Centrifuge modelling of unsaturated soils

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Abstract. The use of centrifuge modeling where unsaturated soils are involved is more limited than its use in problems involving dry sand or saturated clays. This limitation is certainly due to the well-known experimental complexities related with unsaturated soils that increase in centrifuge modeling; on the other hand, few data is available about the scaling laws for unsaturated soils. In this paper, physical modeling of unsaturated soil problems in centrifuge is evaluated considering some phenomena involved in the behavior of those materials such as water migration, expansion and collapse. An overview of the scaling laws that has to be used is presented, with a selection of geotechnical problems studied on small-scale models in centrifuge.

Keywords: Centrifuge, unsaturated soils, scaling laws, geotechnical works

1. Introduction

Centrifuge modelling is a valuable geotechnical tool used to extrapolate the results from a reduced scale model to a full-scale prototype. The key point of centrifuge modelling is to conserve the same stresses within the soil at both scales: model and prototype. This condition is achievable increasing the gravity acceleration, \( g \), in the same relationship than the reduction in the length scale, \( 1/N \). This condition is possible in a centrifuge using a centrifugal acceleration of \( N \times g \); so the acceleration within the soil in the model becomes \( N_g \) and then the vertical stresses are conserved. As the purpose of centrifuge modelling is to extrapolate the results from a small-scale model to a full-scale prototype, the extrapolation of all the parameters controlling the behavior of a geotechnical work requires the use of scaling laws. These scaling laws are a set of relationships connecting the behavior of the model and the prototype for any length scale.

Some geotechnical works can be studied using reduced scale models under \( 1 \times g \) acceleration (without centrifuge), or using calibration chambers reproducing the stress in the prototype (i.e. deep tunnels, deep piles, etc.). However, centrifuge models are particularly useful in problems for which the boundary condition imposed at the free surface or the stress gradient play an important role on the structure’s behavior.

For geotechnical works located near the soil surface the variation of water content of the soil due to the interaction with the atmosphere could be as relevant as the stress gradient due to the massic forces. Table 1 presents a qualitative evaluation of the role of the stress gradients due to the massic forces and the role of the boundary condition imposed at the free surface for different geotechnical works. This table highlights the potential of centrifuge modeling techniques to study the performance of some geotechnical works like embankments, slopes, shallow foundations or retaining structures on unsaturated soils whereas for deep tunnels and very deep foundations the relevance of the gradients decrease with depth, as a result these problems can be studied without using centrifuge. However, it is important to note that Table 1 generalizes each geotechnical problem; in fact, it is possible to found numerous cases where the role of the gradients and boundary conditions are different from the presented in the table. Nevertheless,
Table 1

<table>
<thead>
<tr>
<th>Geotechnical work</th>
<th>Role of the mass</th>
<th>Role of the boundary condition over the free surface</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stress gradient</td>
<td>Atmospheric stress</td>
<td></td>
</tr>
<tr>
<td>Embankments</td>
<td>↑</td>
<td>↑</td>
<td>1</td>
</tr>
<tr>
<td>Slopes</td>
<td>↑</td>
<td>↑</td>
<td>1</td>
</tr>
<tr>
<td>Shallow foundations</td>
<td>→</td>
<td>↑</td>
<td>2</td>
</tr>
<tr>
<td>Deep foundations</td>
<td>→</td>
<td>↓</td>
<td>2</td>
</tr>
<tr>
<td>Retaining structures</td>
<td>↑</td>
<td>→</td>
<td>1</td>
</tr>
<tr>
<td>Shallow tunnels</td>
<td>↑</td>
<td>↓</td>
<td>3</td>
</tr>
<tr>
<td>Deep tunnels</td>
<td>↓</td>
<td>↓</td>
<td>3</td>
</tr>
</tbody>
</table>

Qualitative evaluation: ↑ strong effect; → small effect; ↓ negligible effect

1. the active forces on embankments, slopes and retaining structures depend on the forces due to mass, the strength of the soils depends on the suction whose gradient is controlled by the atmospheric boundary condition.
2. shallow and deep foundations on expansive or collapsing soils are affected by the variation of suction whose gradients depend on the atmospheric boundary condition; for deep foundations the relevance of the gradients depend on the length of the pile.
3. the stability of shallow tunnels depends on the forces due to mass and its gradients; depending on the depth of the tunnel, the boundary condition at the free surface is more or less important. For deep tunnels the effect of the free surface is negligible and therefore centrifuge modeling is less relevant.

Table 1 is useful to evaluate, for each geotechnical problem, the relevance of using a centrifuge and the source of errors when using reduced scale models without a centrifuge.

Nowadays centrifuge modeling is widely used to study the performance of different type of geotechnical works involving saturated clays or dry sands. The use of centrifuge modeling where unsaturated soils are involved is more limited than its use in problems involving dry sand or saturated clays. This limitation is certainly due to the well-known experimental complexities related with unsaturated soils such as the measurement of suction and water content as well as the control of the hydraulic boundary conditions; complexities that increase in centrifuge modeling; on the other hand, few data is available about the scaling laws for unsaturated soils [1].

This paper presents the results of several authors on scaling laws related to unsaturated soils as well as the results of some centrifuge models.

It is important to note that centrifuges can be used as geotechnical tools in two ways:

(i) Geo-centrifuges may be used simply as a tool that increases the gravity accelerating some related parameters such as the flow velocity. In this case, the terminology used to describe those tests is centrifuge testing. In unsaturated soils domain, this technique is widely used to obtain the water retention curve [2], and more recently to measure the unsaturated water permeability [3].

(ii) Another way in using centrifuges is to extrapolate the results from the small scale model to the full scale prototype; this methodology is described as centrifuge modeling. In this case the scaling laws must be fulfilled.

Despite the relevance of obtaining the water characteristic curve and the unsaturated water permeability using centrifuges, this paper focuses only on centrifuge modeling.

2. Scaling laws for centrifuge modeling with unsaturated soils

2.1. General considerations

As described, experiments carried out using geotechnical centrifuge generally fall in two categories: tests that uses the centrifuge acceleration as a tool to measure physical properties of the soil such as suction or permeability (centrifuge testing); and tests that intends to reproduce a boundary value problem on a reduced scale model to extrapolate the results to the full scale geotechnical work (centrifuge modeling). This last category makes use of scaling laws that relates the results of centrifuge experiments (the model scale) to the problem under investigation (the prototype scale).
Fig. 1. Capillary rise in prototype and in model scales.

The inspectional analysis\cite{4} is a powerful method to analyze the scaling laws, providing that the mathematical description of the process is well established. This analysis involves mapping the equation controlling the process in a non dimensional form and find out the invariance of the physical law under changes of scale \cite{5}.

Modeling of models \cite{6} can be used when the theoretical model is not available or to validate the scaling laws obtained using inspectional analysis. This technique involves measures at different acceleration levels so that scaling laws can be inferred directly from the measured data.

2.2. Validation of scaling laws for non-deformable unsaturated soils

Prototype-model scaling in unsaturated porous media that undergoes negligible consolidation in the centrifuge has received much attention previously. Several authors have presented scaling analysis for water flow in unsaturated soils \cite{4, 7–16}.

Three processes are relevant on scaling laws of flow in unsaturated soils having negligible volume changes: the flow velocity or the discharge, the dynamics of the evolution of water content or saturation, and the water content profile at equilibrium.

The suction curve of a soil depends on its fabric and mineralogy, these two factors controls the pore sizes and the adsorption of water. Furthermore the ions in water affect the osmotic suction.

Due to the complexities of unsaturated soils, it is better to use in the model the same soil than a prototype. In other words, to use a soil having the same mineralogy, reproducing the same fabric and uses the same water. Under these conditions the suction curve is conserved and the profile of water content at equilibrium in a soil column is scaled using the scale of ratio for length, $N$. In fact, considering a soil column subjected to capillary infiltration (Fig. 1), the pore water pressure in two homologous points $A_p$ and $A_m$, located at heights $Z_p$ and $Z_m = Z_p/N$ are the same:

\begin{equation}
    u_p = \rho_w g Z_p
\end{equation}

\begin{equation}
    u_m = \rho_w g N Z_m = \rho_w g \frac{Z_p}{N}
\end{equation}
Considerable efforts have been done during the NECER project (Network of European Centrifuges forEnvironmental Geotechnics Research) [17], to validate the scaling law for capillary rise. For example, Fig. 2a shows the results reported in [15] concerning the profile of water content in capillary rise for different centrifuge levels. Fig. 2b shows the good agreement for all the results reported in prototype scale using $1/N$ as scale factor.

The flow of water in unsaturated soil is commonly described using generalized Darcy’s law [18–21]:

$$v_w = \frac{K_w}{\partial H/\partial z}$$  \hspace{1cm} (3)

In this equation $H$ is the hydraulic potential, $K_w$ is the unsaturated permeability, and $z$ the direction of flow. The coefficient of permeability, $K_w$, is a function of any two of three possible volume-mass properties [22]:

$$K_w = K_w(S, e) \text{ or } K_w = K_w(e, w) \text{ or } K_w = K_w(w, S)$$  \hspace{1cm} (4)

If the pore water pressure is conserved in homothetic points, the flow of water in the prototype and in the model are:

$$v_{wp} = K_w(z_p) \frac{\partial H}{\partial z_p}, \text{ and } v_{wm} = K_w(z_m) \frac{\partial H}{\partial z_m}$$  \hspace{1cm} (5)

If furthermore the model and the prototype have the same permeability, $K_w(z_p) = K_w(z_m)$, which is the case because the void ratio and the pore water pressure in $z_p$ and $z_m$ are the same in the model and the prototype, then:

$$v_{wm} = N v_{wp}$$  \hspace{1cm} (6)

Figure 3 shows the results reported in [15] concerning the flow of water during drainage tests of initially saturated samples at different $g$ levels. Good agreement is observed using $N$ as a scaling factor.

The evolution of volumetric water content in unsaturated soils is done by the Richard’s equation [20]:

$$\frac{\partial \theta_w}{\partial t} = \frac{\partial}{\partial z} \left[ K_w(z) \frac{\partial H}{\partial z} \right]$$  \hspace{1cm} (7)
In this case, the double derivation about \( z \) implies that the evolution of the water content as a function of time is scaled with a factor of \( N^2 \) [23]:

\[
\left( \frac{\partial \theta_w}{\partial t} \right)_m = N^2 \left( \frac{\partial \theta_w}{\partial t} \right)_p
\]  

(8)

Figure 4 shows the results reported in [11] concerning the dynamic of the capillary rise as a function of time, and the good agreement of the \( N^2 \) scale factor.
2.3. Validation of scaling laws for deformable soils

Most of the studies relating scaling laws applied for unsaturated soils have been done on sands having little or negligible volumetric changes during the infiltration process. Concerning unsaturated deformable soils Bear et al. [24] studied the centrifugal filtration in unsaturated deformable soil, and Caicedo et al. [25] presented a validation of the scaling laws for expansive soils.

Volumetric changes in unsaturated soils depend on the variation of total stress and suction [26]. Discussions about the best way to take into account suction and total stress in volumetric changes on unsaturated soils are very active. However most authors agree with taking separately the role of total stress and suction, for example the following equation is proposed in [26]:

\[
\frac{\partial \varepsilon_v}{\partial t} = m_1 \frac{\partial (\sigma - u_a)}{\partial t} + m_2 \frac{\partial (u_a - u_w)}{\partial t}
\]  

(9)

where \( m_1 \) is the coefficient of volume change with respect to a change in net stress \( (\sigma - u) \); and \( m_2 \) is the coefficient of volume change with respect to a change in matric suction \( (u_a - u_w) \). The change of matric suction with time is controlled by equation 7. As a result if the change in net stress is null, the relationship of volumetric strain between the model and the prototype is:

\[
\left( \frac{\partial \varepsilon_v}{\partial t} \right)_m = N^2 \left( \frac{\partial \varepsilon_v}{\partial t} \right)_p
\]  

(10)

This result is valid too when there is a change in net stress, however if the change in net stress is the result of an external load, the loading velocity must be scaled with a factor of \( N^2 \).

The displacement \( \Delta H \) of the surface of a soil column is:

\[
\Delta H = \int \varepsilon_v \, dz
\]  

(11)

As the volumetric change is conserved in the model and in the prototype, the scale factor for displacement is:

\[
\Delta H_p = N \Delta H_m
\]  

(12)

The results reported in [25] confirm these scaling relationships for expansive soils. These results concern the modeling of a 10 m high soil column of over consolidated lacustrine soil from the “Sabana de Bogotá”. The expansion was measured when applying a total inundation of 1 m high at the start of modeling (Fig. 5). The soil column was modeled with accelerations of \( 100 \times g, 200 \times g, \) and \( 400 \times g \).

Figure 5 shows the expansion displacement in prototype scale using \( N \) and \( N^2 \) as scale factors for heave and time respectively.

These curves show the good agreement obtained using these scale factors.

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Fig. 5. Prototype expansion for different \( g \) levels.
Concerning expansion, the water content profile is an appropriate parameter that reflects the evolution of expansion in depth. To verify that the soil conditions along the depth are the same in the different models, the water content was measured taking out soil samples at different depth and at the same prototype time for the 200 \times g and 400 \times g models. Figure 6 shows the good agreement of the water content profile for the tests performed at 200 \times g and 400 \times g.

3. Scaling laws for soil atmosphere interaction

Boundary conditions in centrifuge models must be adapted to follow the scale relationships. One particular boundary condition, relevant when simulating the effect of weather on soils, is the case of the interaction between soil and atmosphere; this interaction involves complex processes of heat and mass transport. Furthermore, in the case of compressible soils these phenomena are coupled with volumetric changes.

Heat and water transport in soils are diffusion processes, that are controlled by the same scale relationships: the transport events occur \( N^2 \) faster in the model than in the prototype, and the flow of mass and heat in the model increases \( N \) times with respect to the prototype [27].

4. Scaling laws for heat exchanges in centrifuge

Heat exchange at the soil surface includes different processes of heat transport: sensible heat, \( q_{\text{sens}} \); radiation, \( q_{\text{rad}} \); thermal emission, \( q_{\text{th}} \); and convection \( q_{\text{conv}} \). Sensible heat flux in the body must be in equilibrium with external conditions to which it is exposed, then:

\[
q_{\text{sens}} = q_{\text{rad}} + q_{\text{th}} + q_{\text{conv}}
\]  

(13)

Also rain and evaporation modify the heat balance as follows [28]:

\[
q_{\text{sens}} = q_{\text{rad}} + q_{\text{th}} + q_{\text{conv}} + q_{\text{rain}} + q_{\text{evap}}
\]  

(14)

where \( q_{\text{rain}} \) is the sensible heat due to falling rain, and \( q_{\text{evap}} \) is the latent heat due to evaporation given by:
\[ q_{\text{rain}} = \frac{dm_{\text{rain}}}{dt} c_p(T_{\text{rain}} - T_s) \]  
(15)

\[ q_{\text{evap}} = \frac{dm_{\text{vap}}}{dt} L_v \]  
(16)

where \( m_{\text{rain}} \) is the rainfall mass per unit area, \( T_{\text{rain}} \) is the temperature of water in rain, \( T_s \) is the temperature at the soil surface, and \( c_p \) is the specific heat capacity of water, \( L_v \) is the latent heat of vaporization and \( m_{\text{vap}} \) is the mass of evaporation per unit area.

The sensible heat flux within the soil, \( q_{\text{sens}} \), represents the heat conduction through the boundary of the soil mass, this component is described by Fourier’s Law, which in one dimension is:

\[ q_{\text{sens}} = -K_T \frac{dT}{dx} \]  
(17)

where \( K_T \) is the thermal conductivity of the body, \( T \) is the temperature and \( x \) the flux direction.

To increase temperature gradients, heat flux at the boundary of the reduced scale model in centrifuge must grow \( N \) times with regard to prototype conditions, then:

\[ \frac{q_{\text{m}}}{q_{\text{p}}} = N \]  
(18)

where the superscripts \( m \) and \( p \) denote the characteristics in the model and the prototype respectively, then:

\[ q_{\text{rad}}^m + q_{\text{th}}^m + q_{\text{conv}}^m + q_{\text{rad}}^m + q_{\text{evap}}^m = N \left( q_{\text{rad}}^p + q_{\text{th}}^p + q_{\text{conv}}^p + q_{\text{rad}}^p + q_{\text{evap}}^p \right) \]  
(19)

Equation 19 can be satisfied increasing heat fluxes in numerous combinations; however, due to the non-linearity of most of the relationships involved in Eq. 19, a straightforward solution could be increasing all components:

\[ q_{\text{rad}}^m = Nq_{\text{rad}}^p, \quad q_{\text{th}}^m = Nq_{\text{th}}^p, \quad q_{\text{conv}}^m = Nq_{\text{conv}}^p, \quad q_{\text{rad}}^m = Nq_{\text{rad}}^p, \quad q_{\text{evap}}^m = Nq_{\text{evap}}^p \]  
(20)

The scaling relationships for each component of the heat flow can be obtained analyzing the physical principles of each factor included in equation 20.

Heat flux due to radiation can be obtained using the mean absorptivity coefficient, \( \alpha \), as follows:

\[ q_{\text{rad}} = \alpha I \]  
(21)

Then, the scaling law for radiation heat flux is:

\[ \alpha^m I^m = N\alpha^p I^p \]  
(22)

And the irradiance in the model, \( I^m \), becomes:

\[ I^m = N \frac{\alpha^p}{\alpha^m} I^p \]  
(23)

As the body heats, it radiates energy, and therefore some heat escapes. The heat flux due to this radiation is given by the Stefan-Boltzmann equation:

\[ q_{\text{th}} = \varepsilon \sigma T_s^4 \]  
(24)

where \( \varepsilon \) is the emissivity coefficient of the surface, \( \sigma \) is the Stefan-Boltzmann constant, \( T_s \) is the temperature at the surface of the body. As a result the scaling law for the thermal emission of the surface becomes:

\[ \varepsilon^m \sigma (T_s^m)^4 = N\varepsilon^p \sigma (T_s^p)^4 \]  
(25)
Then the temperature at the surface of the model, \( T_m^s \), becomes:

\[
T_m^s = \left[ N \varepsilon_p \varepsilon_m \right]^{1/4} T_p^s
\]  
(26)

Heat fluxes due to free and forced convection are the result of the interaction between the soil and the air at the surface. As a result, heat transport is coupled with fluid mechanics and then the heat flow is proportional to the difference between the temperature at the surface of the body, \( T_s \), and the fluid temperature \( T_a \) affected by a convection coefficient, \( h_c \), that depends on the Nusselt number, \( N_u \):

\[
q_{conv} = h_c (T_a - T_s)
\]  
(27)

\[
h_c = \frac{N_u k}{L}
\]  
(28)

where \( k \) is the fluid thermal conductivity and \( L \) is the characteristic length of the body. Therefore the scaling law for convection becomes:

\[
N_{m}^{Nu} \frac{k}{L} (T_a^m - T_s^m) = N \frac{k}{L} (T_a^p - T_s^p)
\]  
(29)

Then the Nusselt number in the model, considering that \( L_m/L_p = 1/N \), must be:

\[
N_{m}^{Nu} = \frac{N_p^{Nu} (T_a^p - T_s^p)}{(T_a^m - T_s^m)}
\]  
(30)

Concerning the sensible and latent heat flux due to rain and evaporation given by equations 15 and 16, the following scaling laws apply:

\[
\frac{dm_{rain}^{m}}{dt} = N \frac{(T_a^m - T_s^m)}{(T_a^m - T_s^m)} \frac{dm_{rain}^{p}}{dt} \frac{dm_{rain}^{p}}{dt}
\]  
(31)

And

\[
\frac{dm_{vap}^{m}}{dt} = N \frac{dm_{vap}^{m}}{dt}
\]  
(32)

The flow of water near the soil surface is controlled by infiltration and evaporation. Each of these processes is the result of the equilibrium between the atmospheric variables and the soil capacity to accept water coming from the atmosphere during rainfall, or to provide vapor to the atmosphere on evaporation.

Rainfall is characterized by its intensity, \( R \), duration \( D \), and frequency \( F \). Considering the scale relationships for time and water flow, the scaling laws for the rainfall characteristics are as follows:

\[
R_m = N R_p
\]

\[
D_m = \frac{1}{N^2} D_p
\]  
(34)

\[
F_m = N^2 F_p
\]

Establishing scaling laws for evaporation is more difficult than for infiltration. Inspectional analysis can be performed from the different approaches to assess potential evaporation: i.e. Dalton, Penman, and Bowen ratio approaches. John Dalton expressed a relationship to predict the rate of evaporation from a body of water [29]. This
was really just a general relationship that said, in essence, that the evaporation rate depended on the difference between the saturation vapor pressure at the evaporating surface and the vapor pressure in the mass of air above the surface (this difference is known as the vapor pressure deficit, VPD); afterwards, this relationship was affected by the rate at which the wind carried away the evaporated molecules. Thus the expression to obtain the evaporation rate according to Dalton is as follows:

$$\frac{dm_{vap}}{dt} = f(V_v,e^*_{s} - e_a)$$ (35)

where $e^*_{s}$ is the saturation vapor pressure at the temperature of the water surface, $e_a$ is the vapor pressure in air and $f(V_v)$ is function of the mean wind velocity $V_v$.

One of the main limitations of the Dalton equation is the requirement of knowing the temperature at the surface, that is necessary to assess $e^*_{s}$. To circumvent this limitation other approaches like Penman [30] and Bowen Ratio combine different physics and empirical relationships to obtain potential evaporation. However for physical and numerical modeling the temperature at the surface is easily obtained, then the Dalton approach seems to be suitable to analyze evaporation in centrifuge models.

There are dozens of variations of Dalton’s Law, each with its own unique set of constants and exponents, empirically calibrated to specific evaporation experiments. Most of these equations are in the following general form [31]:

$$\frac{dm_{vap}}{dt} = (A_s + B_s V_v)(e^*_{s} - e_a)$$ (36)

where $A_s$ and $B_s$ are empirically calibrated constants.

Dalton’s Law provides some insights about the mechanisms to increase $N$ times potential evaporation in an artificial atmosphere over the model. Considering Eq. 20:

$$\frac{dm_{vap}}{dm_{vap}} = \frac{A_s + B_s V_v^n}{A_s + B_s V_p^n} (\frac{e^*_{s} - e_a}{e^*_{s} - e_a})^n = N$$ (37)

Different combinations of wind velocity and vapor pressure deficit satisfy Eq. 38; however, a trivial solution is obtained by keeping the same wind velocity in the model and the prototype and by growing the vapor pressure deficit $N$ times:

$$\frac{V_v^n}{V_p^n} = 1, \quad \text{and} \quad \frac{(e^*_{s} - e_a)^n}{(e^*_{s} - e_a)^n} = N$$ (38)

Finding other combinations of wind velocity and VPD satisfying Eq. 39 is possible; however, it requires further research.

Increased VPD is possible by increasing the temperature at the surface of the model and reducing the vapor pressure on the atmosphere by either reducing its temperature or its relative humidity.

Table 2 summarizes the scaling laws that are useful to model problems involving soil-atmosphere interaction according to the previous analysis.

### 5. Applications

The use of geotechnical centrifuges for modeling geotechnical works involving unsaturated soils is increasing. In this paper some examples are presented in a succinct form to illustrate the possibilities of this technology.

#### 5.1. Centrifuge modelling of slope stability

The possibilities of using centrifuge modeling to study the performance of slopes and its interaction with rain is described in [32–35], among others. Here an example of the effect of rain on a slope made of volcanic ashes is described.
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Table 2
Summary of scaling laws for soil-atmosphere interaction in centrifuge

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model/prototype</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry and mass</td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>$L^m/L^p$</td>
</tr>
<tr>
<td>Area</td>
<td>$A^m/A^p$</td>
</tr>
<tr>
<td>Volume</td>
<td>$V^m/V^p$</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho^m/\rho^p$</td>
</tr>
<tr>
<td>Mass</td>
<td>$M^m/M^p$</td>
</tr>
<tr>
<td>Diffusion (heat and mass transport)</td>
<td></td>
</tr>
<tr>
<td>Time</td>
<td>$t^m/t^p$</td>
</tr>
<tr>
<td>Gradient</td>
<td>$(\partial T^m/\partial t^m)/(\partial T^p/\partial t^p)$</td>
</tr>
<tr>
<td>Flux</td>
<td>$q^m/q^p$</td>
</tr>
<tr>
<td>Soil-atmosphere interface (heat flux)</td>
<td></td>
</tr>
<tr>
<td>Irradiance</td>
<td>$I^m/I^p$</td>
</tr>
<tr>
<td>Thermal emission</td>
<td>$T^m/T^p$</td>
</tr>
<tr>
<td>Convection (Nusselt number)</td>
<td>$N^c_m/N^c_p$</td>
</tr>
<tr>
<td>Sensible heat flux (rain)</td>
<td>$d\rho m_{rain}/d\rho m_{rain}$</td>
</tr>
<tr>
<td>Latent heat flux (evaporation)</td>
<td>$d\rho m_{sat}/d\rho m_{sat}$</td>
</tr>
<tr>
<td>Soil-atmosphere interface (rain)</td>
<td>$d\rho m_{rain}/d\rho m_{rain}$</td>
</tr>
<tr>
<td>Rate of rain per unit area</td>
<td>$D_m/D_p$</td>
</tr>
<tr>
<td>Frequency</td>
<td>$F_m/F_p$</td>
</tr>
<tr>
<td>Soil-atmosphere interface (evaporation)</td>
<td></td>
</tr>
<tr>
<td>Rate of mass evaporated per unit area</td>
<td>$d\rho m_{sat}/d\rho m_{sat}$</td>
</tr>
<tr>
<td>Wind velocity</td>
<td>$V^m/V^p$</td>
</tr>
<tr>
<td>Deficit of vapour pressure</td>
<td>$(e^m_s-e^p_s)/(e^m_s-e^p_s)$</td>
</tr>
</tbody>
</table>

*Other combinations of wind velocity and vapour pressure deficit are possible to satisfy the scaling law concerning the rate of mass evaporated per unit surface. *For $T^m_s = T^p_s$ and $T^m_a = T^p_a$. |

Table 3
Geomechanical properties of the volcanic ash soil used in the test

<table>
<thead>
<tr>
<th>Property</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid limit, $w_l$</td>
<td>%</td>
<td>51</td>
</tr>
<tr>
<td>Plastic limit, $w_p$</td>
<td>%</td>
<td>35</td>
</tr>
<tr>
<td>Water content, $w$</td>
<td>%</td>
<td>56</td>
</tr>
<tr>
<td>Dry unit weight, $\gamma_d$</td>
<td>kN/m³</td>
<td>8.7</td>
</tr>
<tr>
<td>Cohesion, $c$</td>
<td>kPa</td>
<td>12.7</td>
</tr>
<tr>
<td>Friction angle, $\phi$</td>
<td>deg</td>
<td>35</td>
</tr>
</tbody>
</table>

The full scale equivalent prototype model is a slope of 10 m high having an inclination of 45° made of natural soils of the central cordillera in Colombia. Soils of superficial formations in the Central Cordillera of Colombia are of volcanic and residual type and are highly complex in terms of their geotechnical behavior. The volcanic ash chosen for this study consists in pyroclastic soils deposited as a result of explosive eruptions which took place between the Pleistocene and the Holocene. At a microstructural level, the volcanic ash consists in a matrix whose components reflect agglomeration with a film of amorphous cementation material around the particles. As a result of this form of deposition and cementation, these soils have a high void ratio and are frequently found under partial saturation. These two factors combined confer high sensitivity characteristics to these soils and make them prone to collapse under certain loading conditions. The geomechanical properties of this soil are summarized in Table 3.

As the volcanic ash is deposited by air, it forms a relatively homogeneous deposit, and furthermore its cohesion permits to obtain undisturbed samples. For this reason, the model was prepared in the strongbox from one block of natural soil.
Model testing was performed at a scaling factor of $N = 50$; a schematic diagram of the model is shown as Fig. 7. A small scale slope model (height 200 mm, 45° slope angle and width of 200 mm) was contained within a centrifuge strongbox.

A set of eight nozzles were used, the injection pressure of water and the number of opened nozzles are controlled to reproduce a typical rain pattern registered in the zone, Fig. 8. The control system to produce rain and to register the volume of water is described in [36].

Image analysis was used to calculate soil displacements from a sequence of digital images captured during the test. Also two LVDT’s displacement sensors are placed on top and at the middle of the slope. At the end of the test the centrifuge was stopped, the apparatus disassembled and samples were taken from the model to measure moisture content and suction using a chilled mirror apparatus (WP4).

Figure 9 shows the relationships between displacement of the slope measured on the two points described before and the cumulated rain. As shown in this figure, a very good description of the displacement with respect to the cumulated rain is obtained using a power equation having an exponent of 0.5. This result appears because infiltration is a diffusion process that can be described by a Laplace equation which can be represented by a linear relationship using the square root of time.

Figure 10 shows the curves of equal suction calculated using the post-test measurements. It is observed that for 90 days of rain, the saturation front reaches only 2.5 m at the bottom of the slope and less than that value at the top and at the middle of the slope. Despite this low water infiltration, vertical and horizontal displacements appear within the slope as a result of the changes in suction.

This example illustrates the possibilities of centrifuge modeling to study slope stability and its interaction with rain and in general with climatic conditions.

5.2. Shallow foundations

This example has been developed in the framework of the European project MUSE (Mechanics of Unsaturated Soils for Engineering, RTN – Marie Curie Actions, EU). A centrifuge model of a circular shallow foundation (1.5 m diameter prototype) resting on an unsaturated loosely compacted silt mass (15 m high prototype) has been submitted to $50 \times g$ [37]. The tests were carried out in the centrifuge facility at the French institute of science and technology for transport, IFSTTAR, Nantes.
The objective of the program was firstly to check the feasibility of carrying out a centrifuge test on an unsaturated fine-grained soil and secondly to investigate the effects of suction changes on the ultimate bearing capacity in unsaturated soils. The silt used in this study is an aeolian silt taken from a site located near the village of Jossigny, France. To induce a possible collapse, the silt was compacted at 90% of the optimum on the dry side. Water level was changed in-flight and suction changes were recorded by means of high capacity tensiometers (HCT) placed at three different heights along the soil mass. HCTs appeared to provide reasonably good suction.
change measurements under a 50×g acceleration. Changes in the water table height were carried out by using a Mariotte bottle [38], in order to obtain three different configurations of test: water table at the bottom of the soil mass, in the middle and at the surface. Several hours (8 to 10 hours) of rotation are needed to reach equilibrium on the water content profile.

Collapse of the compacted silt was observed during wetting. The results (Fig. 11) show the role played by the unsaturation on the bearing capacity. It is also linked to the CPT’s profiles recorded in flight. For saturated media, the suction is equal to zero at the surface and the bearing capacity is less than 1% of the maximum bearing capacity observed when the water table is 15 m below the surface. In the intermediate case, for suction at the soil surface of about 25%, the bearing capacity reaches 70% of the maximum one.

The values of bearing capacity measured were compared with predictions and it is concluded that experimental results are consistent with the bearing capacity equations, if appropriate values of cohesion are used. This kind of test can also be used as an appropriate benchmarking reference for numerical simulations.

5.3. Thermomechanical coupling

An example of the modeling of a soil atmosphere interaction problem in centrifuge during drying was presented in [36]. This test consists in applying two cycles of heating and cooling over a centrifuge model made of compacted
kaolin. For this test \( N = 20 \), and heating and cooling of soil is produced only by convection. Temperature, suction, relative humidity and settlement are measured during the cycles.

The soil is placed into the adiabatic strongbox and compacted under a vertical stress of 2.5 MPa; this stress enables obtaining a homogeneous material at the desired dry density [39].

Figure 12 shows the variation of temperature with depth measured during the second cycle of heating and cooling. The temperature at the surface of the model varies between 19\(^\circ\)C and 32\(^\circ\)C whereas the penetration depth, for which temperature variations are significant is reduced to 12 cm (2.4 m in prototype scale).

Figure 13(a) shows the temperature in the artificial atmosphere and at the soil surface measured during the cycles. As described, on this test heating and cooling of soil is made by convection; as a result, to increase the temperature at soil surface up to 30\(^\circ\)C it’s necessary to rise the air temperature up to 38\(^\circ\)C. The behavior on cooling is similar than in heating, in this case the air temperature at the final of each cooling cycle is lower than the temperature on the surface of the soil.

Figure 13(b) shows the measurements of relative humidity in artificial atmosphere above the model. At the end of each heating cycle the relative humidity decreases to 55%, while at the end of cooling cycles it grows up to 88%. In addition, two condensation events are measured during the commutation from cooling to heating, these events appear since the commutation was made stopping the centrifuge and therefore the change in air pressure provokes condensation.

The temperature measurements at the surface of the model made by the infra red sensor allow calculating the deficit of vapor pressure, VPD, this deficit is presented in Fig. 13(c). As observed, the maximum value of VPD is 4kPa and is obtained at the final of the heating cycle. The value of VPD = 4kPa on the model at \( N = 20 \) correspond to a VPD = 200 Pa on the prototype. This value could be insufficient for most sites, for example extreme values of VPD up to 6kPa with most of the values in the range of 0<VPD<1 kPa are reported in [40].

Figure 13(d) shows the settlement at the surface of the model measured using a laser sensor. As described, the soil used for this model is compacted kaolin, which exhibits few volumetric changes on drying. This behavior appears due to the high compaction level with low water content. As a result, the measured settlement is the combination of volumetric strains due to drying and thermal strains. These two mechanisms act in the same direction during cooling and in opposite direction during heating, for this reason the slope of the settlement curve is higher during the two cooling cycles.
5.4. Embankment

The relevance of the oedometer tests for the prediction of wetting-induced deformations in embankments is examined in [41]. Single- and double oedometer tests are carried out following different loading path (Fig. 14). Path 1 corresponds to an oedometer test for unsaturated soil, that should correspond to the embankment construction. Path 2 follows conventional oedometer test, and Path 3 combine Path 2 and a soaking process to simulate the inundation of compacted fills occurring via, for instance, flooding.

The vertical stress influence on the deformation type and amplitude is confirmed by the results of the single-oedometer tests according to path 3 (Fig. 14). Swelling occurs within specimens inundated with a vertical stress of 3 kPa, but, when the vertical stress is higher than 100 kPa, settlements occur all the more extensively since the initial void ratio is high. Therefore, the void ratio after inundation does not depend on the void ratio after compaction; in fact, for both conditions the final state goes to the void ratio corresponding to the compression saturated oedometer test. As observed in Fig. 15, when the final void ratio after collapse of loose specimens is represented in the oedometric plot, the void ratio decreases and the results drop on the compression curve of the saturated oedometer test.
This means that the conventional compression curve of the saturated oedometer test can be used to predict the void ratio after collapse and, consequently, the collapse deformations, similar results are presented in [42].

A comparison between laboratory tests and geotechnical centrifuge modelling at 100 × g conducted to examine an inundated embankment made of a sand-clay mixture is made. A 20-cm high embankment model is built and instrumented (Fig. 16). The material is compacted on the “dry side” of the optimum Proctor curve at a low compaction rate in order to emphasize settlement phenomena. The inundation simulation is conducted in two successive sequences during centrifuge flight up to a water table of five centimetres.

The cumulative displacements as a function of depth in the model (Fig. 17) show that:

- The application of the centrifuge force generates a crest settlement (2 millimetres) less extensive than the one predicted with the unsaturated oedometer tests (3.5 millimetres). This relatively low value for the model scale would represent a difference of fifteen centimetres on a full size embankment, which is quite significant. However, this settlement would occur during the construction of the embankment and, therefore, would not be actually observed;
The first inundation sequence presents a cumulative displacement profile similar to the unsaturated oedometer profile. The small shift between both curves might be the consequence of the saturation of the lowest layer, most probably by capillary rise. The difference between both curves on the crest also reaches 1.5 millimetres at model scale;

The second inundation sequence presents a profile, which is very close to the saturated oedometer profile in the three first layers. Above, the fourth layer, only, is partly affected by the inundation. Extensive settlements, consistent with the predictions, are observed in the third and in part of the fourth layer. This suggests that the inundation event come with notable and a very fast capillary rise causing the saturation of the soil on a height of approximately five centimetres in a few hours. Previous research works have shown that, in sand materials capillary rises occur very rapidly under the effect of macro-gravity because of an increase of the capillary velocity by \( N \) times [16], which can explain the fast capillary rises observed in layers 3 and 4.
A reasonable consistency is observed between oedometer and centrifuge tests. Assuming that the oedometer tests are representative of embankment settlements far from the slope (no horizontal displacements observed nor allowed), these tests are satisfactory for prediction. In the shallower zones and under the slopes, the stress ratio may notably modify performances [43], and this approach should not be used.

The prediction is associated with uncertainty, since model scale discrepancies of one to two millimetres are observed between laboratory and centrifuge tests, corresponding to differences between ten and twenty centimetres at full scale, i.e., 10 to 20 percent of the settlement. It gives the order of magnitude of the prediction uncertainty obtained with the oedometer tests.

6. Conclusions

In this paper, physical modeling of unsaturated soils in centrifuge has been analyzed based on some examples from the literature, both considered in the cases of non-compressible and compressible soil. An overview of the scaling laws involved for such a topic has been presented, including soil atmosphere interaction and heat exchanges. A set of experiments linked with engineering applications on slope stability, shallow foundation, thermo mechanical coupling and embankments illustrate the possibilities of the use of centrifuge modeling for unsaturated soils. Based on the results of these tests some conclusions can be outlined:

- Centrifuge modelling is a powerful tool to study slope stability because because those problems are controlled by massic forces. Furthermore, centrifuge modelling offer the possibility of studying rain induced landslides taking advantage of the increase in the water flow velocity that applies for diffusion processes in centrifuge. However, to increase the quality of the experimental results, it is necessary to use miniature devices to measure suction and water content.

- Another geotechnical work that can be studied in centrifuge is the bearing capacity of foundations on unsaturated soils. Studies about this problem shows that the bearing capacity of shallow foundations on unsaturated soils can be calculated using classical bearing capacity equations, using an appropriate value of cohesion depending on the level of suction. Bearing capacity tests in centrifuge will certainly provide high quality data as benchmarking reference for numerical simulations.

- Centrifuge modelling has proved its applicability for geotechnical works dealing with collapsible and/or compacted soils. In fact, this paper presents the case of the collapse of an embankment subjected to inundation, the results of this test show the well performance of the double oedometer test to predict the displacements due to collapse in this kind of geotechnical works.

- The results presented in this paper about the thermo mechanical coupling show the possibility of using centrifuge to study problems controlled by diffusion processes and its coupling with the soil mechanical behavior. Also in this field, centrifuge modelling will be a powerful tool to obtain data that to be used as reference for numerical models.

In summary, centrifuge modelling appears as a valuable tool to study boundary value problems that can help for a better understanding of the mechanics of unsaturated soils and to validate both theoretical and physical models.

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