Interval valued neutrosophic sets and multi-attribute decision-making based on generalized weighted aggregation operator

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Abstract. Neutrosophic sets are powerful logics designed to facilitate understanding of indeterminate and inconsistent information; many types of incomplete or complete information can be expressed as interval valued neutrosophic sets (IVNSs). This paper proposes improved aggregation operation rules for IVNSs, and extends the generalized weighted aggregation (GWA) operator to work congruently with IVNS data. The aggregated results are also expressed as IVNSs, which are characterized by truth-membership degree, indeterminacy-membership degree, and falsity-membership degree. The proposed method is proved to be the maximum approximation to the original data, and maintains most of the information during data processing. Then, a method is proposed to determine the ranking orders for all alternatives according to their comparative advantage matrices based on their general score, degree of accuracy and degree of certainty expressed by the aggregated IVNSs. Finally, a numerical example is provided to illustrate the applicability and efficiency of the proposed approach.

Keywords: Interval valued neutrosophic sets, multi-attribute decision making, generalized weighted aggregation, fuzzy sets

1. Introduction

Due to the complexity and uncertainty of objective things, and the ambiguity of human thinking, Zadeh proposed a remarkable theory of fuzzy sets (FSs) in 1965 [13]. Since then, the FS theory has been successfully applied to various fields of multi-attribute decision-making. Moreover, extended FSs were developed, such as the interval valued fuzzy sets (IVFSs) [5], intuitionistic fuzzy sets (IFSSs) [12], interval valued intuitionistic fuzzy sets (IVIFSs) [11], hesitant fuzzy sets (HFSs) [18], etc. Although FS theory has been developed and generalized, it cannot account for all possible uncertainties in a variety of physical problems. For instance, when an expert is asked a question, he or she may think that the possibility of a true answer is equal to 0.6, the possibility of a false answer is 0.4, and the degree of their uncertainty is 0.2. This issue is beyond the scope of FSs and IFSSs. Hence, Smarandache proposed neutrosophic logic and neutrosophic sets (NSs) in 1999 [1]. A NS is a set in which each element of the universe has respective degrees of truth, indeterminacy, and falsity, which lie in the nonstandard unit interval of $\left[0, 1\right]$. This method represents an extension of the standard interval $[0,1]$ used for IFSSs. The uncertainty presented here, (i.e., the indeterminacy factor) is independent of the truth and falsity values. This extended IFS theory to account for uncertain information.

Many scholars have begun to study the practical application of NSs in multi-attribute decision-making. Wang, et al. [2] and Ye [9] defined the aggregation operators for single valued neutrosophic set (SNSs),

All of the aforementioned definitions of aggregation operators observe certain rationalities, but some are inconsistent with the reality logic. Due to the realistic differences among the characteristics of truth-membership degree, indeterminacy-membership degree, and falsity-membership degree, operation rules must not only consider the basic operation rules, but also consider the realistic significance of operators. In many multi-attribute decision-making applications, the weight of each attribute is determined according to its importance. The generalized weighted aggregation operator is utilized to obtain the aggregated value for all attributes of each alternative, and rank the limited alternatives or select the best by comparing the advantage degrees of the aggregated IVNSs representing each alternative. This paper proposes improved operation rules for IVNSs, and defines the generalized weighted aggregation operator for IVNSs based on a traditional GWA operator. Due to the maximum approximation to original values, the aggregated value based on the GWA operator can most effectively reflect the multi-attribute values while maintaining the greatest amount of information. Compared to the multi-attribute decision-making method based on distance, this method maintains the integrity of information during the calculation process. Meanwhile, by choosing different parameters, decision makers can obtain ranking order results according to their own risk attitudes.

The rest of this paper is organized as follows. In Section 2, some concepts and operations of IVNSs are briefly introduced. Based on the basic operation rules, the GWA operator is extended to IVNSs. In Section 3, a decision-making method is developed for IVNSs based on a GWA operator. In Section 4, an example is presented to illustrate the proposed method, and comparison analysis and discussion are provided. Finally, Section 5 concludes the paper.

2. IVNS concepts and operators

Definition 1. [8] Let X be a space of points (objects) with generic elements in X. An interval valued neutrosophic set (IVNS) A in X is characterized by truth-membership function T_A(x), indeterminacy-membership function I_A(x) and falsity-membership function F_A(x). For each point x in X, T_A(x), I_A(x), F_A(x) ε [0,1].

A = \{x, [T_A(x), I_A(x), F_A(x)]\},

\{F_A(x), T_A(x)\} > x ε X

An IVNS is also defined as A = ([T_A, I_A, F_A]), ([T'_A, I'_A, F'_A]).

Definition 2. The complement of an interval valued neutrosophic set A = ([T_A, I_A, F_A], [T'_A, I'_A, F'_A]) is defined by

A' = ([T_A, I_A, F_A], [T'_A, I'_A, F'_A])

(2)

The maximum of an IVNS is <[1,1],[0,0],[0,0]>, and the minimum is <[0,0],[0,0],[1,1]>.

This definition varies from the definition of IVNSs presented in previous literature; when a cost IVNS is transformed into a benefit IVNS, the truth-membership and falsity-membership are altered, while the indeterminacy-membership is unchanged.

Definition 3. [19] Definition of advantage comparison for interval values. Suppose that there are n interval numbers \(b_i = [a_i', a_i''] (i = 1, 2, \ldots, n)\) and that the comparison of each interval number \(\tilde{a}_i\) to all interval numbers \(\tilde{a}_j\) (j = 1, 2, \ldots, n) is formulated by

\[ p_{ij}(\tilde{a}_i \geq \tilde{a}_j) = \max \left\{ 1 - \left[ \frac{\tilde{a}_j' - \tilde{a}_i'}{\tilde{a}_j'' - \tilde{a}_i''} \right] 0, 0 \right\} \]

Then a complementary matrix can be constructed as follows:
\[ P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mn} \end{bmatrix} \]

where \( p_{ij} \geq 0, \ p_{ji} + p_{ij} = 1, \ p_{ii} = 0.5 \).

**Definition 4.** Based on the score function and accuracy function of IVIES, the score function, accuracy function and certainty function of an IVNS \( A = (T_A, I_A, F_A) \) are defined as follows.

1. \( s(A) = T_A + 1 - F_A + 1 - T_A \)
2. \( a(A) = \min \{T_A - F_A, T_A - F_A\} \)
3. \( c(A) = \max \{T_A - F_A, T_A - F_A\} \)

**Definition 5.** \( \lambda \) Let \( A \) and \( B \) be two IVNSs. The comparison approach can be defined as follows.

If \( p(s(A) \geq s(B)) > 0.5 \), then \( A \) is superior to \( B \), denoted by \( A > B \).

If \( p(s(A) > s(B)) = 0.5 \), and \( p(a(A) \geq a(B)) > 0.5 \), then \( A \) is superior to \( B \), denoted by \( A > B \).

If \( p(s(A) \geq s(B)) = 0.5 \), and \( p(a(A) > a(B)) > 0.5 \), then \( A \) is indifferent to \( B \), denoted by \( A \sim B \).

**Theorem 1.** Let \( A, B, C \) be three IVNSs, \( \lambda, \lambda_1, \lambda_2 > 0 \).

The following equations are true.

1. \( A + B = B + A \)
2. \( A - B = B - A \)
3. \( \lambda(A + B) = \lambda A + \lambda B \)
4. \( (A - B)^\lambda = A^\lambda - B^\lambda \)
5. \( \lambda_1 A + \lambda_2 A = (\lambda_1 + \lambda_2) A \)
6. \( A^{\lambda_1} - A^{\lambda_2} = A^{(\lambda_1 + \lambda_2)} \)
7. \( (A + B) + C = A + (B + C) \)
8. \( (A - B) - C = A - (B - C) \)

**Proof 1.** According to definition 6, (1) and (2) are obvious. The others are proven as follows.

**Proof 2. Formula (3):**

\[ \lambda(A + B) = \lambda < [T_A + T_B, T_A + T_B, T_A + T_B, T_A + T_B], \\
\quad [I_A + I_B, I_A + I_B, I_A + I_B, I_A + I_B] > \]

\[ = \lambda < [1 - (1 - T_A), 1 - (1 - T_A), 1 - (1 - T_A), 1 - (1 - T_A)], \\
\quad \lambda \cdot [1 - (1 - T_B), 1 - (1 - T_B), 1 - (1 - T_B), 1 - (1 - T_B)] > \]

\[ = \lambda < [1 - (1 - T_A)^\lambda, 1 - (1 - T_A)^\lambda, 1 - (1 - T_A)^\lambda, 1 - (1 - T_A)^\lambda], \\
\quad \lambda \cdot [1 - (1 - T_B)^\lambda, 1 - (1 - T_B)^\lambda, 1 - (1 - T_B)^\lambda, 1 - (1 - T_B)^\lambda] > \]

\[ = \lambda < [1 - (1 - T_A)^\lambda, 1 - (1 - T_A)^\lambda, 1 - (1 - T_A)^\lambda, 1 - (1 - T_A)^\lambda], \\
\quad \lambda \cdot [1 - (1 - T_B)^\lambda, 1 - (1 - T_B)^\lambda, 1 - (1 - T_B)^\lambda, 1 - (1 - T_B)^\lambda] > \]

\[ = \lambda < [1 - (1 - T_A)^\lambda, 1 - (1 - T_A)^\lambda, 1 - (1 - T_A)^\lambda, 1 - (1 - T_A)^\lambda], \\
\quad \lambda \cdot [1 - (1 - T_B)^\lambda, 1 - (1 - T_B)^\lambda, 1 - (1 - T_B)^\lambda, 1 - (1 - T_B)^\lambda] > \]
Proof 3. Formula (4):

\[
(A \cdot B)^\triangledown = <\left(\Delta_4 A, \Delta_4 B\right), \left(\Delta_5 A, \Delta_5 B\right), \left(\Delta_6 A, \Delta_6 B\right), \left(\Delta_7 A, \Delta_7 B\right), \left(\Delta_8 A, \Delta_8 B\right)>^\triangledown
\]

Proof 4. Formula (5):

\[
\lambda_1 A + \lambda_2 B
\]

\[
\lambda_1 A + \lambda_2 B
\]

\[
\lambda_1 A + \lambda_2 B
\]
Proof 5. Formula (6):

\[
\begin{align*}
A^{(i)} \cdot A^{(j)} &= \left(\left(\left[\left(T_{A}^{(i)}\right)^{T_{A}^{(i)}}, \left[1 - (1 - T_{A}^{(i)}), 1 - (1 - T_{A}^{(i)})\right]\right], \left[1 - (1 - T_{A}^{(i)}), 1 - (1 - T_{A}^{(i)})\right]\right)\right) \cdot \left(\left[\left(T_{B}^{(j)}\right)^{T_{B}^{(j)}}, \left[1 - (1 - T_{B}^{(j)}), 1 - (1 - T_{B}^{(j)})\right]\right], \left[1 - (1 - T_{B}^{(j)}), 1 - (1 - T_{B}^{(j)})\right]\right) \\
&= \left(\left(\left[\left(T_{A}^{(i)}\right)^{T_{A}^{(i)}}, \left[1 - (1 - T_{A}^{(i)}), 1 - (1 - T_{A}^{(i)})\right]\right], \left[1 - (1 - T_{A}^{(i)}), 1 - (1 - T_{A}^{(i)})\right]\right)\right) \cdot \left(\left[\left(T_{A}^{(i + 1)}\right)^{T_{A}^{(i + 1)}}, \left[1 - (1 - T_{A}^{(i + 1)}), 1 - (1 - T_{A}^{(i + 1)})\right]\right], \left[1 - (1 - T_{A}^{(i + 1)}), 1 - (1 - T_{A}^{(i + 1)})\right]\right) \\
&\cdot \left(\left[\left(T_{B}^{(j)}\right)^{T_{B}^{(j)}}, \left[1 - (1 - T_{B}^{(j)}), 1 - (1 - T_{B}^{(j)})\right]\right], \left[1 - (1 - T_{B}^{(j)}), 1 - (1 - T_{B}^{(j)})\right]\right) \\
&= A^{(i + 1)} \cdot A^{(j)} \\
&= A^{(i + j)}
\end{align*}
\]

Proof 6. Formula (7):

\[
(A + B) + C = \left(\left(\left[\left(T_{A}^{(i)} + T_{A}^{(i)}\right)^{T_{A}^{(i)}}, \left[1 - (1 - T_{A}^{(i)}), 1 - (1 - T_{A}^{(i)})\right]\right], \left[1 - (1 - T_{A}^{(i)}), 1 - (1 - T_{A}^{(i)})\right]\right)\right) \cdot \left(\left[\left(T_{A}^{(i)} + T_{A}^{(i)}\right)^{T_{A}^{(i)}}, \left[1 - (1 - T_{A}^{(i)}), 1 - (1 - T_{A}^{(i)})\right]\right], \left[1 - (1 - T_{A}^{(i)}), 1 - (1 - T_{A}^{(i)})\right]\right) \\
\cdot \left(\left[\left(T_{A}^{(i)} + T_{A}^{(i)}\right)^{T_{A}^{(i)}}, \left[1 - (1 - T_{A}^{(i)}), 1 - (1 - T_{A}^{(i)})\right]\right], \left[1 - (1 - T_{A}^{(i)}), 1 - (1 - T_{A}^{(i)})\right]\right) \\
= A + \left(\left(\left[\left(T_{A}^{(i)} + T_{A}^{(i)}\right)^{T_{A}^{(i)}}, \left[1 - (1 - T_{A}^{(i)}), 1 - (1 - T_{A}^{(i)})\right]\right], \left[1 - (1 - T_{A}^{(i)}), 1 - (1 - T_{A}^{(i)})\right]\right)\right) \\
= A + (B + C)
\]
\[ \text{Definition 7.} \]

Let \( A_j = \{ F_{1,j}, F_{2,j}, \ldots, F_{n,j} \} \) be a collection of IVNSs, and \( W = (w_1, w_2, \ldots, w_n) \) be the weight vector of \( A_j \), with \( w_j \geq 0 \) and \( \sum_{j=1}^{n} w_j = 1 \). Their generalized weighted aggregation GWA [16] \( A^* \rightarrow A \) is defined as follows.

\[
Z = \text{IVNSGWA}(A_1, A_2, \ldots, A_k) = \left( \sum_{j=1}^{n} w_j A^*_j \right)^{1/2}
\]

\[
= \left( \sum_{j=1}^{n} w_j \left( \left[ \left( T_{1,j} T_{2,j} T_{3,j} \right) \cdot \left[ F_{1,j} + F_{2,j} + F_{3,j} - F_{4,j} \right] \cdot \left[ F_{1,j} + F_{2,j} + F_{3,j} - F_{4,j} \right] \cdot \left[ F_{1,j} + F_{2,j} + F_{3,j} - F_{4,j} \right] \right) \right)^{1/2}\right.
\]

\[
= \left( \sum_{j=1}^{n} w_j \left[ \left[ 1 - (1 - F_{1,j}) (1 - F_{2,j}) \cdot (1 - F_{1,j}) (1 - F_{2,j}) \right] \cdot \left[ 1 - (1 - F_{1,j}) (1 - F_{2,j}) \cdot (1 - F_{1,j}) (1 - F_{2,j}) \right] \cdot \left[ 1 - (1 - F_{1,j}) (1 - F_{2,j}) \cdot (1 - F_{1,j}) (1 - F_{2,j}) \right] \cdot \left[ 1 - (1 - F_{1,j}) (1 - F_{2,j}) \cdot (1 - F_{1,j}) (1 - F_{2,j}) \right] \right)^{1/2}\right.
\]

\[
= \left( \left[ \prod_{j=1}^{n} (1 - (1 - F_{1,j}) (1 - F_{2,j}) \cdot (1 - F_{1,j}) (1 - F_{2,j}) \cdot (1 - F_{1,j}) (1 - F_{2,j}) \cdot (1 - F_{1,j}) (1 - F_{2,j})\right)^{1/2}\right)
\]

\[
= \left( \left[ \prod_{j=1}^{n} (1 - (1 - F_{1,j}) (1 - F_{2,j}) \cdot (1 - F_{1,j}) (1 - F_{2,j}) \cdot (1 - F_{1,j}) (1 - F_{2,j}) \cdot (1 - F_{1,j}) (1 - F_{2,j})\right)^{1/2}\right)
\]

\[
= \left( \left[ \prod_{j=1}^{n} (1 - (1 - F_{1,j}) (1 - F_{2,j}) \cdot (1 - F_{1,j}) (1 - F_{2,j}) \cdot (1 - F_{1,j}) (1 - F_{2,j}) \cdot (1 - F_{1,j}) (1 - F_{2,j})\right)^{1/2}\right)
\]
When $\lambda \to 0$

$$Z = \text{IVNSGW} \left( A_1, \ldots, A_n \right) = \prod_{j=1}^{n} \lambda_{A_j}^2$$

$$= \prod_{j=1}^{n} \left( \left[ (T_j^B)^{\nu_j}, (F_j^B)^{\nu_j} \right], \left[ 1 - (1 - T_j^B)^{\nu_j}, 1 - (1 - F_j^B)^{\nu_j} \right], \left[ 1 - (1 - T_j^F)^{\nu_j}, 1 - (1 - F_j^F)^{\nu_j} \right] \right)$$

$$= \left( \left[ \prod_{j=1}^{n} (T_j^B)^{\nu_j}, \prod_{j=1}^{n} (F_j^B)^{\nu_j} \right], \left[ \prod_{j=1}^{n} (1 - T_j^B)^{\nu_j}, 1 - \prod_{j=1}^{n} (1 - T_j^F)^{\nu_j} \right], \left[ 1 - \prod_{j=1}^{n} (1 - F_j^F)^{\nu_j}, 1 - \prod_{j=1}^{n} (1 - F_j^F)^{\nu_j} \right] \right)$$

When $\lambda = 1$

$$Z = \text{IVNSGW} \left( A_1, \ldots, A_n \right) = \sum_{j=1}^{n} w_j A_j$$

$$= \sum_{j=1}^{n} \left( \left[ (T_j^B)^{\nu_j}, (F_j^B)^{\nu_j} \right], \left[ (T_j^F)^{\nu_j}, (F_j^F)^{\nu_j} \right] \right)$$

$$= \left( \left[ \prod_{j=1}^{n} (T_j^B)^{\nu_j}, \prod_{j=1}^{n} (F_j^B)^{\nu_j} \right], \left[ \prod_{j=1}^{n} (1 - T_j^B)^{\nu_j}, 1 - \prod_{j=1}^{n} (1 - T_j^F)^{\nu_j} \right], \left[ \prod_{j=1}^{n} (F_j^F)^{\nu_j}, \prod_{j=1}^{n} (F_j^F)^{\nu_j} \right] \right)$$

When $\lambda = 1$, Formula (19) is reduced to Formula (21).

3. Multi-attribute decision-making method based on IVNSGW operators

Definition 7 indicates that:

(1) When $\lambda \to 0$, IVNSGW$(A_1, A_2, \ldots, A_n) = \prod_{j=1}^{n} \lambda_{A_j}^2$, the standard aggregation operator IVNSGW is reduced to weighted geometric aggregation operator (WG).

(2) When $\lambda = 1$, IVNSGW$(A_1, A_2, \ldots, A_n) = \sum_{j=1}^{n} w_j A_j$, the standard aggregation operator IVNSGW is reduced to weighted arithmetic aggregation operator (WA).

(3) Let $Z = \text{IVNSGW} \left( A_1, A_2, \ldots, A_n \right)$. When $Z = \left( \sum_{j=1}^{n} w_j A_j \right)^{\frac{1}{n}}$, the value of $Z = A_j$ is at its minimum. Therefore, the aggregation operator IVNSGW is the greatest approximation representing $n$ attributes.

A multi-attribute decision-making problem represents a process by which to find the best alternative from all of feasible alternatives, in which all alternatives can be evaluated according to a number of attributes or criteria. Let $A = \{A_1, A_2, \ldots, A_n\}$ be a set of attributes and $C = \{C_1, C_2, \ldots, C_n\}$ be a set of alternatives. Assume that the vector of the weight $w = (w_1, w_2, \ldots, w_n)$, where $w_j \in [0, 1]$, and $\sum_{j=1}^{n} w_j = 1$. In this case, the characteristic of attribute $C_j$ of alternative $A_i$ is represented by an IVNSGW $\rho_{ij}$, $[T_{ij}, T_{ij}^*, F_{ij}, F_{ij}^*]$. Here, $[T_{ij}, T_{ij}^*, F_{ij}, F_{ij}^*]$ respectively indicate the degree to which alternative $A_i$ satisfies the attribute $C_j$, the indeterminacy degree to which the alternative $A_i$ satisfies or does not satisfy the attribute $C_j$, and the degree to which the alternative $A_i$ does not satisfy the attribute $C_j$. Therefore, an interval valued neutrosophic decision matrix can be elicited: $R = [r_{ij}]_{n \times n}$.

The steps of interval valued neutrosophic multi-attribute decision-making based on IVNSGW can be presented as follows.

**Step 1.** Construction of standard decision matrix.

If $C_j$ is a benefit attribute, $b_j = (T_{ij}^*, T_{ij}^*, F_{ij}^*, F_{ij}^*)$.

If $C_j$ is a cost attribute, $b_j = (T_{ij}^*, T_{ij}^*, F_{ij}^*, F_{ij}^*)$.

Thus, a standard decision matrix is obtained, $B = [b_{ij}]_{n \times n}$.

**Step 2.** Calculation of the aggregation value $Z_i$ for alternative $a_i$. See Definition 8.

**Step 3.** Construction of dominance matrices.

Based on Definition 4, calculate score $s(Z_i)$, degree of accuracy $a(Z_i)$ and degree of certainty $c(Z_i)$ for alternative $a_i$. Then, according to Definition 3, construct the
score dominance matrix $P_r$, accuracy dominance matrix $P_a$ and certainty dominance matrix $P_c$ as follows.

$$
\begin{pmatrix}
\frac{ps_1}{\sum ps_{ij}}, & \frac{ps_2}{\sum ps_{ij}}, & \cdots, & \frac{ps_n}{\sum ps_{ij}} \\
\frac{pa_1}{\sum pa_{ij}}, & \frac{pa_2}{\sum pa_{ij}}, & \cdots, & \frac{pa_n}{\sum pa_{ij}} \\
\frac{pc_1}{\sum pc_{ij}}, & \frac{pc_2}{\sum pc_{ij}}, & \cdots, & \frac{pc_n}{\sum pc_{ij}}
\end{pmatrix}
$$

Step 4. Calculation of the total score $p_{si}$, total accuracy $p_{ai}$ and total certainty $p_{ci}$ for alternative $a_i$.

$$
p_{si} = \sum_{j=1}^{m} p_{sij}, \quad p_{ai} = \sum_{j=1}^{m} p_{a_{ij}}, \quad p_{ci} = \sum_{j=1}^{m} p_{c_{ij}}
$$

Step 5. According to Definition 5, determine the ranking order for each alternative.

4. Illustrative example

In this section, an example of the multi-attribute decision-making problem of alternatives is used to demonstrate the application and effectiveness of the proposed decision-making method.

Let us consider the decision-making problem adapted from Chi and Liu, in which an investment company wants to invest in the best option [15].

There are four possible alternatives in which to invest, expressed as $\{A_1, A_2, A_3, A_4\}$, where $A_1$ is a bookshop, $A_2$ is a chemical plant, $A_3$ is a supermarket, and $A_4$ is a food company.

The investment company must make a decision according to the following three attributes: $C_1$ is the earning estimate analysis, $C_2$ is the growth analysis, and $C_3$ is the environmental impact analysis for the alternatives; $C_1$ and $C_2$ are benefit attributes, while $C_3$ is a cost attribute. The weight vector of the attribute is given by $w = (0.2, 0.25, 0.55)$. The four possible alternatives are evaluated according to the above three attributes by IVNSs, as shown in the following interval valued neutrosophic decision matrix $R$.

$R = \begin{pmatrix}
\langle [0.4, 0.5], [0.2, 0.3], [0.3, 0.4] \rangle \\
\langle [0.6, 0.7], [0.1, 0.2], [0.2, 0.3] \rangle \\
\langle [0.3, 0.6], [0.2, 0.3], [0.3, 0.4] \rangle \\
\langle [0.7, 0.8], [0.0, 0.1], [0.1, 0.2] \rangle \\
\langle [0.4, 0.6], [0.1, 0.3], [0.2, 0.4] \rangle \\
\langle [0.6, 0.7], [0.1, 0.2], [0.2, 0.3] \rangle \\
\langle [0.5, 0.6], [0.2, 0.3], [0.3, 0.4] \rangle \\
\langle [0.6, 0.7], [0.1, 0.2], [0.1, 0.3] \rangle \\
\langle [0.4, 0.5], [0.2, 0.3], [0.7, 0.9] \rangle \\
\langle [0.8, 0.9], [0.3, 0.5], [0.3, 0.6] \rangle \\
\langle [0.7, 0.9], [0.2, 0.4], [0.4, 0.5] \rangle \\
\langle [0.8, 0.9], [0.3, 0.4], [0.6, 0.7] \rangle
\end{pmatrix}$

Each IVNS value in the matrix represents the assessment of the corresponding attribute of a certain alternative. For instance, the first value $\langle [0.4, 0.5], [0.2, 0.3], [0.3, 0.4] \rangle$ in matrix $R$ represents the possibility of earning of alternative $A_1$, which lies between 0.4 and 0.5; the uncertainty of earning of alternative $A_1$, which lies between 0.2 and 0.3; and the possibility of loss of alternative $A_1$, which lies between 0.3 and 0.4.

(1) Construction of standard decision matrix $B$.

$$
B = \begin{pmatrix}
\langle [0.4, 0.5], [0.2, 0.3], [0.3, 0.4] \rangle \\
\langle [0.6, 0.7], [0.1, 0.2], [0.2, 0.3] \rangle \\
\langle [0.3, 0.6], [0.2, 0.3], [0.3, 0.4] \rangle \\
\langle [0.7, 0.8], [0.0, 0.1], [0.1, 0.2] \rangle \\
\langle [0.4, 0.6], [0.1, 0.3], [0.2, 0.4] \rangle \\
\langle [0.6, 0.7], [0.1, 0.2], [0.2, 0.3] \rangle \\
\langle [0.5, 0.6], [0.2, 0.3], [0.3, 0.4] \rangle \\
\langle [0.6, 0.7], [0.1, 0.2], [0.1, 0.3] \rangle \\
\langle [0.4, 0.5], [0.2, 0.3], [0.7, 0.9] \rangle \\
\langle [0.8, 0.9], [0.3, 0.5], [0.3, 0.6] \rangle \\
\langle [0.7, 0.9], [0.2, 0.4], [0.4, 0.5] \rangle \\
\langle [0.8, 0.9], [0.3, 0.4], [0.6, 0.7] \rangle
\end{pmatrix}
$$

Since $C_3$ is a cost attribute, it is transformed into a benefit attribute according to Definition 2.

(2) Calculation of aggregation value $Z_i$ for alternative $a_i$.

According to formula (22), let $\lambda = 1$. Thus:
Finance matrices are obtained. Ranking order is expressed as follows. According to Definition 3, the respective dominance matrices were calculated. The ranking orders are listed in Table 1. As indicated by Table 1, the total score of each alternative varies for different values of parameter $\lambda$, while the ranking order remains unchanged. In fact, $\lambda$ generally represents the attitude of decision makers; smaller $\lambda$ indicates more conservative attitudes, while larger $\lambda$ indicates more optimistic attitudes.

The ranking order results are consistent with results reported in previous literature [8, 15], and the results are not impacted by different values of $\lambda$. Therefore, the method proposed in this paper is applicable and effective. However, the results are not consistent with results from other literature [4, 6] even with appropriately-adjusted weight vectors. Analysis indicates that Zhang, et al., did not standardize cost attribute data. Although Wang and Li utilized standardized cost attribute data, the definition of complement operator is different from the definition utilized in this paper [6].

Next, according to Definition 3, the respective dominance matrices are obtained.

$$S_{A,C}(Z) = \begin{bmatrix} 1.6101,2.1245,0.0679,0.1033 & 0.5000 & 0.5577 \\ 0.0542,2.4191 & 0.4153 & 0.4423 & 0.6636 & 0.7720 \\ 1.7840,2.1948,0.2069,0.2942 & 0.5293 & 0.7172 \\ 2.1335,2.6333,0.4479,0.5259 & 0.6824 & 0.7898 \end{bmatrix}$$

(5) Ranking the alternatives. The scores of the three alternatives vary, so the ranking order is $A_4 > A_2 > A_3 > A_1$ according to $p_r$, which is identical to the ranking order used in the [15] TOPSIS method.

In order to discuss the impact of parameter $\lambda$, the ranking order under different $\lambda$ were calculated. The ranking orders are listed in Table 1.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>Total score $p_{\lambda}$</th>
<th>Ranking order</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rightarrow 0$</td>
<td>$[0.8069,2.7001,1.3887,2.8504]$</td>
<td>$A_4 &gt; A_2 &gt; A_3 &gt; A_1$</td>
</tr>
<tr>
<td>0.1</td>
<td>$[0.9307,2.5411,1.4095,3.0223]$</td>
<td>$A_4 &gt; A_2 &gt; A_3 &gt; A_1$</td>
</tr>
<tr>
<td>0.3</td>
<td>$[0.9347,2.5791,1.3959,3.0680]$</td>
<td>$A_4 &gt; A_2 &gt; A_3 &gt; A_1$</td>
</tr>
<tr>
<td>0.5</td>
<td>$[0.9385,2.5276,1.3934,3.0913]$</td>
<td>$A_4 &gt; A_2 &gt; A_3 &gt; A_1$</td>
</tr>
<tr>
<td>1.0</td>
<td>$[0.9467,2.5721,1.3793,3.1019]$</td>
<td>$A_4 &gt; A_2 &gt; A_3 &gt; A_1$</td>
</tr>
<tr>
<td>1.4</td>
<td>$[0.9516,2.5661,1.3643,3.1190]$</td>
<td>$A_4 &gt; A_2 &gt; A_3 &gt; A_1$</td>
</tr>
<tr>
<td>1.8</td>
<td>$[0.9548,2.5665,1.3613,3.1175]$</td>
<td>$A_4 &gt; A_2 &gt; A_3 &gt; A_1$</td>
</tr>
<tr>
<td>2.0</td>
<td>$[0.9560,2.5655,1.3575,3.1210]$</td>
<td>$A_4 &gt; A_2 &gt; A_3 &gt; A_1$</td>
</tr>
<tr>
<td>5.0</td>
<td>$[0.8425,2.5644,1.3623,3.1569]$</td>
<td>$A_4 &gt; A_2 &gt; A_3 &gt; A_1$</td>
</tr>
<tr>
<td>10</td>
<td>$[0.8402,2.5815,1.3463,3.1837]$</td>
<td>$A_4 &gt; A_2 &gt; A_3 &gt; A_1$</td>
</tr>
</tbody>
</table>

5. Conclusion

This paper proposed improved aggregation operation rules for interval valued neutrosophic sets, and extended the generalized aggregation method to IVNSs based on the defined operators. Additionally, the appli-
cation of the IVNS decision-making model based on IVNSGW A operator was detailed. The application of the IVNS multi-attribute decision-making method can help people make a correct decision out of available alternatives in indeterminate and inconsistent information environments. Compared to the multi-attribute decision-making method based on distance, this method maintains the integrity of information during calculation. Additionally, by choosing different parameters, decision makers can obtain ranking order results according to their own risk attitudes.

This paper only considered the attribute decision-making problems with IVNS data. Considering that other data types can be expressed as IVNSs, this paper can also be easily extended to account for problems which deal with multiple data types.

Acknowledgments

The authors are indebted to anonymous reviewers for their very insightful comments and constructive suggestions, which help ameliorate the quality of this paper. This work supported by the National Research Foundation of Korea Grant funded by the Korean Government (NRF-2014S1A2A2027622), the National Natural Science Foundation of China (17112076, 71171099 and 71201071), the Joint Research of The NSFC-NRF Scientific Cooperation Program (71411140250), the Fund of Ministry of education of Humanities and Social Sciences (14YJAZH025), the Fund of China Nation Tourism Administration (15TACK003), and the Natural Science Foundation of Shandong Province (ZR2013GM003).

References