Research on the assessment of psycholinguistic teaching effect with triangular fuzzy information

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Abstract. The effect and quality of education is the fundamental of higher education, and instructional reform is the core one of all reforms in higher education. In order to motivate the instructional reform and enhance the quality of education, the department of education in China has evaluated and rewarded the study performance in instructional practice of educators in higher learning instructions since 1989. This means has pushed enormously the process of the instructional reform in higher learning institutions and enhanced the effect and quality of higher education. However, evaluating and rewarding the excellent study performance of educators in higher learning institutions still depends on the qualitative evaluation by the expert’s intelligence and experience. In this paper, we investigate the multiple attribute decision making problems for evaluating the psycholinguistic teaching effect with triangular fuzzy information. Motivated by the idea of geometric Bonferroni mean and Einstein operations, we develop the triangular fuzzy Einstein geometric Bonferroni mean (TFEGBM) operator and triangular fuzzy weighted Einstein geometric Bonferroni mean (TFWEGBM) operator for aggregating the triangular fuzzy information. Then, based on the TFWEGBM operator, we develop the procedure for multiple attribute decision making with the triangular fuzzy information. Finally, a practical example for evaluating the psycholinguistic teaching effect is given to verify the developed approach.

Keywords: Multiple attribute decision making, triangular fuzzy numbers, Geometric Bonferroni mean, triangular fuzzy Einstein geometric Bonferroni mean (TFEGBM) operator, triangular fuzzy weighted Einstein geometric Bonferroni mean (TFWEGBM) operator, psycholinguistic teaching effect

1. Introduction

Fuzzy multiple attribute decision making (FMADM) is an important part of modern decision science [1–4]. It assumes that there exists a set of alternatives with multiple attributes which a decision maker (DM) should evaluate and analyze. The aim of FMADM is to find the most desirable alternative or rank the feasible alternatives for supporting decision makings. As an active research area, FMADM problems have been tried to be solved by some classical methods such as the simple additive weighting method (SAW), the analytic hierarchy process (AHP), and the technique for order preference by similarity to ideal solution (TOPSIS) [5–7]. Xu [8] and Fan and Wang [9] developed the fuzzy ordered weighted averaging (FOWA) operator. Xu [10] introduced the fuzzy ordered weighted geometric (FOWG) operator. Xu and Wu [11] proposed the fuzzy induced ordered weighted averaging (FIOWA) operator. Xu and Da [12] developed the fuzzy induced ordered weighted geometric (FIOWG) operator. Xu [13] developed the fuzzy weighted harmonic mean (FWHM) operator, fuzzy ordered
weighted harmonic mean (FOWHM) operator, fuzzy
hybrid harmonic mean (FHHM) operator. Wei [14]
proposed the fuzzy ordered weighted harmonic
averaging (FOWHA) operator. Wei [15] developed
the fuzzy induced ordered weighted harmonic
mean (FIOWHM) operator. Wei [16] proposed the
generalized triangular fuzzy correlated averaging
operator and applied these operators to multiple
attribute decision making. Merigo [17] presented
the fuzzy probabilistic ordered weighted averaging
(FPOWA) operator. Merigo and Casanovas [18]
presented the fuzzy generalized hybrid averaging
(FGHA) operator, the fuzzy induced generalized
hybrid averaging (FIGHA) operator, the Quasi-FHA
operator and the Quasi-FIHA operator. Merigo
and Gil-Lafuente [19] proposed the fuzzy induced
generalized ordered weighted averaging (FIGOWA)
and the fuzzy induced quasi-arithmetic OWA
(Quasi-FIGOWA) operator. Xu [20] developed some
fuzzy ordered distance measures, such as linguistic
ordered weighted distance measure, uncertain
ordered weighted distance measure, linguistic hybrid
weighted distance measure, and uncertain hybrid
weighted distance measure, etc. Zhao et al. [21]
proposed fuzzy prioritized operators for multiple
attribute group decision making. Wei et al. [22]
proposed fuzzy power aggregating operators for
multiple attribute group decision making. Zhu et al.
[23] developed the triangular fuzzy weighted Bonferroni
mean (TFWBM) operator for multiple attribute
decision making under the triangular fuzzy environ-
ments. Guo et al. [24] developed the triangular fuzzy
gometric Bonferroni mean (TFGBM) operator and
triangular fuzzy weighted geometric Bonferroni
mean (TFWGBM) operator, based on which we
design two procedure for multiple attribute decision
making under the triangular fuzzy environments.

In this section, we briefly describe some basic con-
cepts and basic operational laws related to triangular
fuzzy numbers.

**Definition 1.** [33] A triangular fuzzy numbers \( \tilde{a} \) can be defined by a triplet \((a^L, a^M, a^U)\). The membership function \( \mu_{\tilde{a}}(x) \) is defined as:

\[
\mu_{\tilde{a}}(x) = \begin{cases} 
0, & x < a^L, \\
\frac{x-a^L}{a^M-a^L}, & a^L \leq x \leq a^M, \\
\frac{x-a^U}{a^U-a^M}, & a^M \leq x \leq a^U, \\
0, & x \geq a^U.
\end{cases}
\]

where \(0 < a^L \leq a^M \leq a^U\), \(a^L\) and \(a^U\) stand for the lower and upper values of the support of \(\tilde{a}\), respectively, and \(a^M\) for the modal value.

**Definition 2.** [33] Basic operational laws related to triangular fuzzy numbers:

\[
\tilde{a} \oplus \tilde{b} = [a^L, a^M, a^U] \oplus [b^L, b^M, b^U] = [a^L + b^L, a^M + b^M, a^U + b^U]
\]
\[
\tilde{a} \otimes \tilde{b} = [a^L, a^M, a^U] \otimes [b^L, b^M, b^U]
\approx [a^L b^L, a^M b^M, a^U b^U]
\]
\[
\lambda \otimes \tilde{a} = \lambda \otimes [a^L, a^M, a^U] = [\lambda a^L, \lambda a^M, \lambda a^U], \quad \lambda > 0.
\]

**Definition 3.** [13] Let \( \tilde{b} = [b^L, b^M, b^U] \) and \( \tilde{a} = [a^L, a^M, a^U] \) be two triangular fuzzy numbers, then the degree of possibility of \( a \geq b \) is defined as

\[
p(a \geq b) = \lambda \max \left\{ 1 - \max \left[ \frac{b^M - a^L}{a^U - a^L + b^M - b^U}, 0 \right], 0 \right\}
\]
\[
+ (1 - \lambda) \max \left\{ 1 - \max \left[ \frac{b^U - a^M}{a^U - a^M + b^U - b^M}, 0 \right], 0 \right\}
\]

(2)

Einstein operations [28] includes the Einstein product and Einstein sum, which are examples of t-norms and t-conorms, respectively. Einstein product \( \otimes_e \) is a t-norm and Einstein sum \( \oplus_e \) is a t-conorm, where

\[
a \otimes_e b = \frac{a + b}{1 + a \cdot b},
\]
\[
a \oplus_e b = \frac{a \cdot b}{1 + (1 - a) \cdot (1 - b)},
\]
\[
\forall (a, b) \in [0, 1]^2.
\]

In the following, Zhu et al. [25] studied on the geometric Bonferroni mean (GBM).

**Definition 4.** [25] Let \( p, q \geq 0 \) and \( a_i (i = 1, 2, \ldots, n) \) refers to a collection of non-negative real numbers. Afterwards, the aggregation functions

\[
\text{GBM}^{p,q} (a_1, a_2, \ldots, a_n) = \frac{1}{p + q} \left( \prod_{i,j=1}^{n} (p a_i + q a_j) \right) \frac{1}{n(n-1)}
\]

is called the geometric Bonferroni mean (GBM) operator.

3. Triangular fuzzy Einstein geometric
   Bonferroni mean operators

The geometric Bonferroni mean [25–27] and Einstein operations [28–32], however, have usually been used in situations where the input arguments are the non-negative real numbers. We shall extend the GBM operators and Einstein operations to accommodate the situations where the input arguments are triangular fuzzy numbers. In this section, we shall investigate the GBM operators and Einstein operations under triangular fuzzy environments. Then, we shall propose the triangular fuzzy Einstein geometric Bonferroni mean (TFEGBM) operator as follows:

**Definition 5.** Let \( \tilde{a}_i = [a^L_i, a^M_i, a^U_i] (i = 1, 2, \ldots, n) \) be a set of triangular fuzzy numbers, and let \( p, q > 0 \).

If

\[
\text{TFEGBM}^{p,q} (\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n) = \frac{1}{p + q} \left( \prod_{i,j=1}^{n} (p a_i + q a_j) \right) \frac{1}{n(n-1)}
\]

\[
= \left[ \frac{1}{p + q} \left( \prod_{i,j=1 \atop i \neq j}^{n} (p a_i^L + q a_j^L) \right) \right] \frac{1}{n(n-1)}
\]
\[
= \left[ \frac{1}{p + q} \left( \prod_{i,j=1 \atop i \neq j}^{n} (p a_i^U + q a_j^U) \right) \right] \frac{1}{n(n-1)}
\]
\[
= \left[ \frac{1}{p + q} \left( \prod_{i,j=1 \atop i \neq j}^{n} (p a_i^M + q a_j^M) \right) \right] \frac{1}{n(n-1)}
\]

(4)
then $TFEGBM^{p,q}$ is called the triangular fuzzy Einstein geometric Bonferroni mean (TFEGBM) operator.

It can be easily proved that the TFEGBM operator has the following properties.

**Theorem 1. (Boundedness)** Let $\tilde{a}_i = [a^L_i, a^M_i, a^U_i]$ $(i = 1, 2, \ldots, n)$ be a set of triangular fuzzy numbers, and let

$$\tilde{a}^- = \min_j \tilde{a}_j, \tilde{a}^+ = \min_j \tilde{a}_j$$

Then

$$\tilde{a}^- \leq TFEGBM^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \leq \tilde{a}^+ \quad (5)$$

**Theorem 2. (Monotonicity)** Let $\tilde{a}_i = [a^L_i, a^M_i, a^U_i]$ $(i = 1, 2, \ldots, n)$ and $\tilde{a}'_i = [a'^L_i, a'^M_i, a'^U_i]$ $(i = 1, 2, \ldots, n)$ be two sets of triangular fuzzy numbers, if $\tilde{a}_j \leq \tilde{a}'_j$, for all $j$, then

$$TFEGBM^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \leq TFEGBM^{p,q}(\tilde{a}'_1, \tilde{a}'_2, \ldots, \tilde{a}'_n). \quad (6)$$

**Theorem 3. (Idempotency)** Let $\tilde{a}_i = [a^L_i, a^M_i, a^U_i]$ $(i = 1, 2, \ldots, n)$ be a set of triangular fuzzy numbers. If all $\tilde{a}_j$ $(\tilde{a}_j = [a^L_j, a^M_j, a^U_j])$ are equal, i.e. $\tilde{a}_j = [a^L_j, a^M_j, a^U_j] = \tilde{a}(\tilde{a} = [a^L, a^M, a^U])$ for all $j$, then

$$TFEGBM^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \tilde{a} \quad (7)$$

**Theorem 4. (Commutativity)** Let $\tilde{a}_i = [a^L_i, a^M_i, a^U_i]$ $(i = 1, 2, \ldots, n)$ and $\tilde{a}'_i = [a'^L_i, a'^M_i, a'^U_i]$ $(i = 1, 2, \ldots, n)$ be two sets of triangular fuzzy numbers, where $\tilde{a}'_i = [a'^L_i, a'^M_i, a'^U_i]$ $(i = 1, 2, \ldots, n)$ is any permutation of $\tilde{a}_i = [a^L_i, a^M_i, a^U_i]$ $(i = 1, 2, \ldots, n)$, then

$$TFEGBM^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = TFEGBM^{p,q}(\tilde{a}'_1, \tilde{a}'_2, \ldots, \tilde{a}'_n) \quad (8)$$

Considering that the input arguments may have different importance, here we define the triangular fuzzy weighted Einstein geometric Bonferroni mean (TFWEGBM) operator.

**Definition 6.** $\tilde{a}_i = [a^L_i, a^M_i, a^U_i]$ $(i = 1, 2, \ldots, n)$ be a set of triangular fuzzy numbers and $p, q > 0$, $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector of $\tilde{a}_i = [a^L_i, a^M_i, a^U_i]$ $(i = 1, 2, \ldots, n)$, where $w_i$ indicates the importance degree of $\tilde{a}_i$, satisfying $w_i > 0(i = 1, 2, \ldots, n)$, and $\sum_{i=1}^n w_i = 1$. If

$$TFWEGBM^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left(\frac{1}{p+q} \sum_{i,j=1}^n \left( p a^L_{i} w_i + q a^L_{j} w_j \right) \right)^{\frac{1}{p+q}} \quad (9)$$

then $TFWEGBM^{p,q}$ is called the triangular fuzzy weighted Einstein geometric Bonferroni mean (TFWEGBM) operator.

4. **An approach to multiple attribute decision making with triangular fuzzy information**

In this section, we shall utilize the triangular fuzzy weighted Einstein geometric Bonferroni mean
(TFWEGBM) operator to multiple attribute decision making for evaluating the psycholinguistic teaching effect with triangular fuzzy information. Let \( A = \{ A_1, A_2, \cdots, A_m \} \) be a discrete set of alternatives, \( G = \{ G_1, G_2, \cdots, G_n \} \) be the set of attributes, whose weight vector is \( \omega = (\omega_1, \omega_2, \cdots, \omega_n) \), with \( \omega_j \geq 0 \), \( j = 1, 2, \cdots, n \), \( \sum_{j=1}^{n} \omega_j = 1 \). Suppose that \( R = (\tilde{a}_{ij})_{m \times n} \) is the decision making matrix, where \( \tilde{a}_{ij} \) is the preference value, which take the form of triangular fuzzy numbers, given by the decision maker, for the alternative \( A_i \in A \) with respect to the attribute \( G_j \in G \).

Then, we utilize the TFWEGBM operator to solve multiple attribute decision making problems for evaluating the psycholinguistic teaching effect with triangular fuzzy information, which can be described as following:

**Step 1.** Utilize the decision information given in matrix \( \tilde{R} \), and the TFWEGBM operator (in general, we can take \( p = q = 1 \))

\[
\tilde{r}_i = (r_i^L, r_i^M, r_i^U) = TFWEGBM_{\theta, q}^p (\tilde{r}_{i1}, \tilde{r}_{i2}, \cdots, \tilde{r}_{in})
\]

\[
= \left( \frac{1}{p + q} \prod_{k,l=1 \atop k \neq l}^{n} (p w_{k} R_{ik} + q w_{l} R_{il}) \right)^{1/(n-1)},
\]

\[
= \left[ \frac{1}{p + q} \prod_{k,l=1 \atop k \neq l}^{n} (p w_{k} R_{ik}^L + q w_{l} R_{il}^L) \right]^{1/(n-1)}, \cdot \left[ \frac{1}{p + q} \prod_{k,l=1 \atop k \neq l}^{n} (p w_{k} R_{ik}^M + q w_{l} R_{il}^M) \right]^{1/(n-1)},
\]

\[
= \left[ \frac{1}{p + q} \prod_{k,l=1 \atop k \neq l}^{n} (p w_{k} R_{ik}^U + q w_{l} R_{il}^U) \right]^{1/(n-1)}, \cdot \left[ \frac{1}{p + q} \prod_{k,l=1 \atop k \neq l}^{n} \left( \frac{p w_{k} R_{ik}^M + q w_{l} R_{il}^M}{1 + p w_{k} R_{ik}^L \cdot q w_{l} R_{il}^L} \right) \right]^{1/(n-1)},
\]

\[
i = 1, 2, \cdots, m, j = 1, 2, \cdots, n.
\]

**Step 2.** To rank these overall preference values \( \tilde{r}_i (i = 1, 2, \cdots, m) \), we first compare each \( \tilde{r}_i \) with all the \( \tilde{r}_j (j = 1, 2, \cdots, m) \) by using Equation (2). For simplicity, we let \( p_{ij} = p (\tilde{r}_i \geq \tilde{r}_j) \), then we develop a complementary matrix as \( P = (p_{ij})_{m \times m} \), where \( p_{ij} \geq 0 \), \( p_{ij} + p_{ji} = 1 \), \( p_{ii} = 0.5 \), \( i, j = 1, 2, \cdots, n \). Summing all the elements in each line of matrix \( P \), we have

\[
p_i = \sum_{j=1}^{m} p_{ij}, i = 1, 2, \cdots, m.
\]

**Step 3.** Rank all the alternatives \( A_i (i = 1, 2, \cdots, m) \) and select the best one(s) in accordance with the collective overall preference values \( p_i (i = 1, 2, \cdots, m) \).

5. Numerical example

The traditional way of teaching features that teachers impart knowledge to students in classrooms where students may have difficulties in good reception of knowledge due to various factors such as local environment, learning peers and other external interruptions. In fact, every kind of learning must prioritize students’ knowledge acceptance, however, in many classes, the students in the back are more likely to ignore what the teacher have transmitted in classes. In the information era, computer technology has become a great helper in bringing magic to classrooms. In the network setting, web-based learning technology can make learning transcend the con-
The expert team selects four attributes to evaluate the five possible college psycholinguistic schools: (1) G1 is the environment of teaching and studying; (2) G2 is the management of teaching information; (3) G3 is the curriculum design and target; (4) G4 is the empathy and the teaching practice. The five possible five possible college psycholinguistic schools A_i (i = 1, 2, ··· , 5) are to be evaluated using the triangular fuzzy numbers by the decision makers under the above four attributes (whose weighting vector is ω = (0.2, 0.1, 0.3, 0.4)), and construct the following matrix A = 〈a_ij〉_{5×4} is shown in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>G1</th>
<th>G2</th>
<th>G3</th>
<th>G4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1</td>
<td>(0.68,0.70,0.71)</td>
<td>(0.63,0.66,0.68)</td>
<td>(0.50,0.52,0.55)</td>
<td>(0.66,0.68,0.69)</td>
</tr>
<tr>
<td>A_2</td>
<td>(0.55,0.58,0.61)</td>
<td>(0.70,0.71,0.73)</td>
<td>(0.71,0.72,0.73)</td>
<td>(0.70,0.72,0.73)</td>
</tr>
<tr>
<td>A_3</td>
<td>(0.51,0.52,0.54)</td>
<td>(0.51,0.52,0.54)</td>
<td>(0.56,0.58,0.61)</td>
<td>(0.67,0.70,0.72)</td>
</tr>
<tr>
<td>A_4</td>
<td>(0.53,0.58,0.63)</td>
<td>(0.45,0.49,0.53)</td>
<td>(0.64,0.66,0.69)</td>
<td>(0.35,0.38,0.41)</td>
</tr>
<tr>
<td>A_5</td>
<td>(0.61,0.63,0.66)</td>
<td>(0.40,0.50,0.56)</td>
<td>(0.33,0.40,0.43)</td>
<td>(0.42,0.45,0.48)</td>
</tr>
</tbody>
</table>

In the following, in order to select the most desirable college psycholinguistic schools, we utilize the TFWEGBM operator to solve the multiple attribute decision making problems for psycholinguistic teaching effect evaluation with triangular fuzzy information, which can be described as following:

Firstly, aggregate all triangular fuzzy preference values 〈r_{ij}〉 (j = 1, 2, ··· , n) by using the TFWEGBM to derive the overall triangular fuzzy preference values 〈r_i〉 (i = 1, 2, 3, 4, 5) of the college psycholinguistic schools A_i.

\[
\begin{align*}
\bar{r}_1 &= [0.148, 0.167, 0.186] \\
\bar{r}_2 &= [0.183, 0.207, 0.231] \\
\bar{r}_3 &= [0.204, 0.225, 0.241] \\
\bar{r}_4 &= [0.198, 0.205, 0.237] \\
\bar{r}_5 &= [0.186, 0.209, 0.232]
\end{align*}
\]

Then, rank all the college psycholinguistic schools A_i (i = 1, 2, ··· , 5) in accordance with the preference degree r_i (i = 1, 2, ··· , 5): A_3 > A_4 > A_2 > A_5 > A_1, and thus the most desirable college psycholinguistic school is A_3.

6. Conclusion

With the booming of China’s economy in the 21st century, the quality of higher education is facing many challenges, such as the grand scale of development, inadequate investment in higher education, to name a few. New challenges ahead need new moves and policy planning. Among them, how to evaluate the teaching effect becomes a critical point in teaching-learning activities. Moreover, the development of multi-disciplinary science has become a huge opportunity with which we could be enlightened. Colleges and universities must have high-quality education to develop high-quality talent. Teaching supervision plays an important role in evaluating teaching process and teaching quality. Even, it can function to help monitoring school management in colleges. Teaching assessment is a key link of teaching-effect and teaching-management. In this paper, we investigate the multiple attribute decision making problems for evaluating the psycholinguistic teaching effect with triangular fuzzy information. Motivated by the idea of geometric Bonferroni mean and Einstein operations, we develop the triangular fuzzy Einstein geometric Bonferroni mean (TFEGBM) operator and triangular fuzzy weighted Einstein geometric Bonferroni mean (TFWEGBM) operator for aggregating the triangular fuzzy information. Then, based on the TFWEGBM operator, we develop the procedure for multiple attribute decision making with the triangular fuzzy information. Finally, a practical example for evaluating the psycholinguistic teaching effect is given to verify the developed approach. In our future studies, our results may be further extended by using the traditional uncertain and fuzzy decision making theories and techniques [34–49].
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References


