The design and performance of the adaptive stock market index

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Abstract. The stock market index is one of the main tools used by investors and financial managers to describe the market and compare the returns on specific investments. Common approaches to index calculation rely on a company’s market value generating a weighted average as the index. This work presents new methods of computing adaptive stock market indices based on dynamical properties of the underlying index constituents, and introduces measures to evaluate their performance. The premise behind this work is that the influence of each stock on other stocks should be a major factor in determining the weight given to each stock in the index composition. The methodologies presented here provide the means to construct a dynamic adaptive index, which can be used as a benchmark for the underlying dynamics of the market. We investigate the components of the S&P500 index, and the components of the TA25 index, representing a large (NYSE) and a small (TASE) developed market, respectively. We focus our study on periods before and during the 2008 Sub-prime mortgage crisis. Our results provide evidence that the adaptive-indices provide an effective tool for policy and decision makers to monitor the stability and dynamics of the markets, and identify bubble formation and their ensuing collapse.

Keywords: Financial markets, partial correlation, stock influence, adaptive-indices

1. Introduction

A stock market index is a tool that is used to measure the value of a given section of a specific stock market. It is calculated from the prices of selected stocks, and used by investors and financial managers to describe the market, and to compare the return on specific investments. There are many types of indices; an index might be global – meaning it does not consider the region of the world stocks are from, such as The S&P Global 100 index, or national – which represents performance of stocks from a given country. Additional classes of indices include sectorial indices, which are composed by stocks from a specific sector of the market (such as Wilshire US REIT or Morgan Stanley Biotech Index). An index may also be classified according to the method used to determine its price; in a price-weighted index the price of each component stock is the only consideration when determining the value of the index. In contrast, a capitalization-weighted index factors in the size of the company (Broby, 2007).

Indices are useful in a number of ways to investors (see for example Wu, Chou, Yang, and Ong (2007)). First, the indices provide an historical perspective of stock market performance, giving investors more insight into their investment decisions. Investors who do not know which individual stocks to invest in can use indexing as a method of choosing their stock investments. By aiming to match the performance of the market, investors can invest in index mutual funds or index exchange-traded funds (ETFs) that track the performance of the indices with which they are aligned (Kevin, 2013). This form of trading gives investors the opportunity to do as well as the markets and not
The use of capitalization-weighted indices is often justified by the central conclusion of modern-portfolio-theory that the optimal investment strategy for any investor is to hold the market portfolio, the capitalization-weighted portfolio of all assets. However, the capitalization-weighting index has been subject to severe criticism, pointing out that the mechanics of capitalization-weighting leads to trend-following strategies that provide an inefficient risk-return trade-off (Haugen & Baker, 1991; Amenc, Goltz, & Le Sourd, 2006; Hsu & Campollo, 2006).

Financial markets exhibit several of the properties that characterize complex systems; they are open systems in which many subunits interact nonlinearly in the presence of feedback (Mantega and Stanley, 2000). Mauboussin (2002) has discussed key features of the stock market as a complex adaptive system, arguing that such an approach can provide new and useful perspectives in areas like risk management. In the rapidly growing field of econophysics, financial markets have been investigated using such tools as cross correlations, eigenvalue analysis and hierarchical clustering (Noh, 2000; Utugi, Ino & Oshikawa, 2004; Mantegna, 1999; Garas & Argyakis, 2007; Jung et al., 2006; Labrousse, Cizeau, Boucaud & Potters, 1999; Cornielje, Tumminello, Lillo, Micciche & Mantegna, 2005; Plerou et al., 2002; Gopikrishnan et al., 2000; Pafka & Kondor, 2004).

Recently new methods were introduced on a stock market. Such rules have also been used to predict future prices (John, Miller & Kerber, 1996; Last, Klein & Kandel, 2001). Additional work includes investigations of artificial markets; for example, Bak, Paczuski and Shubik (1997) used agent-based models to show how agents in a market might imitate each other’s behavior and explored the influence of expert agents in the market. They presented theories, which suggest that bubbles appear as a result of a low number of rational traders in such scenarios. This study concluded that the classical economic models provide a poor representation of stock trading. Further work by Setzú (2007) used simulation of trading to show the effects of taxes on the market. Finally, previous work by Boyer, Gibson and Lorent (1999), has shown the importance of investigating a dataset of sufficient length in time, as changes in correlations over time or significantly underperform. The second benefit of stock market indices is that they provide a yardstick with which investors can compare the performance of their individual stock portfolios. Individual investors with professionally managed portfolios can use the indices to determine how well their managers are doing in managing their money.
across regimes could not be detected reliably by splitting a sample according to the realized values of the investigated data.

Thus, considering the important role a market index plays in financial markets, and the complex features exhibited by financial markets, the aim of this work is to introduce a new class of adaptive market indices. We will introduce new methods to calculate a dynamic adaptive stock market index, based on stocks cohesion level (stock correlation, see Shapira et al., 2009) and inter-stock influence using different statistical methods, and evaluate their performance. We focus on the period of the sub-mortgage crisis of 2008, and present how examining the statistical measures of adaptive indices can provide new information on market dynamics. The indices introduced here are calculating in a dynamic and adaptive manner, and the weights and contribution of components is updated on a regular basis, which is significantly different from the current accepted practice. The weights attributed to the components of the indices are calculated using measures of correlation and partial correlation, which will be introduced below. We find that these new indices, especially in the smaller investigated market, can better identify anomalies. These might be eventually developed into instruments that help data scientist reveal underlying dynamics of the market not visible with conventional tools.

2. Materials and methods

2.1. Data


The S&P (Standard & Poor) 500 is a stock market index based on the market capitalizations of 500 leading companies publicly traded in the NYSE. The TA (Tel Aviv) 25 is a capitalization-weighted index of the largest 25 stocks in the Tel Aviv Stock Exchange, which was launched on January 1st, 1992. The stocks composing the index are selected twice a year – June 15th and December 15th, while their weights are changed every day according to the market capitalization of the companies they represent (TASE Website, http://www.tase.co.il, 2013). The S&P 500 index is one the most followed stock market indices in the world, established in 1957. The index composition is compiled by a committee reviewing several parameters of companies: market capitalization, liquidity, domicile, public float, sector classification, financial viability, length of time publicly traded and listing exchange and is periodically, typically in response to acquisitions, or to keep the index up to date as various companies grow or shrink in value. The S&P 500 is different from most US stock market indices in its diversity. Both the TA25 and S&P 500 indices, which we study in this work, are free float capitalization indices. The Free Float Capitalization-Weighted Index is a market index for which the weight of each stock is determined according to the total market value of their outstanding shares (Broby, 2007). A major difference between the two markets is the index calculation type; while the S&P 500 index is a total-return-index – i.e. an index which assumes all cash distributions from the companies are reinvested, the TA 25 index is a price index, which only takes into account the company price movement (capital gains or losses). For this work, we used the price index quotes of the S&P 500 index to avoid artifacts caused by this difference. However, the S&P 500 has made the transition to a float-weighted index in the year 2005 – meaning that it only takes into account the shares offered for public trading when calculating market capitalization.

For both investigated indices, the original index composition is frequently updated (an example of this can be shown in an appendix 3, in which we present a timeline of the composition of the TA25 index throughout the time of this study). Since our method requires consistency of the stocks identities, we choose the stocks for each market that were traded throughout the entire period investigated (which make up about 80% of the composition of each index).

2.2. Construction of adaptive-indices

In this work, we present new methodologies to calculate dynamic adaptive market indices. The new indices introduced are constructed using a weighted average on the stock prices, with weights calculated using the observed correlations between stocks. We introduce here two such methods for adaptive-index calculation, which we dub the Influence Score and Bare Influence Score, which we will explain in this section.

The correlation and influence of stocks on each other we use, is based on the correlation between the
logarithmic price changes of pairs of stocks. The logarithmic price change is calculated in the following way:
\[
r(t) = \ln \left( \frac{Y(t) + \Delta t}{Y(t)} \right) = \ln(Y(t + \Delta t)) - \ln(Y(t)),
\]
where \( Y(t) \) is the price on a given day \( t \), and \( \Delta t \) is the time horizon investigated, which is 1 day in this study.

Next, we calculate the Pearson pairwise correlation coefficients (Pearson, 1895):
\[
C(i, j) = \frac{(r(i) - \mu(i))(r(j) - \mu(j))}{\sigma(i)\sigma(j)},
\]
where \( r(i) \) and \( r(j) \) are the logarithmic price changes for stocks \( i \) and \( j \), \( \mu(i) \) and \( \mu(j) \) denote the corresponding means, \( \sigma(i) \) and \( \sigma(j) \) are the corresponding standard deviations (STD) and \( \langle \cdot \rangle \) denotes the average over time. Note that \( C(i, j) \) is a symmetric square matrix and \( C(i, i) = 1 \) for all \( i \). This describes the correlation between sets of variables – in our case, data regarding the returns of stocks. This is done of course without the index, since it doesn’t represent a stock.

Next we use the resulting correlation matrix to compute the partial correlation matrix. The first order partial correlation coefficient is a statistical measure indicating how a third variable affects the correlation between two other variables (Baha, 2004; Kenett et al., 2010, 2011; Shapira et al., 2009). The partial correlation between variable \( j \) and \( k \) with respect to a third variable \( i \), \( \rho(k, j) \), is defined as
\[
\rho(k, j) = \frac{C(k, j) - C(k, i) \cdot C(j, i)}{\sqrt{(1 - C^2(k, i)) \cdot (1 - C^2(j, i))}}
\]
where \( j \) and \( k \) are two stocks, and \( i \) represents the stock which effect we want to remove. The relative effect of variable \( j \) on the correlation \( C(i, k) \), is given by (Kenett, Pretis, Gun-Gershgonen & Ben-Jacob (2012), Madi et al. (2011); Kenett, Kenett, Ben-Jacob and Faust (2011)):
\[
d(k, j) = C(k, j) - \rho(k, j).
\]

This transformation avoids the trivial case where variable \( j \) appears to strongly affect the correlation \( C(i, k) \), mainly because \( C(i, j) \), \( C(i, k) \) and \( C(j, k) \) have small values. We note that this quantity can be viewed either as the correlation dependency of \( C(i, k) \) on variable \( j \) (the term used here) or as the correlation influence of variable \( j \) on the correlation \( C(i, k) \). Next, we define the total influence of variable \( j \) on variable \( i \), or the dependency \( D(i, j) \) of variable \( i \) on variable \( j \) to be
\[
D(i, j) = \frac{1}{N - 1} \sum_{k \neq i, j} d(k, j).
\]

As defined, \( D(i, j) \) is a measure of the average influence of variable \( j \) on the correlations \( C(i, k) \), over all variables \( k \) not equal to \( j \). All variable dependencies define a dependency matrix \( D \) whose \( i, j \) element is the dependency of variable \( i \) on variable \( j \). It is important to note that while the correlation matrix \( C \) is a symmetric matrix, the dependency matrix \( D \) is nonsymmetrical, since the influence of variable \( j \) on variable \( i \) is in general not equal to the influence of variable \( i \) on variable \( j \). Finally, we sort the variables according to the system level influence of each variable on the correlations between all other pairs. The influence score is simply defined as the sum of the influence of \( j \) on all other variables \( i \) not equal to \( j \):
\[
influence(j) = \sum_{i \neq j} D(i, j).
\]

Finally, normalizing the influence values to a weight vector creates the influence score:
\[
influence \text{ score} = \frac{\text{influence}}{\sum_{k} \text{influence}(k)}.
\]

The price of the index is given as the weighted average of the stocks prices using the weights given by Equation 7:
\[
influence \text{ score index (t)} = \frac{\sum_{j=1}^{N} \text{influence}(j) \cdot \text{stock}(j)_{\text{price}}(t)}{\sum_{j=1}^{N} \text{influence}(k)}.
\]
where $\rho(k, j|i)$ is as defined in Equation 3. We then continue in the following manner:

$$d(k, j|i, m) = \rho(k, j|i) - \rho(k, j, m)$$  \hspace{1cm} (10)

$$D(k, m|i) = \frac{1}{N-1} \sum_{k \neq j} d(k, j|i, m)$$  \hspace{1cm} (11)

$$\text{bare influence}(k) = \frac{N-1}{\sum_{k=m} \sum_{j=1}^N D(k, m|i)}$$  \hspace{1cm} (12)

$$\text{bare influence score} = \frac{\sum_{j=1}^N \text{bare influence}(j)}{\sum_{j=1}^N \text{bare influence}(j)}$$  \hspace{1cm} (13)

The final index price is calculated by:

$$\text{bare influence score index } (t) = \sum_{j=1}^N \text{bare influence}(j) \times \text{stock}(j)_{\text{price}}(t)$$  \hspace{1cm} (14)

with the $\text{stock}(j)_{\text{price}}$ as the time series of stock $j$ prices, similar to the calculations made for Equation 8.

Finally, we introduce three other adaptive indices, which we use as control to compare against the influence indices. We use a Simple Correlation method to calculate a new index, by taking the correlation of the stocks with the original index and using it as the weighing factor for each stock – denoted as the Correlation index. The two additional types of control indices: Equal Weights using an average of the data from each stock and Random Weights using a randomly selected weights average for each stock. The random weights are recalculated every time we would change the weights in the other methods (and not set once for the entire period). The price data of the generated adaptive-indices is normalized in order to make easier comparisons between data of different adaptive indices by proportion and scale.

2.3. Adaptive-index calculation methods

The price of the adaptive indices was derived from the original stock prices using the weights calculated using a simple linear multiplication. The analysis is done using a 66-day time window from which data is taken to calculate a single set of weights. This means that to calculate a weight set, we use the data taken from the 66 days immediately preceding it. The calculations are made in two methods for comparison. A Sliding Window analysis Ya-Lun, (1963) in which the weights were recalculated every day and a Sequential Method where the weights were used for sequences of 66 days at a time (here, 66 is the length of the sequence the weights are used for, not to be confused with the 66 days window used to make up the weights) and only then recalculated using a new time window. The comparison between the two methods of calculation was used to investigate the effectiveness of the way in which weights are currently calculated (in a manner similar to the Sequential Method) with the effectiveness of the adaptive method (Sliding Window). It is important to note that all methods use only back data in order to calculate the weights and use them in forward decision-making, and thus can actually be implemented in real time.

Table 1 presents a summary of the indices we researched during this work, differing from each other by method of correlation calculation and method of weight calculation. Each of these was calculated and studied for each method of price generation – Sliding Window and Sequential method.

2.4. Measuring performance of indices

For each adaptive-index we calculate both the price of the index and the return. We use the aforementioned measurements to examine its efficiency of use in real world environment. We then proceed to perform several statistical tests on these time series in order to detect phenomena with insights on the market.
We test the standard deviation of the price of the index over time in order to determine its stability. We also test statistical measures on the return time series -- the Variance and Mean of each sliding window frame. These statistical measures are calculated for 66 days periods made by a sliding window by one day, and the results are presented for the last day of each window.

We also measure correlation between the new adaptive indices and the original indices for each market. To enable measuring of how this correlation changes for different periods in time, we use time window of 22 days, corresponding to one work month, for which the correlation is calculated (see Kenett et al., 2011) with full overlap.

To gain insights regarding the value our adaptive indices would have as virtual portfolios in which investors might invest, we use the classic Sharpe ratio. This is a measure used by investors in order to measure the excess return (or risk premium) per unit of deviation in an investment asset or a trading strategy, typically referred to as risk. Essentially – a Sharpe Ratio on an asset for a given period is meant to show how profitable its return was in proportion to the risk taken investing in it – all this in comparison with investing in the market index, as shown by Sharpe (1998).

We calculate the value of the Sharpe ratio for each adaptive index for trading periods of 132 days (corresponding to six months of trading, which is used for the composition updates of the real indices) according to the next formula:

\[ S = \frac{E[R_a - R_b]}{\sqrt{\text{VAR}_a - \text{VAR}_b}} \]  \hspace{1cm} (15)

where \( R_a \) and \( R_b \) are the returns of the adaptive-index and the original market index, respectively, and \( \text{VAR}_a \) and \( \text{VAR}_b \) are the variances of the indices.

We also examine the beta value for stocks for each different index used as a benchmark. The beta, according to the capital asset pricing model (CAPM), is a key measure of the risk from exposure to the general market movement as opposed to idiosyncratic factors, thus determining the connection between the stocks and the index as a benchmark (Sharpe, 1964). This beta value is the defining measure of the relationship between the asset (e.g. the stock) and the stock market index, as it defines the systemic risk associated to the market factors the index represents as a benchmark. The CAPM suggests that an investor’s cost of equity capital is determined by the beta (Chong et al., 2013). We calculate this beta value for each stock separately based on its return and the return of the index used as a baseline. This beta is calculated for periods of 132 days, using a sliding window with full overlap (each window is advanced by one day). The beta is given by:

\[ \beta = \frac{\text{Cov}(R_i, R_s)}{\text{VAR}(R_s)} \]  \hspace{1cm} (16)

where \( R_i \) is the return of the stock and \( R_s \) is the return of the index. \( \text{Cov} \) is the covariance operator between the two and \( \text{VAR} \) is the variance operator. To get a clear picture of both the beta and its stability throughout the stocks, we study the average beta values for all stocks.

Finally, we make use of a power spectrum analysis to make some further examination of the results for the adaptive indices, while using the Discrete Fourier Transform (DFT) to convert the data to the frequency domain. Put simply, as opposed to a time domain representation (e.g. the return or price time series of the indices we study) that shows how the value changes over time, the frequency domain shows how the original time series can be represented as a combination of purely periodic functions with different periods of repetition length, by setting the value of change over a single period each of these functions would have. For example, the frequency domain for a time series that is purely periodic would be the measure in which it is changed for its period length, and 0 for all other period lengths. This representation of the information, allows the observer to notice periodic phenomena of a time series, as well as its stability. To make the conversion, we use the discrete version since we use a discrete dataset of sampled information. Note that since the data we are processing is not a digital signal, rather financial time series, the frequency we are discussing is in fact sampling cycles per day, where the value ranges between 0 and 1. For example, 0.2 sampling cycles a day mean we sample the market once every 5 days.

The DFT to convert values to the cycles/day (frequency) domain is given by:

\[ X_f = \sum_{t=0}^{N} x_t e^{-i2\pi ft}, \]  \hspace{1cm} (17)

where \( N \) represents the amount of days sampled, and \( f \) is the frequency of sampling between 0 and 1. We then use the results of Equation (8) to calculate the power of the series given by:

\[ Y(f) = |X(f)|^2. \]  \hspace{1cm} (18)
This is done on both Sharpe ratio values and the return time series.

3. Results

The adaptive indices introduced above have been calculated for the entire investigated time period. We will focus on the time period corresponding to the subprime financial crisis, mainly for the period of 2007–2009. We will make use of the tests described above to study the performance of the adaptive indices at this time period, and compare it to what is observed for the original index.

3.1. Price and return comparison

Figure 1 shows the time series for both price and returns for the TA25 opposed to the S&P500 Influence Score and Bare Influence Score adaptive indices along with the original.

Studying the time series generated for both the price and returns of the indices, the Bare Influence adaptive index displays a very unique phenomenon in the Tel Aviv Stock Exchange analysis. The behavior of the price time series in the period from late 2007 to early 2009 is extremely different from what is seen in the time series for other indices. The time series exhibits in this period a very steep increase starting late in 2007, well before a noticeable occurrence is detectable in the time series of the original market index. Also, the returns reduce significantly in their absolute value from the middle of 2008, while other indices start showing an increase in the returns absolute value.

3.2. Statistical measures

Next, we study the statistical properties of the introduced index. In Fig. 1 we present the standard deviation of the adaptive indices, in comparison to the original indices. We observe that for the small market indices (TA25), the adaptive-indices show a steep rise in the STD value for a period starting in late 2007 – in a scale more noticeable than the original or the equal weights index. The random adaptive-index shows a great anomaly for that time, as expected larger than that of most of the indices, but surprisingly the Bare Influence Score adaptive-index shows a rise twice as steep and precisely in the mentioned time. Again, this does not occur for the larger market or in any of the sequential based indices.

Figure 3 shows the statistical measures (variance and mean) of the return time series in sliding time windows of 66 days. The small market indices (TA25) exhibits a very clear distinction from the large one in its behavior during the period between late in 2007 and early 2009. Taking a close look at the variance of sliding window (sliding by one day at a time) periods of 66 days, the Bare Influence Score adaptive index displays a gradual rise starting from March 2007. This is a period that is well before the peak of the crisis. This fact is more significant when compared with the other indices for the same market – both the other adaptive indices (Influence Score, Equal weights, Simple Correlation and Random weights) and the original index – which shows no significant change in the variance size throughout the period. In the larger market indices (S&P500) the Bare Influence Score index shows a greater variance as well, however a less significant anomaly is registered for the other indices as well. The anomaly in this market only occurs after (early 2009) the stock market crash, tough.

In the small market indices (TA25), the variance changes correlate with mean values, for the Bare Influence Score adaptive-index. As seen before (Fig. 1) the value of the return for the Bare Influence Score drops significantly after the crisis, which is also very visible in Fig. 3. The change between the mean values of this index for the small market is shown here to be dramatic much more than its corresponding counterpart for the large market or the sequential data, as shown in Appendix 1.

In support with these findings, we also show in Appendix 2 similar calculations made for a diminished set of randomly selected 30 stocks from the stocks composing the larger market index (S&P500). This is to test whether the behavior is a factor of the market, or simply a factor of having a small number of participating stocks. The results shown are more similar on average to the case of the larger market than the smaller market.

The sequential results show extreme consistency throughout the time series in both markets for both measurements between all new indices and the original (also shown in Appendix 1).

3.3. Correlation with the original index time series

Figure 4 shows the correlation of the Influence and Bare Influence Score adaptive indices correlation with the original index in the return time series, for a sliding window (sliding by one day) of 22 days. It is very
clear that for the small market indices, and the Bare Influence Score adaptive-index, the results show a distinction for the period in the time of the crisis, after early 2008. The S&P500 indices also show a growing rise in the correlation with the original index for the Bare Influence Score index, however – one that starts well before the crisis (middle of 2007) and lasts consistently for almost 5 years and therefore might not be considered a singular event.

The Equal weights and Simple Correlation weights adaptive indices both seem to be extremely correlated with the original index in both markets. The sequential calculations results also show a very similar high correlation with the original index.

As for the random adaptive-index, a distinctly high correlation can be seen in the S&P500 case for an extremely long period of time after the crisis. For the TA25 case, the behavior is not as noticeable, as shown in Fig. 5.

3.4. Beta values

Studying the values of the beta coefficients for the different indices further highlights the importance of the Bare Influence adaptive index. For the adaptive indices calculated, the results of the beta were more stable than the original index, and for the TA case the results were extremely stable, showing a very
high fluctuation after the crisis period (late 2008).

The results for the sequential cases, as can be seen in Appendix 1, were similar to the original results.

Figure 6 shows the average beta values for the stocks using the original index and the Bare Influence adaptive indices as benchmarks for both the TA25 and S&P500 cases, along with error bars that show the standard deviation of the beta values between the different stocks.

3.5. Sharpe ratio

As for the Sharpe Ratio value calculated for the adaptive indices, the Bare Influence Score adaptive index of the TA25 shows a very unique drop in its value in late 2008. A similar phenomenon is seen in the S&P500 adaptive index of the same kind, however only a few months later and not as distinct from the rest of the time period as the TA25 case. This can be seen in Fig. 7. The figure compares the Influence and Bare Influence Score values for both markets and shows the anomalies displayed in the Bare Influence case and not in the Influence case. The graphs for the Sharpe Ratio values of the control adaptive indices do not have the same dramatic changes.

Other than the phenomena detected for the Bare Influence Score adaptive-index, another phenomenon observed is the fact that the Sharpe Ratio for the adaptive indices seems to be periodic. We single out the Equal Weights adaptive-index and present its Sharpe Ratio to display this, for two reasons; first, the phenomenon is more clear in its values than in others, and second of all, the Equal Weights index can emulate a real life stock portfolio which takes the amount of assets of the investors and divides it equally in a selected set of stocks. Figure 8 shows the Sharpe Ratio values both in the time and frequency domain (power spectrum) for both markets to display its periodic behavior. Appendix 4 shows the results for the random weights indices as a reference.

4. Discussion

The premise behind the use of the Influence Score and Bare Influence Score values in the creation of a stock market adaptive-index was that the weight each stock gets as a component in the index price should not
be based on its market capitalization, but on the influence it has on other stocks. As explained before, the Bare Influence method for calculating this influence, removes the cohesion effect of the index on the stock market, thus allowing for a more accurate assessment of the proper weight the stock should get as component of the index.

Another major difference in the method we used to generate the adaptive-index opposed to the way the original stock market index is calculated, is its adaptive nature. The weights in the indices for which the distinct results occurred were recalculated daily, where the stock market original index has its weights recalculated only after a period of a few months. These results were not recreated for the indices calculated with the same weight generating method, using the Sequential calculation, for which weights were recalculated after a sequence of 66 days, similarly to the way weights change for the original index.

The dynamic adaptive-indices presented here, and specifically the Bare Influence adaptive-index, exhibit two trends related to the underlying dynamics of the stock markets; the statistical measures of the Bare Influence adaptive index in the TA25 case showed early signs of the stock market being influenced by outside financial stimulations, well before similar signs were shown in the adaptive index made up by stocks of the larger market index. Additionally, the behavior of the Sharpe Ratio of the equal weights adaptive-index, found to be periodic, can potentially be a practical phenomenon used in the future to optimize portfolio creation by investors.

Combining the two methods illustrated above was a determining factor in generating the Bare Influence adaptive-index. The statistical measures showing anomalies in the smaller, TA market, appeared well before the general stock market in NY was affected. The variance and mean of the returns showing the
increase, as well as the standard deviation for the specific period in late 2007 of the TASE are such anomalies. These results are further supported by the presence of stable values of the beta coefficient, particularly for the Bare Influence case, more specifically in the small market of the TA25 index. It should be noted that the removal of the index influence in the calculation of the weights also reduces the effect of market fluctuations. The results found in the TA market and not in the NY market show the effectiveness of this method on a smaller, less diversified market, rather than a large, stable and varied market. It further provides insights into the differences between these two since this issue was compared against a similar sample size for the NY case, this can be interpreted as being a signature effect of the nature of the TA stock market. This provides important information for the development of new monitoring tools for both markets. Furthermore, the introduced adaptive index methodolog-

Finally, the periodic nature of the Sharpe Ratio for an index made of equal weights distribution of stocks shown in Fig. 8, can be further examined as a measure for optimizing investment portfolios. As mentioned, the other adaptive-indices Sharpe Ratio has also shown a periodic nature, however not as prominently. Further-
Fig. 8. Sharpe Value of 132 days trading periods for the equal weights adaptive-indices in both the time and frequency domain and in both markets. (a) TA25 values of time series, (b) TA25 values of frequency domain, (c) S&P500 time series, (d) S&P frequency domain. The periodic behavior of the values is clearly displayed in these graphs.

more, unlike the Equal weights adaptive index, there is no available ETF for investing in these indices, whilst the Equal weights index is essentially an applicable portfolio an investor can place his capital in – simply distributing his investment equally between selected stocks. What the results shown in Fig. 8 showed, is that in both markets (however, more prominently in the larger NY market) such a portfolio, for trading periods of 132 days on a sliding window (by one day), has a Sharpe Ratio, which acts almost periodically. The frequency domain analysis made this feature observable.

In summary, the methods presented here provide the means to develop adaptive dynamic market indices. These will provide policy and decision-makers critical with information on the stability and dynamics of the market, and can provide further information on bubble formation and collapse. Future work will focus on additional markets, and the information provided by comparing the dynamics of the adaptive indices of different markets.

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References


Appendix 1 – Calculations made using the sequential weight method

As presented in Section 2.3, the adaptive indices are defined using a sliding window (by one day) approach. Here we focus on the differences between the results of the sliding window calculation methods used to generate the adaptive indices. In the moving window approach, the weights making up the index price from the stock prices are recalculated and changed daily, while in the sequential method, in which the weights are changed only once every 66 days. The sequential calculation method is more resembling of the current methods used to make up most of the original stock market indices, specifically the TA25 and S&P500 on which we focused in this research.

Equations 19 and 20 show how the sequential index calculations are made in comparison with the adaptive...
Fig. 10. The standard deviation of the sequential calculated indices in comparison to the original index for both markets: TA 25 (a & c) and S&P 500 (b & d) in the sequential case. Panels (a) and (b) show the Influence Score adaptive-index (blue) and the Bare Influence Score adaptive-index (red) with the original index (black). Panels (c) and (d) show the control adaptive-indices, random weights (blue) and equal weights (red). Summary: (a): TA25 Influence adaptive indices, (b): S&P500 Influence adaptive indices, (c): TA25 Control adaptive indices, (d): S&P500 Control adaptive indices.

Fig. 11. Statistical measures for the indices for sliding windows for the sequential case. (a) Variance for TA25, (b) Variance for S&P500, (c) Mean for TA25, (d) Mean for S&P500. The values are of the return time series of the various indices marked by abbreviation (Org – original, Equ – Equal weights, Simp – simple correlation, Inf – Influence Score, BlInf – Bare Influence Score).
Fig. 12. Average beta values for the stocks in the sample, with either the original index, or a new sequential index used as a benchmark. (a) TA25 Original index, (b) TA25 Bare Influence sequential index, (c) S&P500 Original index, (d) S&P500 Bare Influence sequential index.

Fig. 13. Price and return time series of the original indices and the influence score indices and bare influence indices of the randomly selected subsample set of the S&P500. (a) Influence Score adaptive-index return, (b) Bare Influence adaptive-index return, (c) Original index return, (d) Influence Score adaptive-index price, (e) Bare Influence adaptive-index price, (f) Original index price.
index calculations. Simply put, the Influence and Bare Influence values used to make up the weights for the index price are calculated in the same manner as the adaptive indices, but are kept constant for a given time period. This is given by:

\[
\text{influence score index}_{\text{sequential}} = \frac{\text{influence score}(\Delta t)}{\text{stock price}_j(t)}
\]  

(19)

and

\[
\text{bare influence score index}_{\text{sequential}} = \frac{\text{bare influence score}(\Delta t)}{\text{stock price}_j(t)}
\]  

(20)

where influence score and bare influence score are the values calculated in Equations 7 and 13, respectively, and \( \Delta t = 66 \). Thus, unlike in the moving window calculation method, here the weights are kept constant for the \( \Delta t \) period.

Figure 9 presents the return and price values for the sequential indices for both the TA25 and S&P500 indices, highlighting the crisis period of 2008 (see for comparison Fig. 1). We observe a strong similarity in price and in return value of the sequentially calculated indices with the original index, which is very different from the results obtained for the adaptive indices.

Figure 10 shows the standard deviation of the sequential calculated indices for both markets. Unlike the results shown for the adaptive indices the values of the Influence, Bare Influence and control indices (see Fig. 2), the Random and Equal weights are almost identical to the values of the original index, throughout the period in both markets.

Figure 11 shows the statistical measures of the sequential calculations made on both markets. The results are consistent throughout the time series in both markets for both measurements between all new indices and the original, different (especially for the TA25 case) from the adaptive index results.

Figure 12 shows the average beta values of the stocks studied in the sequential case. The graphs also show error bars indicating the standard deviation between the beta values of the different stocks. Unlike the adaptive case, the results for the sequential indices are shown to be very similar in stability and value to the original case.
Fig. 15. Stock composition of the TA25 index of the period studied in this work. Blue means the stock was included, red means the stock was not.
Appendix 2 – Calculations on a diminished set of S&P500 stocks

In order to test various statistical values (such as the mean and variance of the adaptive indices) we made calculations using a subsample set of 30 stocks randomly selected from the stocks making up the S&P500 index as if it were a small market separate from the NYSE. The goal for this test was to examine whether the difference between the two markets results was because of having fewer stocks in the sample (in which case the results of this test would be closer to the TA25 results) or because the stocks came from markets of varying size (in which case the results would be closer to the S&P500 results). Figure 13 presents the return and price values for the index made using this method.

Figure 14 shows an example of calculations made on a randomly selected set of 30 stocks compared with the calculations made for the adaptive indices on both markets, as shown in the article. Looking closely at the scale, the results for the diminished set are closer in value to the results of the S&P500 results, than the TA25 results.

Appendix 3 – TA 25 Composition timeline

Figure 15 shows the composing stocks of the TA25 index over the period studied in this work. The frequent changes make clear the difficulty in keeping up with these changes and using the current stocks consistently in this type of work.

Appendix 4 – Time and frequency domain display of random weights indices

Figure 16 shows the time and frequency domains of the Sharpe Ratio of the indices made of randomly selected weights for both the TA 25 case and the S&P 500. It is clear from both of these, that the value of the Sharpe Ratio in the random case is not of a single component periodic nature as seen in the Equal weights indices, and less prominently in the Influence and Bare Influence indices. The time series of the values shown here is far noisier and made in a composition from several periodic components.