The nets of reason
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History in fast-forward. Logic and argumentation are a natural combination. Though the precise origins of logic are hidden in the mists of antiquity, reflection on patterns in legal or philosophical debate may have been one of the driving forces in the genesis of the discipline. But afterwards, the main emphasis over time shifted to consequence relations in an abstract universe of propositions, and the formal systems to which these give rise. Though contacts were never lost entirely between logic and the realities of discussion and debate, the twentieth century saw a deep split. Perelman and Olbrechts–Tyteca (1958) pointed out how actual reasoning may be more like weaving a piece of cloth from many threads than forging a chain with links in linear mathematical proof style, and rhetoric and informal logic then took their own course. Likewise, Toulmin (1958) made a powerful case of how legal procedure and functional schemas – ‘formalities’ rather than logical form – may be the best paradigm for understanding argumentation. Both critics have inspired follow-up frameworks that continue to flourish today (cf. Walton and Krabbe 1995; van Eemeren and Grootendorst 2004). But this split was not inevitable, and it was not forever. Already Lorenzen (1955) used innovative game-theoretical models of dialogue to investigate the foundations of logic, and in more recent times, Dung (1995) introduced formal models of argumentation in a setting of Artificial Intelligence, which turned out to have strong connections to computational logics.

Gabbay’s programme. Dov Gabbay believes that the interface with argumentation may be the last frontier, where modern logic finds its proper generality and impact on human reasoning practice (cf. Gabbay and Woods 2002). Over the past decade, he has developed a paradigm of networks that applies to reasoning at many levels, from unconscious neural nets in the brain to conscious reasoning of many kinds. As the three papers in this issue show, there is a wealth of substance here, as will be clear to any reader of ‘Equational Approach to Argumentation Networks’, ‘Temporal, Numerical and Meta-Level Dynamics in Argumentation Networks’ and ‘Model and Temporal Argumentation Networks’. Gabbay’s networks unify across different fields, from logic programmes to dynamical systems. They also come with interesting technical properties, including an equational algebraic analysis of connection strength, where stable states can be found by applying the Brouwer fixed-point theorem. Moreover, when network activity is studied as proceeding over time, logic returns at a higher level, too – as a description for the resulting behaviours, and various interesting new modal and temporal languages have emerged in this investigation by Gabbay and his coworkers, as demonstrated in the papers just mentioned.

Clearly, this is an immense intellectual space to explore, and at the same time, Gabbay has engaged in an impressive community-building effort, through a stream of books and papers, including handbooks that pull separate research clans together. If you asked me, my stated opinion

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would be that I totally agree with this vision, and am happy to endorse and support it. Maybe this invited commentary should just stop here. But the editors have asked me for a few comments on logic and argumentation from my own perspective. In what follows, I shall therefore do so, raising some issues that intrigue me – though without any attempt at definitive statements, since my thoughts are in flux.

**Argumentation in logics of agency.** Well, first of all, my own interest in the area is a bit special. It is focused on understanding something often left out in the semantic study of rational agency where I have been active in recent years. Here is one example out of many. We model beliefs, but seldom the reasons for those beliefs. We model belief change, but we tend to ignore the fact that many belief changes in our lives come from pressure by others in discussion. I am interested in a richer structure of reasons, and that even in two ways: as support for our current beliefs, but just as well, as hooks for undermining these beliefs. Explanation is at the same time vulnerability. And that is as things should be. After all, rationality and being ‘reasonable’ mean being able to provide reasons, and being swayed by them. Formal proof is a non-contender for modelling this, except in extreme cases. How to model this fine-grained level more precisely is also a central issue in epistemology. So, what logical models should we use for bringing out this richer structure of having and giving reasons?

**Dynamics of reasoning, logic and games.** To me, the main point to be noted is a shift from statics to dynamics. ‘Reasons’ are not just a list of propositions to be ticked off. Crucially, they also involve the activity of reasoning, and one of the most pregnant forms of that is discussion and argumentation. And to understand the latter, standard logical form is not enough, just as Toulmin said. We need to focus on the logic of procedure and process. That is the main thrust of my work on ‘logical dynamics’ in recent years, witness the trilogy of van Benthem (1996, 2011, 2012). By now, there are dynamic logics for the basic actions of information update and belief change, as well as many others connected to these (questions, suggestions, or even commands) that together drive the stream of cognitive reality. While these are single update steps, a powerful model for the longer term temporal process structure of all this are games where agents interact, using strategies towards certain goals. I myself believe that argumentation and conversation are game-like in essential respects – even if we cannot always compute equilibria in a standard economic style. That this is possible at many levels has been shown by authors such as Lorenz and Lorenzen (1978), van Rooij (2004), Hintikka and Sandu (1997), Parikh (2001), or Feinberg (2007). I myself have proposed games that model patterns in argumentation at several places (cf. van Benthem (2001) on legal argumentation and van Benthem (2004) on playing one’s cards in decision making meetings). So, this is not just a metaphor, but a concrete programme.

**Agency versus networks.** But then, I see some differences with the network paradigm. The first is that the process view of argumentation presupposes interactive agency: and agency in the sense of dependent actions in response to new external informational inputs as well as observed actions of others seems absent from Dov Gabbay’s argumentation networks. They are structures of interactive reasons, not of the agents behind them. The second difference is one in the basic notions that one cares about. The process view is about truth, meaning, and deliberation, and it results in game-theoretic strategies under our conscious control. To me, the intuitive thrust of the network view is different: we are essentially talking about dynamical systems that evolve over time, driven by some transition equation creating statistical patterns in the long run.

**Processes and networks.** But in logic, things are never quite what they seem. Here, is a second round of considerations. First, an argumentation network may be viewed as rich structure of
reasons manipulated by agents, and as such, it could be used to provide fine-structure for the game states that I want in my process view. But this is only one way to go. We can also reinterpret the networks in an agent-oriented manner. Think of the nodes as persons engaging in debate, and connection strengths as their complex informational effects on each other. When colleague $X$ is in favour of proposal $p$, I will agree more with $p$, when colleague $Y$ is in favour, I quickly cool off on $p$. Now, the network models the global drift of public opinion, and again, it connects with my process view, this time as a sort of higher level longer term super-structure. This is not just speculation. Concrete instances of such mixed approaches exist in current logical studies of belief networks (Quine and Ullian 1978; Ghosh and Velazquez-Quesada 2007) or social networks (van Eijck and Sietsma 2011; Girard, Liu, and Seligman 2011).

Dynamic logic and dynamical systems. Given all this, there is nothing strange to the juxtaposition of the above two perspectives. It even happens on a larger scale. Think of the transition from classical game theory to evolutionary game theory, where the logical short-term strategic deliberation of ordinary finite games changes into the mathematics of a large-scale and long-term statistical process. Or think of the law, where specific rules constructed by conscious design and adopted by explicit legislation function in a statistical sea of human behaviour perhaps better described by networks. How are the two perspectives related in the final mathematical analysis? Some intriguing results may be found in current studies of cognitive function. Examples are found in d’Avila Garcez, Lamb, and Gabbay (2009), van Lambalgen and Stenning (2007), or Leitgeb (2004), which present precise equivalence theorems between default logics, logic programs, and neural nets. Even so, these are just case studies. The precise mathematical interface between dynamic logic and the theory of dynamical systems seems an area with many unknowns. Indeed, I feel that much of Dov Gabbay’s work in the papers published here provides valuable pointers to such a more comprehensive understanding.

‘Logic of networks’ and ‘logic in networks’. The preceding constituted my Round Two of comparing networks and game-oriented process models for argumentation. Let me now do a final Round Three, shifting the logical perspective still a little bit further. Networks do describe reasoning, even without explicit agents, thereby internalizing logic, so to speak. We could call this the view of logic in networks. But conversely, networks can also be described externally by logical languages (cf. Kremer and Mints (2007) on modal logics of dynamical systems). A good case study in this logic of networks is the analysis of the Dung framework in Grossi (2010). Here, the process of finding stable extensions, or other notions of interest in the AI literature, is analysed uniformly in terms of logical fixed-point formulas in the $\mu$-calculus and beyond, whose standard evaluation games provide argumentation games of ‘analyzing the available evidence’. These logics of networks also turn out to have connections with current dynamic logics for iterated information update and belief revision, and the circle closes again with the above process views. And one can take that dynamic perspective even further. Networks themselves can be made the subject of dynamic logics, if one analyses the realistic phenomenon of how they can be updated with new nodes and links, or changed in other ways under various triggers.

Conclusion. I am not sure that I have arrived at a stable state of deliberation here, let alone a final view on how different logical approaches to argumentation are related. I find the distinction between network and process views a natural one, but the two also form a natural duality, and the cases of harmony I have given can easily be turned into spirals where one view keeps following up on the other. But I admit that the contrast may still be elusive, since I have not defined it in any precise technical sense.
So, let me just end by stating how I see the area of cognitive reality that we are after. We humans live in a tiny range of the total physical scale of magnitude, where our body movements bring a few objects of the right size under our deliberate control. ‘Below’ us is the statistical molecular and atomic reality over which we have no control, ‘above’ us is the large-scale structure of the universe with the same lack of control. Likewise, cognitively, we live in a tiny little personal zone of deliberation and decision described by logical and game-theoretic models, with below us the statistical physics of brain processes, and above us the statistical realities of long-term social group behaviour. I have not yet seen one vision of logic that manages to unify all this, but Dov Gabbay’s programme certainly seems the most ambitious attempt so far.

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