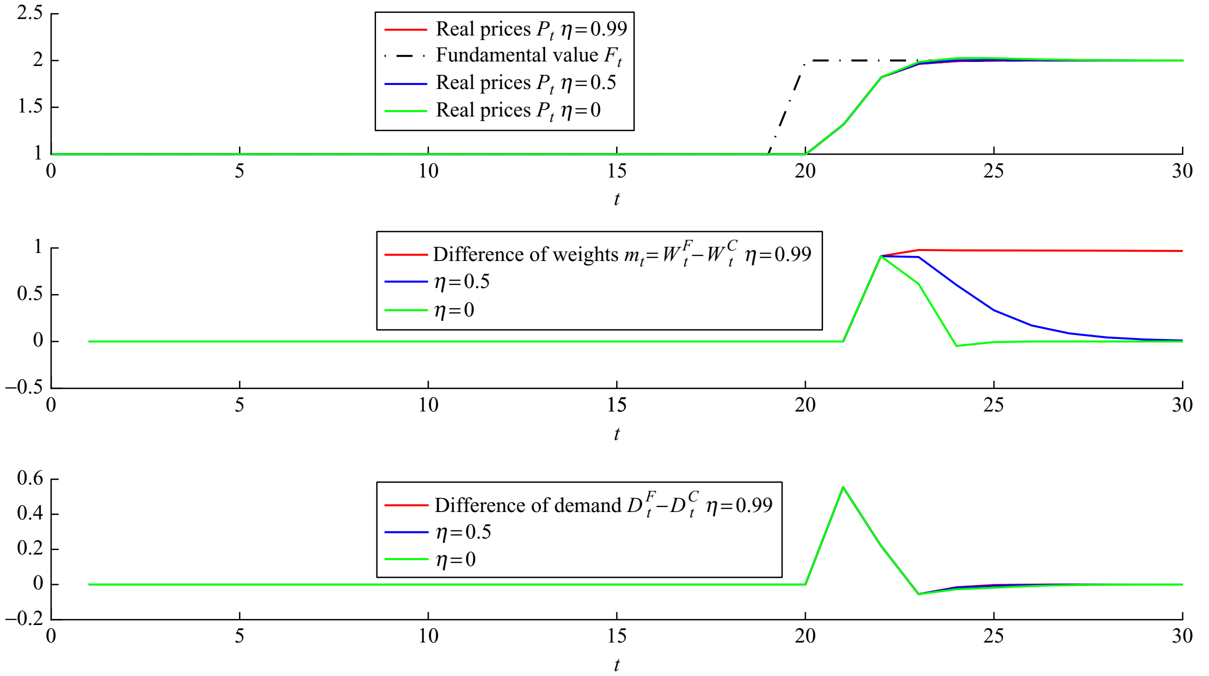

Fig. 6. Reaction of real prices P_t, difference of weights m_t, and difference of demand $D_t^F - D_t^C$ to a step shock in news for the complex model for the underreaction case with variation of memory η

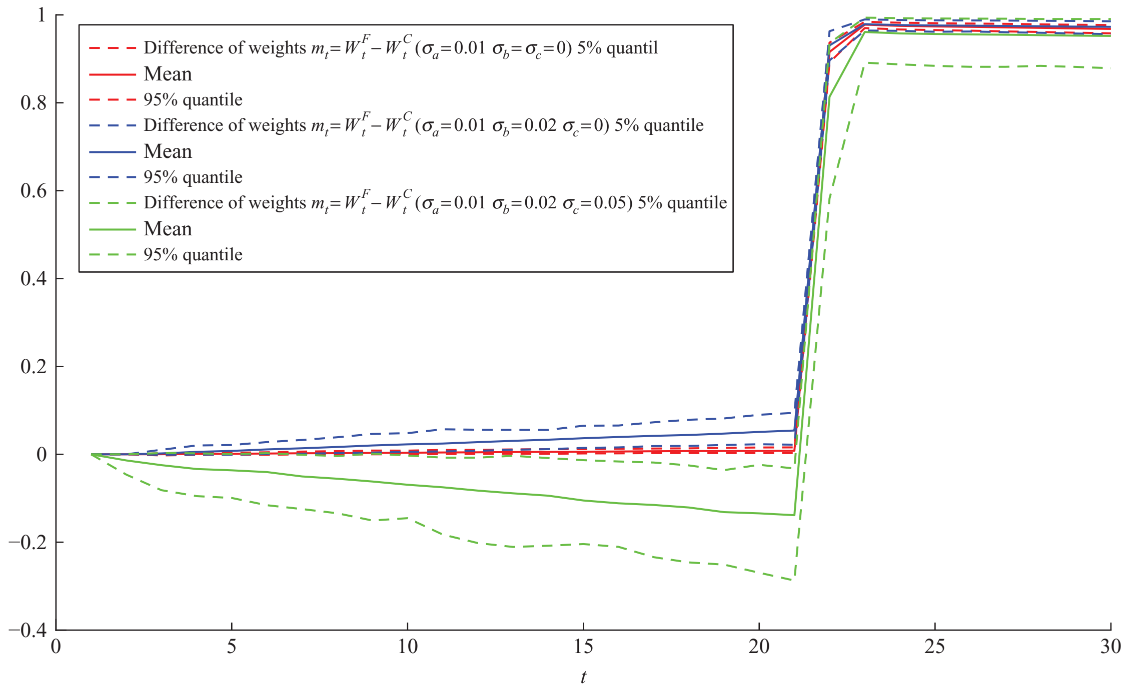

Fig. 7. Difference of weights m_t after a step shock in news for the complex model for the underreaction case with variation of noise σ_i

strategy σ_b, the weight of this strategy increases[20]. Vice versa, the introduction of noise in the chartist strategy σ_c shifts the weights to this strategy[21]. This yields the interpretation that a strategy becomes more attractive if it is noisier. The economic rationale for this would be that the act of trading on a certain strategy confirms the predictions of this strategy and thereby makes it more attractive. Nevertheless, after the news shock the fundamentalist strategy still clearly dominates, even though this domination is not so strong for the case with chartist noise σ_c. This effect of noise seems to be of great importance and should be discussed more intensely in future research.

5. Conclusion

In this paper the phenomenon of under and overreaction to news in financial markets is discussed within the framework of a Heterogeneous Agent Model. This model relies on the idea that market prices are the result of the interaction of fundamental and technical traders both subject to bounded rationality as well as short-term thinking. Furthermore, there is noise in the trading process. An analytical approach of the linearized model confirmed that the existence of finite price adjustment speed and risk-aversion leads to underreaction. A fundamental-only scenario with infinite price adjustment speed (Walrasian auctioneer) on the other hand can replicate the instantaneous adjustment to news fundamentals as predicted by the Efficient Market Hypothesis. Chartist behavior transform an underreaction-only scenario into a scenario with under and overreaction. Consistent with Chiarella et al. (2006), the use of longer moving average rules also leads to systemic instability.

Based on a simulation this paper also shows that the analytical approach overestimates the effect of financial fragility by assuming constant agent weights. In the simulation, news leads to a higher weight of fundamental agents that transforms the system back to its fundamental value. Apart from that, the simulation confirms that high degrees of rationality and long-term thinking decrease the effect of underreaction as well as overreaction and thereby contributing to higher market efficiency.

The simulation also considers the effect of noise. There is evidence that high noise contributes to the attractiveness of a strategy. This should be explored in future research. Apart from that, the autocorrelations frequently measured in empricial studies of under

[20]This is the case, since the blue line is above the red line.
[21]Since the green line is below the blue line.

and overreaction should be studied in this model framework. Since the model is able to derive a closed form solution of the prices and returns, this could be a starting point for calculating a closed form solution of the autocorrelations yielding further insights.

This paper only presents a very simplified analytical approach. Deeper insights might be gained in the case where the model is analyzed in the so-called z-domain developed for difference equations (e.g. Juang, 1994). Furthermore, more realistic moving-average rules, as presented in Brock et al. (1992), should be examined in a simulation-based approach. Further research should also discuss the effect of these rules on statistical properties commonly investigated in HAMs.

Appendix

A. The simplified demand function

The computational model processes log-prices p_t instead of real prices P_t. This has the advantage that in contrast to real prices, which cannot fall below zero, log-prices are not bounded. Recall the following mathematical connection for log-prices:

$$p_{t+1} - p_t = \ln(P_{t+1}) - \ln(P_t) = \ln\left(\frac{P_{t+1}}{P_t}\right) = \ln(1 + r_{t+1}). \tag{33}$$

The first-order Taylor approximation for the ln function yields the following equation:

$$\ln(1 + x) = x + O(x^2). \tag{34}$$

Since we only consider short time periods, we also want to assume that the volatility of returns is constant ($\sigma_{t,r}^2 = \sigma_r^2$) (cp. e.g. Black and Scholes, 1973) and introduce the variable $\rho = RA \cdot \sigma_r^2$. If we now use these results, the demand of group i can be displayed in the following way:

$$\begin{aligned} D_t^i &= \frac{E_t^i(p_{t+1} - p_t) - \ln(1 + r_f)}{RA \cdot \sigma_r^2} \\ &= \frac{E_t^i(\ln(1 + r_{t+1})) - \ln(1 + r_f)}{RA \cdot \sigma_r^2} \\ &\approx \frac{E_i(r_{t+1}) - r_f}{\rho}. \end{aligned} \tag{35}$$

Since the daily risk-free rate is close to zero it is usually neglected (see e.g. Fama, 1998). This results in the fact that in this simplified version the demand equals the expected return scaled down by a constant risk aversion ρ.

B. A very short introduction in linear control theory

The so-called classical linear control analysis studies the behavior of dynamic system and is heavily utilized in aerospace or process engineering (Wingrove and Davis, 2012). Besides that, there is also a long-term relationship between control theory and macroeconomic modeling (Kendrick, 2005). This toolbox enables researches to identify the behavior of complex systems and to derive rules to control them. The classic control theory focuses on linear systems. The analysis is conducted in the frequency domain $s = j\omega$, where ω represents the oscillatory frequency and $j = \sqrt{-1}$ is the imaginary unit. The transformation from the time domain t to the frequency domain s is given by the solution of the Fourier integral (Franklin et al., 2005):

$$y(s) = \int_0^\infty y(t) e^{-st} dt. \tag{36}$$

It can be described by the following symbolism:

$$y(t) \circ\!\!-\!\!\!-\!\!\bullet\, y(s). \tag{37}$$

One of the most important transformations is the one for derivatives (Franklin et al., 2005):

$$\frac{d^n y(t)}{dt^n} \circ\!\!-\!\!\!-\!\!\bullet\, s^n y(s) - \sum_{i=1}^{n} s^{n-i} \left(\frac{d^{(i-1)} f(t)}{dt^{i-1}}\right)_{t=0+}. \tag{38}$$

The transfer function $F(s)$ describes the behavior of a dynamic system y to the input u and is defined in the following way (Franklin et al., 2005, p. 60):

$$F(s) = \frac{y(s)}{u(s)} = \frac{b_0 + b_1 s + \cdots + b_m s^m}{a_0 + a_1 s + \cdots + a_n s^n} = \frac{N(s)}{D(s)}. \tag{39}$$

By setting the denominator to zero ($D(s) = 0$) we can derive the so-called poles or eigenvalues of the system $s_1, s_2, \ldots, s_n$, which describe the homogeneous solution of the system in the time domain (Franklin et al., 2005):

$$y_{hom}(t) = \sum_{i=1}^{n} C_i e^{s_i \cdot t}. \tag{40}$$

The stability condition is that the real part of the eigenvalue is negative ($Re\{s_i\} < 0$) (Franklin et al., 2005).

C. Derivation of the different transfer functions for the special cases with $L_s = 1$ and $L_l = 2, 3$

We start by the derivation of the most simple case with fundamental traders only. By transforming the differential equation into the frequency domain and solving for the transfer function the following result can be derived:

$$\begin{aligned}
\dot{p} &= \frac{\mu}{2\rho}(\alpha(f-p)) \\
&\circ\!\!-\!\!\bullet\, p(s)\left(s + \frac{\mu\alpha}{2\rho}\right) = f(s)\frac{\mu\alpha}{2\rho} \\
&\Rightarrow F(s) = \frac{\mu\alpha}{2\rho s + \mu\alpha} = \frac{1}{\frac{2\rho}{\mu\alpha}s + 1}.
\end{aligned} \tag{41}$$

The chartist demand for the case $L_s = 1$ and $L_l = 2$ can be derived if we consider the following assumption for the second order derivative:

$$\begin{aligned}
\ddot{p} &\approx \dot{p}(t) - \dot{p}(t-1) \\
&\approx (p_{t+1} - p_t) - (p_t - p_{t-1}) \\
&= p_{t+1} - 2p_t + p_{t-1}.
\end{aligned} \tag{42}$$

This results in the following chartist demand D^C:

$$\begin{aligned}
D^C &= \frac{\beta}{2\rho}(p_t - p_{t-1}) \\
&= \frac{\beta}{2\rho}((p_{t+1} - p_t) - (p_{t+1} - 2p_t + p_{t-1})) \\
&= \frac{\beta}{2\rho}(\dot{p} - \ddot{p}).
\end{aligned} \tag{43}$$

Using these results the following transfer function can be derived:

$$\begin{aligned}
\dot{p} &= \frac{\mu}{2\rho}\left[\frac{\beta}{2}(\dot{p} - \ddot{p}) + \alpha(f-p)\right] \\
&\circ\!\!-\!\!\bullet\, p(s)\left(\frac{\mu\beta}{4\rho}s^2 + (1 - \frac{\mu\beta}{4\rho})s + \frac{\mu\alpha}{2\rho}\right) \\
&= f(s)\cdot\frac{\mu\alpha}{2\rho} \\
&\Rightarrow F(s) = \frac{\frac{\mu\alpha}{2\rho}}{\frac{\mu\beta}{4\rho}s^2 + (1 - \frac{\mu\beta}{4\rho})s + \frac{\mu\alpha}{2\rho}} \\
&= \frac{1}{\frac{\beta}{2\alpha}s^2 + \frac{(4\rho - \mu\beta)}{2\mu\alpha}s + 1}.
\end{aligned} \tag{44}$$

Now, the case of $L_s = 1$ and $L_l = 3$ is presented. The transformation of the difference equation into a differential equation requires the following connection:

$$\begin{aligned}
\dddot{p}_t &\approx \ddot{p}(t) - \ddot{p}(t-1) \\
&\approx (p_{t+1} - 2p_t + p_{t-1}) - (p_t - 2p_{t-1} + p_{t-2}) \\
&= p_{t+1} - 3p_t + 3p_{t-1} - p_{t-2}.
\end{aligned} \tag{45}$$

The chartist demand can therefore be described by the following equation:

$$\begin{aligned}
D^C &= \frac{\beta}{\rho}\left(\frac{1}{3}\dddot{p} - \frac{4}{3}\ddot{p} + \dot{p}\right) \\
&= \frac{\beta}{\rho}\left[\left(\frac{1}{3}p_{t+1} - p_t + p_{t-1} - \frac{1}{3}p_{t-2}\right)\right. \\
&\quad \left. + \left(-\frac{4}{3}p_{t+1} + \frac{8}{3}p_t - \frac{4}{3}p_{t-1}\right) + (p_{t+1} - p_t)\right] \\
&= \frac{\beta}{\rho}\left[\frac{2}{3}p_{t+1} - \frac{1}{3}p_t - \frac{1}{3}p_{t-1} - \frac{1}{3}p_{t-2}\right].
\end{aligned} \tag{46}$$

Using this result the transfer function is calculated:

$$\begin{aligned}
\dot{p} &= \frac{\mu}{2\rho}\left[\beta\left(\frac{1}{3}\dddot{p} - \frac{4}{3}\ddot{p} + \dot{p}\right) + \alpha(f-p)\right] \\
&\circ\!\!-\!\!\bullet\, p(s)\left(-\frac{\mu\beta}{6\rho}s^3 + \frac{2}{3}\frac{\mu\beta}{\rho}s^2 + \left(1 - \frac{\mu\beta}{2\rho}\right)s\right. \\
&\quad \left. + \frac{\mu\alpha}{2\rho}\right) = f(s)\cdot\frac{\mu\alpha}{\rho} \\
&\Rightarrow F(s) = \frac{1}{-\frac{\beta}{\alpha 3}s^3 + \frac{4\beta}{3\alpha}s^2 + (\frac{2\rho - \mu\beta}{\mu\alpha})s + 1}.
\end{aligned} \tag{47}$$

This system is always unstable. This system is a so-called PT_3 system, which can be described as serial connection of three PT_1 systems. Mathematically, this can be done by multiplying PT_1 functions:

$$F(s) = \left(\frac{K}{sT+1}\right)^3 = \frac{K^3}{s^3T^3 + 3s^2T^2 + 3sT + 1}. \tag{48}$$

Since the stability condition for the PT_1 system requires $T > 0$ all coefficients of the denominator of the PT_3 function have to be positive as well. In this case the coefficient of s^3 is always negative, thus rendering the system unstable.

D. Stability conditions for the generalized case

This section proofs that the case of $N_s = 1$ and $N_l > 2$ is always unstable. The chartist demand can be presented in the following general way:

$$\begin{aligned} D_t^C &= \frac{\beta}{\rho}\left(p_t - \frac{1}{N_l}\sum_{i=0}^{N_L-1} p_{t-i}\right) \\ &= \frac{\beta}{\rho}\left(\frac{N_l-1}{N_l}p_t - \frac{1}{N_l}\sum_{i=1}^{N_l-1} p_{t-i}\right). \end{aligned} \tag{49}$$

This difference equation can be considered as a differential equation of the following form in the frequency domain, where β_i represent the weight of the i-th derivative:

$$D^C = \frac{\beta}{\rho}\cdot p(s)\sum_{i=1}^{N_L}\beta_i s^i. \tag{50}$$

If we insert this term in the price-equation, the following result can be derived:

$$s\cdot p(s) = \frac{\mu}{2\rho}\left(\alpha(f(s)-p(s)) + \beta\cdot p(s)\sum_{i=1}^{N_L}\beta_i s^i\right)$$

$$\Rightarrow F(s) = \frac{p(s)}{f(s)} = \frac{1}{1 + \frac{2\rho-\mu\beta\beta_1}{\mu\alpha}s - \frac{\beta}{\alpha}\sum_{i=2}^{N_l}\beta_i s^i}. \tag{51}$$

Again, we can consider this as a serial connection of N_l systems of the $PT1$-type:

$$F(s) = \left(\frac{K}{Ts+1}\right)^{N_l}. \tag{52}$$

This results in the fact that all coefficients of the denominator have to be positive[22]. This results in the following stability condition:

$$\beta_i < 0 \quad \text{for} \quad i > 1 \quad \text{and} \quad \beta_1 < \frac{2\rho}{\mu\beta}. \tag{53}$$

The factors β_i are determined by the following system of linear equations that links the difference equation (left side representing equation 49) to the differential equation in the frequency domain (right side related to equation 50):

$$\begin{aligned} \begin{pmatrix} \frac{N_l-1}{N_l} \\ -\frac{1}{N_l} \\ -\frac{1}{N_l} \\ \vdots \\ \vdots \\ -\frac{1}{N_l} \end{pmatrix} &= \begin{bmatrix} -1 & -2 & -3 & -4 & \cdots & \cdots \\ 0 & 1 & 3 & 6 & \cdots & \cdots \\ 0 & 0 & -1 & -4 & \cdots & \cdots \\ 0 & \cdots & 0 & 1 & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 & \pm 1 \end{bmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \vdots \\ \vdots \\ \beta_{N_l} \end{pmatrix} \\ &= \begin{bmatrix} \vec{d}_1^T \\ \vec{d}_2^T \\ \vdots \\ \vdots \\ \vdots \\ \vec{d}_{N_l}^T \end{bmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \vdots \\ \vdots \\ \beta_{N_l} \end{pmatrix}. \end{aligned} \tag{54}$$

The matrix consists of the transformed vectors $\vec{d}_i^T$ that can be derived from Pascal's triangle. The diagonal-elements of the matrix consists of alternating ± 1 and the rows alternate between positive and negative entries. The system can be solved starting from the last equation:

$$-\frac{1}{N_l} = \beta_{N_l}(-1)^{N_l} \Rightarrow \beta_{N_l} = \frac{1}{N_l}(-1)^{1-N_l}. \tag{55}$$

For $N_l > 2$ this gives a result for the preceding value β_{N_l-1}. In this case $\vec{d}_i(j)$ represents the j-th element

[22] Or more precisely of the same sign for $K < 0$ and N_l being an uneven number.

of the i-th vector:

$$\beta_{N_l}\vec{d}_{N_l-1}(N_l) + \beta_{N_l-1}\vec{d}_{N_l-1}(N_l-1) = -\frac{1}{N_l}$$

$$\Rightarrow \beta_{N_l-1} = \left(-\frac{1}{N_l} - \beta_{N_l}\vec{d}_{N_l-1}(N_l)\right) \frac{1}{\vec{d}_{N_l-1}(N_l-1)}. \quad (56)$$

Keeping in mind that $\beta_{N_l} = \frac{1}{N_l}(-1)^{1-N_l}$, $\vec{d}_{N_l-1}(N_l-1) = (-1)^{N_l-1}$ and due to the construction of the matrix $\vec{d}_{N_l-1}(N_l) = N_l(-1)^{N_l-1}$ the following result can be derived:

$$\beta_{N_l-1} = \left(-\frac{1}{N_l} - 1\right)(-1)^{1-N_l} = \left(\frac{-N_l-1}{N_l}\right)(-1)^{1-N_l}. \quad (57)$$

This yields the following result:

$$\frac{\beta_{N_l}}{\beta_{N_l-1}} = \frac{1}{-N_l-1} < 0. \quad (58)$$

This implies that if N_l is an even number, $\beta_{N_l} > 0$ and the system is unstable. If N_l is an uneven number, $\beta_{N_l-1} > 0$ also making the system unstable. This shows that for all $N_l > 2$ the system is unstable.

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