Book Review

Inverse Problems: Mathematical and Analytical Techniques with Applications to Engineering, by Alexander G. Ramm

In the introduction (Chapter 1) of the book, the author explains the need in inverse problems: “Inverse problems are the problems that consist of finding an unknown property of an object, or a medium, from the observation of a response of this object, or medium, to a probing signal. Thus, the theory of inverse problems yields a theoretical basis for remote sensing and non-destructive evaluation.”

Chapter 1 brings various examples of inverse problems; chapter 2 deals with ill-posed problems; chapter 3 is devoted to the one-dimensional inverse scattering and spectral problems; chapter 4 considers inverse obstacle scattering; chapter 5 is dedicated to stability of the solutions of 3D inverse scattering problems with fixed-energy data; chapter 6 treats non-uniqueness and uniqueness results; chapter 7 is concerned with inverse problems of potential theory and other inverse source problems; chapter 9 considers low-frequency inversion; the final, tenth chapter, deals with wave scattering by small bodies of arbitrary shapes. The book ends with bibliographical notes, references and index.

The readers of this journal will be most interested with chapter 3, occupying pages 91-226 of this 442 page book. Author mentions: “There are excellent books by Marchenko V.A. [“Sturm-Liouville Operators and Applications”, Birkhäuser, Basel, 1986] and Levitan B. [“Inverse Sturm-Liouville Problems”, VNU Press, Utrecht, 1987], where inverse spectral and scattering problems are discussed in detail.” Unfortunately, the author overlooks an excellent book by Gladwell G., “Inverse Problems in Vibration”, Kluwer Academic Publishers, 1986 first edition; 2004, second edition]. Author reproduces, among others, now classic result by Borg and Marchenko that two spectra uniquely determine the Sturm-Liouville operator $\ell = -d^2/dx^2 + q(x)$, i.e. the potential $q$ and the boundary conditions at $x = 0$ and $x = 1$ of the type

\[
\begin{align*}
    u'(1) + h_1 u(1) &= 0 \\
    u'(0) &= h_0 u(0)
\end{align*}
\]

where $h_0$ and $h_1$ are constants, and one assumes that the two spectra correspond to the same $h_0$ and two distinct $h_1$.

Author does not deal with the question on how to obtain infinite amount of natural frequencies of the system that are demanded in the above problem. Will Sturm-Liouville operator describe accurately the behaviour of the system at high frequencies? These questions are extremely pertinent to the book whose subtitle contains “applications to engineering.”

The above comment does not diminish the importance of this book. This monograph will be an important reference to all those who deal with inverse problems. It appears that the libraries will be enriched by having this book available to interested researchers.

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