

A conflict analysis framework considering fuzzy preferences

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Abstract. A conflict analysis framework with fuzzy preference can be utilized to model and analyze conflicts in which decision makers (DMs) hold unclear or ambiguous preferences. One important function of this framework is that it can predict different conflict resolutions based on DMs' different fuzzy satisficing thresholds. The fuzzy preference conflict analysis framework extends the option prioritization technique that can efficiently elicit DMs' crisp preferences, also using it to calculate DMs' fuzzy preferences. Through analyzing a water pollution conflict that occurred between the upstream and the downstream areas of a reservoir in China, the capability of the fuzzy preference conflict analysis framework to investigate real-world conflicts is verified, as is the efficiency of the fuzzy option prioritization methodology in representing DMs' fuzzy preferences.

Keywords: Conflict analysis, fuzzy preference, option prioritization, water pollution conflict

1. Introduction

Based on the earlier work of Fraser and Hipel [12] and Howard [13], Kilgour et al. [2] proposed the graph model for conflict resolution (GMCR), a flexible methodology for systematically modeling and analyzing conflicts. GMCR requires only the relative preference information of decision makers (DMs), and it has been utilized to study a variety of conflicts including water resources disputes (Madani and Hipel [7], Getirana and Malta [1], Hipel and Walker [6], Yu et al. [3, 4]).

GMCR can be employed to study a conflict through two steps: first, modeling the conflict within a formal mathematical framework; then, conducting a stability analysis to calculate possible equilibriums of the

conflict. DMs' relative preferences constitute an important factor in both steps. Since DMs may be unclear or unsure about some of their preferences in practice, Bashar et al. [10] developed the concept of fuzzy preferences under the framework of GMCR. A fuzzy preference degree between two states indicates the extent to which the preference for one state over the other is certain. The efficient crisp preference eliciting method, or option prioritization [5, 8, 9], was extended by Bashar et al. [11] to model both crisp and fuzzy preferences. Note that a crisp preference is a clear or unambiguous preference.

This paper utilizes GMCR and option prioritization with consideration of fuzzy preferences to study a water pollution conflict between the upstream and downstream regions of China's Guanting reservoir. Section 2 introduces the fuzzy preference framework for GMCR, as well as the fuzzy option prioritization technique. Section 3 applies these methodologies to

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model and analyze a real-world conflict. Section 4 presents some conclusions.

2. GMCR with fuzzy preferences

In general, the fuzzy preference framework for GMCR is composed of a set of DMs $N = \{1, 2, \dots, i, \dots, n - 1, n\}$, a set of feasible states $S = \{s_1, s_2, \dots, s_k, \dots, s_t, \dots, s_w\}$, a set of fuzzy preferences $\{R_i\}_{i \in N}$ on S for each DM, and a set of oriented arcs $A_i \subseteq S \times S$. Overall, the framework can be described as $G = \langle N, S, \{A_i\}_{i \in N}, \{R_i\}_{i \in N} \rangle$.

2.1. Fuzzy preference and stability definitions

Definition 1. (Fuzzy Preference): A fuzzy preference of DM i over S is a fuzzy relation on S , represented by a matrix $R_i = (r_{kt})_{w \times w}$, with membership function $\mu_R : S \times S \rightarrow [0, 1]$, where $\mu_R(s_k, s_t) = r_{kt}$ denotes the preference degree of state s_k over s_t , satisfying $r_{kt} + r_{tk} = 1$ and $r_{kk} = 0.5$.

The preference degree is the level of certainty that a DM will prefer one state over the other. Note that the fuzzy preference model allows for both crisp and fuzzy preferences between two states. When a DM holds a crisp preference, the preference degrees over all states are 0, 0.5, or 1.

Definition 2. (Fuzzy Relative Strength of Preference (FRSP)): Let r_{kt}^i denote the preference degree of state s_k over s_t for DM i . Then, DM i 's FRSP of state s_k over s_t is defined as $\alpha^i(s_k, s_t) = r_{kt}^i - r_{tk}^i$, where $-1 \leq \alpha^i(s_k, s_t) \leq 1$.

The concept of FRSP is used to measure how strongly a DM prefers one state to the other. In stability analysis, each DM with fuzzy preferences may select a level of FRSP to decide whether to maintain or leave a focal state. The FRSP level represents the DM's fuzzy satisficing threshold.

Definition 3. (Fuzzy Satisficing Threshold (FST)): Let DM i 's FST be denoted as γ_i ($0 < \gamma_i \leq 1$). Then, DM i would prefer to move from state s to s_k if and only if (iff) $\alpha^i(s_k, s) \geq \gamma_i$.

Notice that a DM may have different FSTs at different times, and different DMs in a conflict may hold different FSTs as well.

Bashar et al. [10] provides four fuzzy stability definitions to identify possible equilibriums for the conflict. The fuzzy stability definitions vary predictably accord-

ing to the DMs' FSTs. Generally, a state is fuzzy stable for a DM iff leaving a focal state does not meet the DM's FST. It is necessary to define the movements for a single DM and a coalition of DMs before defining the fuzzy stability concepts.

Definition 4. (Fuzzy Unilateral Improvement List (FUIL)): Let γ be DM i 's FST, and $R_i(s)$ be the set of reachable states from state s for DM i . Then, state $s_k \in R_i(s)$ is called a Fuzzy Unilateral Improvement (FUI) from s for DM i iff $\alpha^i(s_k, s) \geq \gamma_i$. The set of all FUIs from s for DM i is called the FUIL, which is denoted by $\tilde{R}_{i,\gamma}^+(s)$, where $\tilde{R}_{i,\gamma}^+(s) = \{s_k \in R_i(s) : \alpha^i(s_k, s) \geq \gamma_i\}$.

Definition 5. (FUIL for a Coalition of DMs): For $H \in N$, let $H = \{1, 2, \dots, m\}$, $\gamma_H = \{\gamma_1, \gamma_2, \dots, \gamma_m\}$, and $\tilde{\Omega}_{H,\gamma_H}^+(s, s_1)$ denote the set of all last or final DMs who can move in one step from state s to s_1 . The set $\tilde{R}_{H,\gamma_H}^+(s)$ satisfies the following criteria: (i) if $i \in H$ and $s_1 \in \tilde{R}_i^+(s)$, then $s_1 \in \tilde{R}_{H,\gamma_H}^+(s)$ and $i \in \tilde{\Omega}_{H,\gamma_H}^+(s, s_1)$; (ii) if $i \in H$, $s_1 \in \tilde{R}_{H,\gamma_H}^+(s)$, $s_2 \in \tilde{R}_{H,\gamma_H}^+(s_1)$, and $\tilde{\Omega}_{H,\gamma_H}^+(s, s_1) \neq \{i\}$, then $s_2 \in \tilde{R}_{H,\gamma_H}^+(s)$ and $i \in \tilde{\Omega}_{H,\gamma_H}^+(s, s_2)$.

In addition to the above definitions, the fuzzy stability concepts are shown in Table 1.

A state that is fuzzy stable for all DMs under a specific fuzzy stability definition is called a fuzzy equilibrium (FE) under that definition.

2.2. Fuzzy option prioritization

In the option prioritization method set forth in [8], each DM i possesses an ordered list of preference statements $P_i = [\Omega_1, \Omega_2, \dots, \Omega_j, \dots, \Omega_q]$ in which the preference statements that are more important for DM i appear earlier in the list. Each preference statement, which is expressed in terms of options and logical connectives, takes a truth-value of either "True" (T) or "False" (F) at each state.

Denote $\Omega_j(s)$ as the truth-value of preference statement Ω_j at state s , and let $\psi_j(s)$ be the score to state s based upon preference statement Ω_j . Define $\psi_j(s) = \begin{cases} 2^{q-j}, & \text{if } \Omega_j(s) = T \\ 0, & \text{otherwise} \end{cases}$ and $\psi(s) = \sum_{j=1}^q \psi_j(s)$; then, the states can be sorted based on their scores. Specifically, $s_k \succ_i s_t$ iff $\psi(s_k) > \psi(s_t)$, and $s_k \sim_i s_t$ iff $\psi(s_k) = \psi(s_t)$.

In some situations, however, preference statements cannot be judged as simply "true" or "false" at some feasible states. Under these circumstances, Bashar et al.

Table 1
Fuzzy stability definitions

Stability	Definitions
Fuzzy Nash Stability (FNash)	A state s is FNash stable for DM i iff $\tilde{R}_i^+(s) = \phi$
Fuzzy General Metarationality (FGMR)	A state s is FGMR stable for DM i iff for every $s_1 \in \tilde{R}_i^+(s)$, there exists an $s_2 \in R_{N-i}(s_1)$ such that $\alpha^i(s_2, s) < \gamma_i$
Fuzzy Symmetric Metarationality (FSMR)	A state s is FSMR stable for DM i iff for every $s_1 \in \tilde{R}_i^+(s)$, there exists an $s_2 \in R_{N-i}(s_1)$ such that $\alpha^i(s_2, s) < \gamma_i$, and $\alpha^i(s_3, s) < \gamma_i$ for all $s_3 \in R_i(s_2)$
Fuzzy Sequentially Stable (FSEQ)	A state s is FSEQ stable for DM i iff for every $s_1 \in \tilde{R}_i^+(s)$, there exists an $s_2 \in \tilde{R}_{N-i}^+(s_1)$ such that $\alpha^i(s_2, s) < \gamma_i$

[11] used fuzzy truth-values represented by numerical values in the interval $[0, 1]$ to judge the truthfulness of a preference statement at a feasible state. Let $\sigma_j(s) = \sigma(\Omega_j, s)$ denote the fuzzy truth-value of preference statement Ω_j at state $s \in S$. Then, $\sigma_j(s) = 0$ is equivalent to $\Omega_j = F$, while $\sigma_j(s) = 1$ is equivalent to $\Omega_j = T$.

The fuzzy score intervals of states can be compared in pairs to calculate the preference degree over two states using Definition 8.

Definition 8. For $L_k = \tilde{\psi}^U(s_k) - \tilde{\psi}^L(s_k)$ and $L_t = \tilde{\psi}^U(s_t) - \tilde{\psi}^L(s_t)$, one can use the following function (1) to calculate $r(s_k, s_t) = r_{kt}$:

$$r_{kt} = \begin{cases} \max \left\{ \min \left\{ \frac{\tilde{\psi}^U(s_k) - \tilde{\psi}^L(s_t)}{L_k + L_t}, 1 \right\}, 0 \right\}, & \text{if } L_k + L_t \neq 0 \\ \max \left\{ \frac{\tilde{\psi}^U(s_k) - \tilde{\psi}^L(s_k)}{|\tilde{\psi}^U(s_k) - \tilde{\psi}^L(s_t)|}, 0 \right\}, & \text{if } L_k + L_t = 0 \text{ and } \tilde{\psi}(s_k) \neq \tilde{\psi}(s_t) \\ \frac{1}{2}, & \text{if } L_k + L_t = 0 \text{ and } \tilde{\psi}(s_k) = \tilde{\psi}(s_t) \end{cases} \quad (1)$$

Let a lower transformation function be $l(x) = x^p$ and an upper transformation function be $u(x) = 2x - x^p$, where $1 \leq p \leq 2$, $0 \leq x = \sigma_j(s) \leq 1$, and $0 \leq l(x) \leq u(x) \leq 1$. The value of p depends on a DM's level of confidence about his judgment with respect to the truth degree of a preference statement at a state.

Definition 6. (Fuzzy Truth Value Interval): Define $\sigma_j^L(s) = l(\sigma_j(s))$, and $\sigma_j^U(s) = u(\sigma_j(s))$; then, $[\sigma_j^L(s), \sigma_j^U(s)] \subset [0, 1]$ is called the DM's fuzzy truth value interval of Ω_j at state s .

Definition 7. (Fuzzy Score Interval): For $\sigma_j(s) = [\sigma_j^L(s), \sigma_j^U(s)]$, choose a real number $\alpha > 0$ satisfying $\alpha > \alpha^2 + \alpha^2 + \dots + \alpha^q$, such as $0 < \alpha \leq \frac{1}{2}$. Let $\tilde{\psi}_j^L(s) = \alpha^j \sigma_j^L(s)$ and $\tilde{\psi}_j^U(s) = \alpha^j \sigma_j^U(s)$; then, $\tilde{\psi}_j(s) = [\tilde{\psi}_j^L(s), \tilde{\psi}_j^U(s)]$ is called the DM's incremental fuzzy score interval of s for Ω_j . Let $\tilde{\psi}^L(s) = \sum_{j=1}^q \tilde{\psi}_j^L(s)$ and $\tilde{\psi}^U(s) = \sum_{j=1}^q \tilde{\psi}_j^U(s)$; then, $\tilde{\psi}(s) = [\tilde{\psi}^L(s), \tilde{\psi}^U(s)]$ is called the DM's fuzzy score interval of state s .

Notice that the value of α depends on how much truth for a preference statement at a particular state exactly balances truth-value 1 of the next less important preference statement at that state.

When the truth-values are either 0 or 1, the preference is crisp.

3. Application to a water resources conflict

Built on the Yongding River, the Guanting reservoir represents an important source of water for populations downstream, including a quarter of the inhabitants of Beijing. The water quantity and quality downstream has been seriously affected by rapid economic development in the upstream regions, and in 1997 the reservoir lost its function providing drinking water. For various reasons including a lack of clear water resources property rights, the "polluter pays" principle could not be implemented effectively in this situation. Groups upstream were not willing to reduce pollutant emissions without substantial compensation, while groups downstream intended to let the polluters upstream pay for pollution treatment alone. In the absence of external coordination, a cross-border water conflict occurred between upstream and downstream stakeholders.

3.1. DMs and options

In this conflict, the central government's objective is to take measures to promote cooperation between

Table 7
Crisp preference information for DM 2 and DM 3

DMs	Preference rankings
DM 2	$s_{13} \succ s_{11} \succ s_7 \succ s_3 \succ s_{12} \succ s_8 \succ s_4 \succ s_{10} \succ s_6 \succ s_2 \succ s_9 \succ s_5 \succ s_1$
DM 3	$s_{13} \succ s_{11} \succ s_7 \succ s_3 \succ s_{12} \succ s_8 \succ s_4 \succ s_9 \succ s_5 \succ s_1 \succ s_{10} \succ s_6 \succ s_2$

Table 8
Fuzzy stability analysis results

Stability	$\gamma_1 = 0.25, \gamma_2 = \gamma_3 = 1.0$	$\gamma_1 = 0.55, \gamma_2 = \gamma_3 = 1.0$
FNash	s_{10}, s_{11}, s_{13}	s_{11}, s_{13}
FGMR	$s_3, s_4, s_7, s_8, s_{10}, s_{11}, s_{13}$	$s_3, s_4, s_7, s_8, s_{11}, s_{12}, s_{13}$
FSMR	$s_3, s_4, s_7, s_8, s_{10}, s_{11}, s_{13}$	$s_3, s_4, s_7, s_8, s_{11}, s_{12}, s_{13}$
FSEQ	s_{10}, s_{11}, s_{13}	s_{11}, s_{13}

3.4. Fuzzy stability analysis

DMs' FSTs usually play a vital role in fuzzy stability analysis. This paper considers two different FSTs for DM 1: (i) $\gamma_1 = 0.25$ and (ii) $\gamma_1 = 0.55$. Since both DM 2 and DM 3 hold crisp preferences, their FSTs are the same ($\gamma_2 = \gamma_3 = 1.0$). After setting each DM's FST and employing the four fuzzy stability concepts presented in Table 1, the fuzzy stability analysis results of the water pollution conflict can be determined as shown in Table 8.

Table 8 shows that when weaker satisficing criteria for DM 1 ($\gamma_1 = 0.25$) are considered, states s_{10}, s_{11} , and s_{13} are FNash stable states, while states s_3, s_4, s_7 , and s_8 satisfy FGMR and FSMR stability. Under these conditions, the conflict will have more opportunities to stay at state s_{10} , where it cannot be solved effectively. However, under stronger satisficing criteria for DM 1 ($\gamma_1 = 0.55$), state s_{10} disappears from the stable state list, state s_{12} joins the FGMR and FSMR stable state list, and state s_{11} has a greater chance to become an equilibrium for the conflict, which is favorable for all DMs. These results illustrate that the fuzziness of DM 1's preferences is strong and will influence the conflict's development and solutions.

Note that the distinguishable state s_{13} is a stable state that did not occur in reality. In reality, the final outcome of the conflict was state s_{11} , where under the pressure of DM 3's both Encourage and Punish options, DM 1 reduced pollution emissions and DM 2 did not take option Sanction. The above analysis is consistent with the actual trajectory of the conflict, a result that demonstrates the feasibility and applicability of this conflict model.

4. Conclusions

This paper introduces and applies a conflict analysis framework under fuzzy preferences in order to model and analyze a water pollution conflict between the upstream and the downstream areas of the Guanting reservoir. Moreover, this paper introduces the option prioritization methodology that can represent both crisp and fuzzy preferences, utilizing it to calculate DMs' preferences in the water contamination conflict. When applied by practitioners and researchers, the GMCR framework under fuzzy preferences provides the important function of predicting different equilibriums for a conflict by considering DMs' different satisficing behaviors.

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