Algorithmic Finance 2 (2013) 99–109
DOI 10.3233/AF-13018
IOS Press

The strategy approval decision: A Sharpe ratio indifference curve approach

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Abstract. The problem of capital allocation to a set of strategies could be partially avoided or at least greatly simplified with an appropriate strategy approval decision process. This paper proposes such a procedure. We begin by splitting the capital allocation problem into two sequential stages: *strategy approval* and *portfolio optimization*. Then we argue that the goal of the second stage is to beat a naïve benchmark, and the goal of the first stage is to identify which strategies improve the performance of such a naïve benchmark. We believe that this is a sensible approach, as it does not leave all the work to the optimizer, thus adding robustness to the final outcome.

We introduce the concept of the *Sharpe ratio indifference curve*, which represents the space of pairs (candidate strategy's Sharpe ratio, candidate strategy's correlation to the approved set) for which the Sharpe ratio of the expanded approved set remains constant. We show that selecting strategies (or portfolio managers) solely based on past Sharpe ratio will lead to suboptimal outcomes, particularly when we ignore the impact that these decisions will have on the average correlation of the portfolio. Our *strategy approval theorem* proves that, under certain circumstances, it is entirely possible for firms to improve their overall Sharpe ratio by hiring portfolio managers with negative expected performance. Finally, we show that these results have important practical business implications with respect to the way investment firms hire, layoff and structure payouts.

JEL classifications: C02, G11, G14, D53.

Keywords: Portfolio theory, Sharpe ratio, pairwise correlation, indifference curve, diversification, free call option.

1. Introduction

The problem of allocating capital to Portfolio Managers (PMs) or strategies is typically addressed using a variation of Markowitz's (1952) approach.

The views expressed in this publication are the authors' and do not necessarily reflect the opinion of Tudor Investment Corporation. We would like to thank the Managing Editor of *Algorithmic Finance*, Philip Maymin (New York University-Polytechnic Institute), as well as two anonymous referees, for their insightful comments during the peer-review process. We are grateful to Tudor Investment Corporation, Marco Avellaneda (Courant Institute of Mathematical Sciences, New York University), José A. Blanco (UBS), Peter Carr (Morgan Stanley, New York University), José A. Gil Fana (Universidad Complutense de Madrid), David Leinweber (Lawrence Berkeley National Laboratory), Attilio Meucci (Kepos Capital, New York University), Riccardo Rebonato (PIMCO, University of Oxford), José M. Riobóo (Universidad de Santiago de Compostela), Piedad Tolmos (Universidad Juan Carlos I), Luis Viceira (Harvard Business School) and José L. Vilar Zanón (Universidad Complutense de Madrid).

Supported in part by the Director, Office of Computational and Technology Research, Division of Mathematical, Information, and Computational Sciences of the U.S. Department of Energy, under contract number DE-AC02-05CH11231.

This method is agnostic as to the criterion employed to pre-select those PMs. In this paper we will show that the standard procedure used by the investment management industry to hire and layoff PMs may indeed lead to suboptimal capital allocations.

In a series of papers, Sharpe (1966, 1975, 1994) introduced a risk-adjusted measure of investment's performance. This measure, universally known as the Sharpe ratio (SR), has become the gold-standard to evaluate PMs in the investment management industry. It is well known that the Sharpe ratio is the right selection criterion if the investor is restricted to picking only one investment, i.e. when maximum concentration is mandated (Bodie et al., 1995). However, the Sharpe ratio is not necessarily a good criterion when a sequence of individual decisions must be made, such as hiring an additional PM or adding an investment strategy to an existing fund. Most firms address this sequential decision making problem by requiring any candidate manager or strategy to pass several fixed thresholds, including SR (De Souza & Gokcan, 2004) and track record length (Bailey & Lopez de Prado, 2012). Among the PMs or strategies that have passed those thresholds, capital is then allocated following an optimization process.

This implies that the capital allocation process is in practice composed of two distinct stages: *approval* and *optimization*. The problem is, these two stages are carried out independently, potentially leading to incoherent outcomes. Selecting a strategy because it passes a certain SR threshold ignores the candidate strategy's correlation to the set of already existing strategies. The consequence of this incoherence between the approval and optimization stages is that the overall outcome of the capital allocation process may be suboptimal, regardless of the optimization applied in the second stage. For example, approving a strategy with lower SR may introduce more diversification than another strategy with higher SR but also higher correlation to the approved set.

Before considering a candidate strategy for approval, it is critical to determine not only its expected SR, but also its average correlation against the approved set of strategies. A first goal of this paper is to demonstrate that *there is no fixed SR threshold which we should demand for strategy approval*. We must define an Approval benchmark which jointly looks at the candidate's SR *and* how it fits in the existing menu of strategies. This benchmark must be naïve, in the sense that it is pre-optimization. Equal Volatility Weighting is a procedure that has been used in past to benchmark portfolio optimization results (for example, DeMiguel et al., 2009), and this is the one we adopt in our framework.

A second goal of this paper is to formalize the trade-off between a candidate's SR and its correlation to the existing set of strategies, a concept we call the *Sharpe ratio indifference curve*. We will often find situations in which a highly performing candidate strategy should be declined due to its high average correlation with the existing set. Conversely, a low performing candidate strategy may be approved because its diversification potential offsets the negative impact on the average SR. Looking at the combined effect that a candidate's SR and correlation will have on the approved set also addresses a fundamental critique to the "fixed SR threshold" approach currently applied by most investment firms: Such fixed threshold tends to favor higher over lower frequency strategies. But considering the (low) correlation that the latter strategies have with respect to the former, lower frequency strategies will have a fairer chance of being approved under the new approach hereby presented. Our *strategy approval theorem* proves that, under certain circumstances, it is entirely possible for firms to improve their overall SR by hiring PMs with negative expected performance.

A third goal of this paper is to explain how this new strategy approval decision process could lead to new business arrangements in the investment management industry. Emulating the performance of "star-PMs" through a large number of uncorrelated low-SR PMs creates the opportunity for investment firms to internalize features that cannot be appropriated by the individual PMs.

It is worth noting that our use of the term benchmark should not be interpreted in the sense of Jensen (1968), Sortino and van der Meer (1991), Treynor (1966) or Treynor and Black (1973). Our motivation is to define a high performance level which, being the result of a naïve procedure, must be *subsequently improved* by any optimization. Tobin (1958) proposed separating the portfolio construction problem into two steps: optimization of risky assets and the amount of leverage. Like Tobin's, our approach also separates the allocation problem into two stages, but as we will see, both methods are substantively different.

Our results are applicable to a wide range of firms faced with the problem of hiring PMs and allocating them capital, including hedge funds, funds of hedge funds, proprietary trading firms, mutual funds, etc. Although some features specific to these investment vehicles could be integrated in our analysis (e.g., non-Normality of returns, lock-up periods, rebalance frequency, liquidity constraints, etc.), we have not done so in order to keep the framework as generally applicable as possible.

The rest of this paper is structured as follows: Section 2 presents a few propositions on the naïve benchmark's performance. Section 3 introduces the concept of the *SR indifference curve* (strategy approval theorem). Section 4 makes a specific proposal for the process of approving strategies. Section 5 suggests a new business arrangement based of this strategy approval process. Section 6 summarizes our conclusions. The appendices present the mathematical proofs to these propositions.

2. Propositions

Capital allocation to PMs or strategies is typically done without consideration of the strategy approval process or hiring criteria (L'habitant, 2004). This means that the portfolio optimization step may have to deal with PMs or strategies pre-selected according to a set of rules that lead to suboptimal capital allocations, like hiring PMs with investment strategies similar to those already in the platform. This paper is dedicated to show how to approve strategies (or hiring) in a manner that is consistent with the goal of portfolio optimization. We do so by dividing the capital allocation problem into two sequential stages. First, we define a naïve benchmark that must be raised at each hiring. Second, we optimize a portfolio composed of strategies that have passed that naïve benchmark. The output of the second step must beat the output of the first (naïve) step. In this way, we avoid leaving the entire decision to optimization techniques that have been criticized for their lack of robustness Best and Grauer (1991).

The following propositions differ from standard portfolio theory (including Tobin (1958)) in a number of ways:

1. They discuss the allocation of capital across strategies or PMs, rather than assets.
2. They are based on the establishment of an Equal Volatility Weightings (or *naïve*) benchmark.
3. This benchmark allows us to split the capital allocation problem into two sequential subproblems:
   a. **Strategy Approval**: The process by which a candidate strategy is approved to be part of a portfolio.
   b. **Portfolio Optimization**: The process that determines the optimal amount of capital to be allocated to each strategy within a portfolio.
4. Key principles of the approach discussed here are:
   a. The goal of the *Portfolio Optimization* process is to beat the performance of a naïve benchmark.
   b. The goal of the *Strategy Approval* process is to raise the performance of the naïve benchmark as high as possible (ideally, to the point that no portfolio optimization is required at all!).

2.1. Benchmark portfolio

2.1.1. Statement

The performance of an Equal Volatility Weights benchmark (SR_B) is fully characterized in terms of:

1. Number of approved strategies (*S*).
2. Average SR among strategies $(\overline{SR})$.
3. Average off-diagonal correlations among strategies $(\bar{\rho})$.

In particular, adding strategies (*S*) with the same $\overline{SR}$ and $\bar{\rho}$ does improve SR_B.

Sections A.1 and A.2. prove this statement.

2.1.2. Example

Following Eq. (7) in Appendix 2, it will take 16 strategies with $\overline{SR} = 0.75$ and $\bar{\rho} = 0.2$ to obtain a benchmark with SR of 1.5. Should the average individual risk-adjusted performance decay to $\overline{SR} = 0.5$, the benchmark's SR will drop to 1. Figure 1 illustrates the point that, if on top of that performance degradation the individual pairwise correlation raises to $\bar{\rho} = 0.3$, the benchmark's SR will be only 0.85.

2.1.3. Practical implications

This proposition allows us to estimate the benchmark's SR without requiring knowledge of the individual strategy's SRs or their pairwise correlations. Average volatility is not a necessary input. All that is needed is *S*, $\overline{SR}$, and $\bar{\rho}$. This makes possible the simulation of performance degradation or correlation stress-test scenarios, as illustrated in the previous epigraph.

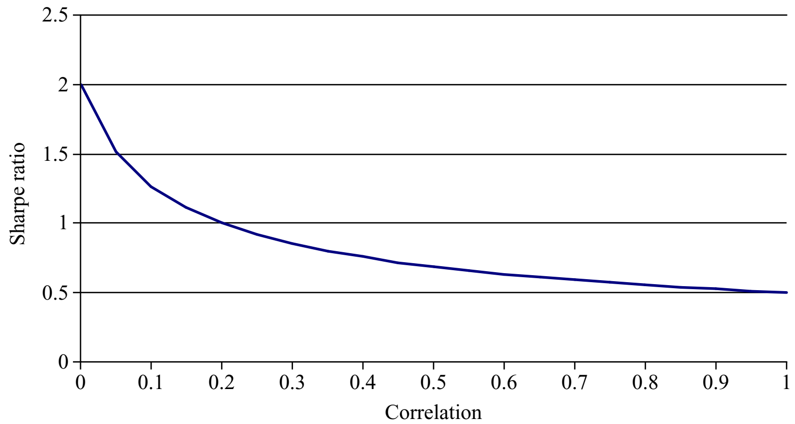

Fig. 1. Benchmark SR as a function of the average correlation. Figure 1 plots the benchmark SR for $S = 16$ strategies with $\overline{SR} = 0.5$, as a function of the average correlation.

2.2. On performance degradation

2.2.1. Statement

The benchmark SR is a *linear* function of the average SR of the individual strategies, and a decreasing convex function of the number of strategies and the average pairwise correlation. This means that, as the number of strategies (*S*) increases, favoring low $\bar{\rho}$ offers a convex payoff which $\overline{SR}$ does not. In the presence of performance degradation, low correlated strategies may be preferable to (supposedly) highly performing ones.

Section A.3 proves this statement.

2.2.2. Example

Figure 2(a) shows the benchmark's SR as a linear function of $\overline{SR}$ for 5 and 25 strategies. Figure 2(b) shows the benchmark's SR as a convex function of $\bar{\rho}$ for 5 and 25 strategies (see Eqs. (8)–(9) in Appendix 3).

2.2.3. Practical implications

This is a critical result. It implies that we may prefer low correlated strategies, even if underperforming, to outperforming but highly correlated strategies. The exact trade-off between these two characteristics will become clearer in Section 3.

2.3. On the maximum achievable benchmark SR

2.3.1. Statement

There is a limit to how much the benchmark SR can be improved by adding strategies. In particular, that limit is fully determined by:

1. Average SR among strategies ($\overline{SR}$).
2. Average off-diagonal correlations among strategies ($\bar{\rho}$).

Section A.4 proves this statement.

2.3.2. Example

Suppose that $\overline{SR} = 0.75$ and $\bar{\rho} = 0.2$. Regardless of how many equivalent strategies are added, the benchmark's SR will not exceed 1.68 (Fig. 3). Higher SRs could still be obtained with a skillful (non-naïve) portfolio optimization process, but are beyond the benchmark's reach (see Eqs. (11)–(12) in Appendix 4).

2.3.3. Practical implications

In the absence of $\overline{SR}$ degradation, it would make little sense increasing the number of strategies (*S*) beyond a certain number. But since $\overline{SR}$ degradation is expected, there is a permanent need for building an inventory of *replacement strategies* (to offset for those decommissioned due to performance degradation or approval error (false positive)). This is consistent with Proposition 2, which offered a theoretical justification for researching as many (low correlated) strategies as possible (the convex payoff due to correlation).

2.4. On the impact of a candidate strategy on the benchmark's SR

2.4.1. Statement

A strategy being considered for approval would have an impact on the benchmark's SR (and thus its naïve targeted performance) that exclusively depends on:

1. Number of approved strategies (*S*).
2. Average SR among strategies ($\overline{SR}$).

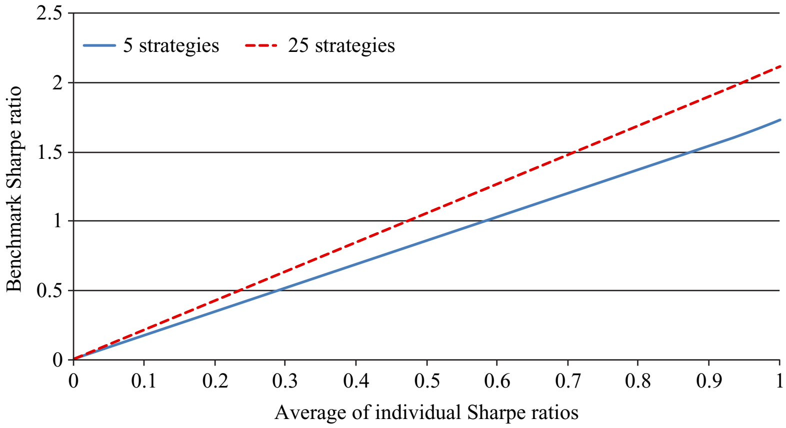

Fig. 2(a). Benchmark SR as a function of $\overline{SR}$. Figure 2(a) demonstrates the linear impact that $\overline{SR}$ has on the benchmark SR.

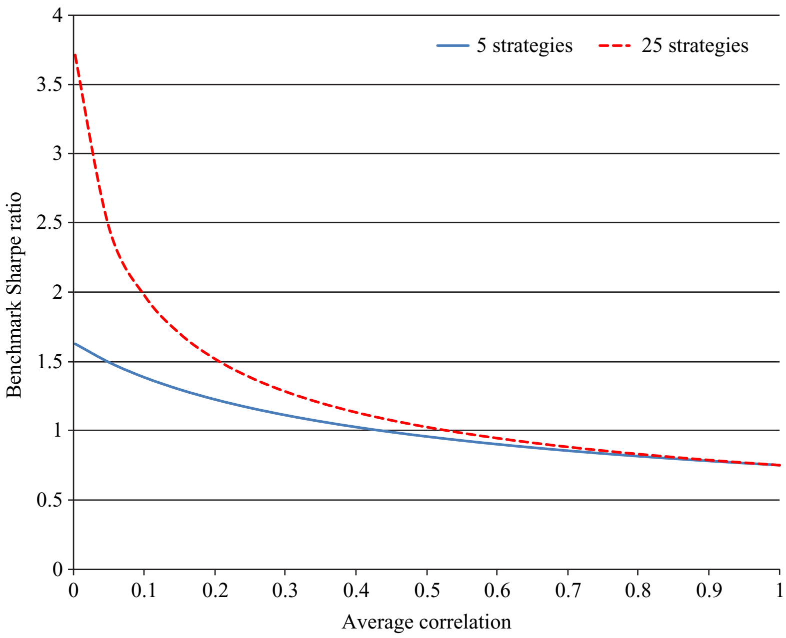

Fig. 2(b). Benchmark SR as a function of $\bar{\rho}$. Figure 2(b) demonstrates the convex impact that $\bar{\rho}$ has on the benchmark SR.

3. Average off-diagonal correlations among strategies ($\bar{\rho}$).
4. Average correlation of the candidate strategies against the approved set ($\bar{\rho}_{S+1}$).
5. The candidate strategy's SR (SR_{S+1}).

Section A.5 proves this statement.

2.4.2. *Example*

Suppose it is the case that $S = 2$, $\overline{SR} = 1$, $\bar{\rho} = 0.1$, thus $SR_B = 1.35$. Consider a third strategy with $SR_3 = 1$ and $\bar{\rho}_3 = 0.1$. Then, applying Eq. (13) in Appendix 5, $SR_B = 1.58$. Adding the third strategy positively impacted the benchmark's SR, even though there was no improvement on $\overline{SR} = 1$, $\bar{\rho} = 0.1$. We knew this from Proposition 1.

Let's turn now to the case where $SR_3 = 0.7$ and $\bar{\rho}_3 = 0.1$. Then, $SR_B = 1.42$. If $SR_3 = 0.7$ but $\bar{\rho}_3 = 0.2$, then $SR_B = 1.35$. Note how adding a "worsening" strategy (which lowers $\overline{SR}$ and increases $\bar{\rho}$) nevertheless did not reduce SR_B, thanks to the diversification gains. We were able to calculate these

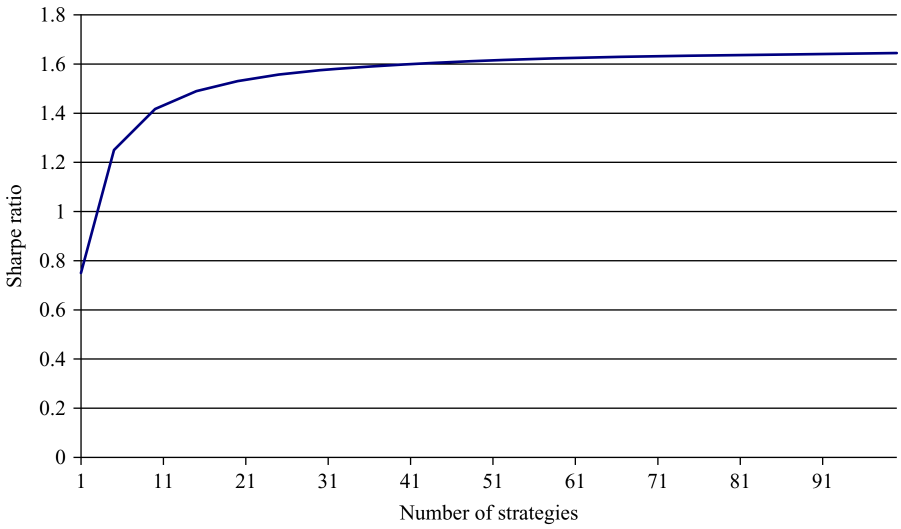

Fig. 3. SR of a portfolio of approved strategies with $\overline{SR} = 0.75$ and $\bar{\rho} = 0.2$. Figure 3 shows how the benchmark Sharpe ratio increases as a result of new strategies being added (keeping constant $\overline{SR} = 0.75$ and $\bar{\rho} = 0.2$).

scenarios without requiring additional knowledge regarding the strategy's risk or pairwise correlations.

2.4.3. *Practical implications*

Proposition 4 shows that $\bar{\rho}_{S+1}$ and SR_{S+1} suffice to determine the new benchmark's SR. In particular, we do not need to know each pairwise correlation, individual SRs or strategies' volatilities, which greatly simplifies simulation exercises.

3. The SR indifference curve (strategy approval theorem)

The previous propositions converge into the following fundamental result.

3.1. *Statement*

There exists a trade-off such that we would be willing to accept a strategy with below average SR if its average correlation to the approved set is below a certain level. This determines an indifference curve as a function of:

1. Number of approved strategies (*S*).
2. Average SR $(\overline{SR})$.
3. Average off-diagonal correlations $(\bar{\rho})$.
4. The candidate strategy's SR (SR_{S+1}).
5. The SR of the benchmark portfolio (SR_B).

where the acceptance threshold in terms of correlation is

$$\bar{\rho}_{S+1} = \frac{1}{2}\left[\frac{(\overline{SR}\cdot S + SR_{S+1})^2}{S\cdot {SR_B}^2} - \frac{S+1}{S} - \bar{\rho}(S-1)\right] \quad (1)$$

Section A.6 proves this statement.

3.2. *Example*

Suppose the same case as in Proposition 4, namely that $S = 2$, $\overline{SR} = 1$, $\bar{\rho} = 0.1$, thus $SR_B = 1.35$. A third strategy with $SR_3 = 1$ and $\bar{\rho}_3 = 0.1$ would lead to $SR_B = 1.58$. The theorem says that, should an alternative third strategy deliver $SR_3 = 1.5$ instead, we would be indifferent for $\bar{\rho}_3 = 0.425$ (see Eq. (1)). Beyond that correlation threshold, the alternative with *higher* SR (i.e., $SR_3 = 1.5$) should be *declined*. Figure 4 shows the entire indifference curve.

More interestingly, we would also be indifferent to a second alternative whereby $SR_3 = -0.1$ and $\bar{\rho}_3 = -0.439$. But why would we ever approve a strategy that very likely will not make any money? Why would a firm hire a PM that loses money? This probably sounds counter-intuitive, but that's where the previous math becomes helpful. The reason is, we are investing in 3 strategies. Overall, we will still have a quite positive

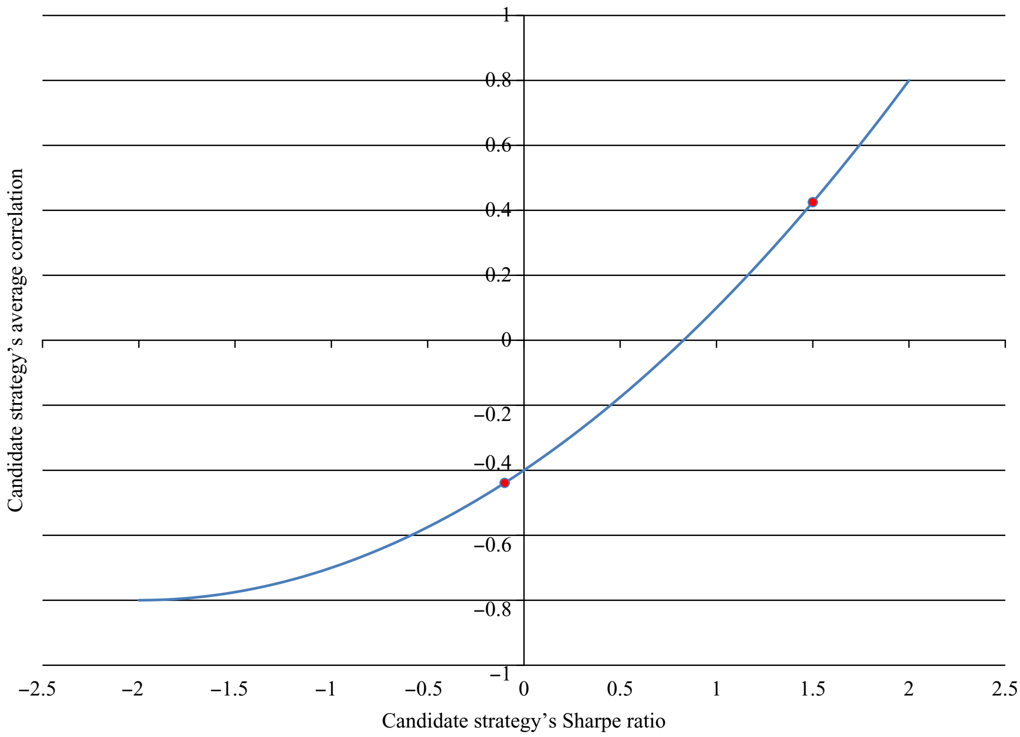

Fig. 4. The Sharpe ratio indifference curve. Indifference curve between a candidate strategy's SR and its average correlation to the approved set (examples marked with red dots).

return. True that this overall return would be *slightly* greater without the third strategy, however without it the standard deviation would also be *much* larger. All things considered, if that third strategy incorporates an average correlation below $\bar{\rho}_3 = -0.439$, it improves the overall SR beyond $SR_B = 1.58$. In this particular example, the third strategy would behave like a call option at a premium equivalent to the $\sigma_3 SR_3$ it costs (in terms of returns) to "buy" it. Naturally, strategies with a $\bar{\rho}_3 = -0.439$ average correlation may be hard to find, but if they presented themselves, we should consider them.

Finally, suppose that the first of the three alternatives is added ($SR_3 = 1,\ \bar{\rho}_3 = 0.1$). This will in turn shift the indifference curve to the left and up (see Fig. 5). It means that pairs of $(SR_{S+1}, \bar{\rho}_{S+1})$ that fall between the red and the blue curve have now become acceptable. As the set of approved strategies pools more risk, it is able to clear room for previously rejected strategies without reducing the benchmark's overall SR.

3.3. *Practical implications*

For every candidate strategy, there exists an infinite group of alternative theoretical candidates whereby all deliver the same benchmark SR. The indifference curve represents the exact trade-off between a candidate strategy's SR and its average correlation against the approved set such that the benchmark's SR is preserved.

This theorem addresses a problem faced by most investment firms: The "fixed SR threshold" strategy selection criterion represents a considerable hurdle for the lower frequency strategies. These strategies tend to have a lower annualized SR, but they also bring lower average correlations, with an overall improvement in diversification. This approach finds the balance between both components, allowing for low frequency strategies to be acceptable under an objective set of requirements.

4. Proposal for a coherent strategy approval process

Before considering a candidate strategy for approval, it is critical to determine not only its expected SR, but also its average correlation against the approved set of strategies. The above results imply that *there is no fixed SR threshold which we should demand*

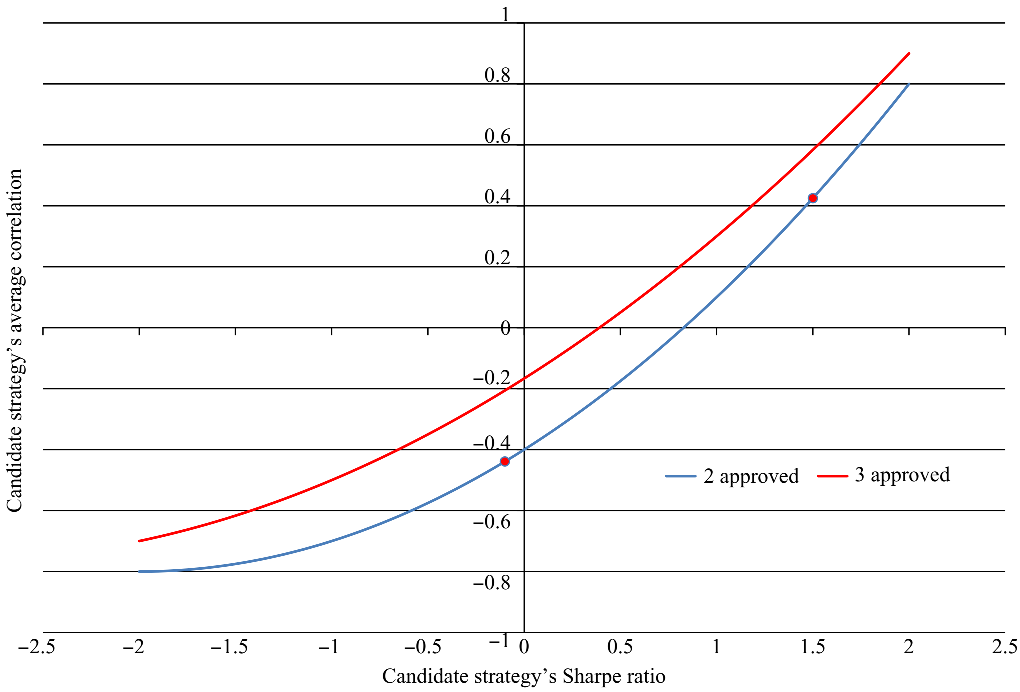

Fig. 5. Sharpe ratio indifference curve dynamics. The indifference curve is not static, and as more risk is pooled, some of the previously rejected strategies become acceptable.

for strategy approval. We must jointly look at the candidate's SR *and* how it fits in the existing menu of strategies by considering:

1. Number of approved strategies (S).
2. Average SR among strategies ($\overline{SR}$).
3. Average off-diagonal correlations among strategies ($\bar{\rho}$).
4. The candidate strategy's SR (SR_{S+1}).
5. Average correlation of the candidate strategies against the approved set ($\bar{\rho}_{S+1}$).

A realistic backtest would reflect transaction costs and market impact when estimating SR_{S+1}, thus incorporating a capacity penalty in this analysis. With these inputs we can then compute SR_B (without the candidate strategy), SR_B^* (including the candidate strategy), and given SR_{S+1} for what $\bar{\rho}_{S+1}$ it occurs that $SR_B = SR_B^*$ (indifference point).

We will often find situations in which a highly performing candidate strategy should be declined due to its high average correlation with the existing set. Conversely, a low performing candidate strategy may be approved because its diversification potential offsets the negative impact on the average SR.

It is important to note that the input variables do not need to be restricted to historical estimates, but can reflect forward looking scenarios. This makes it possible to reset approval thresholds under alternative assumptions on capacity, future correlation, performance degradation, etc.

5. Business implications

Far from being solely a theoretical result, these propositions have a number of very practical business implications. Funds typically pay PMs a percentage of the net profits generated by their strategies. PMs do not share a percentage of the losses, which gives them exposure to the upside only. Funds are therefore writing what is called a *"free call option"* (Gregoriou et al., 2011; Ineichen, 2003; L'habitant, 2004). The true value of the option is proportional to the risks associated with a PM's strategy. The better the PM's strategy, the lower the probability of losses, therefore the cheaper the option given away by the Fund. Conversely, the option offered to an unskilled PM is extremely expensive. For this reason, funds do not evaluate a PM's performance in terms of average annual return, as that would not take into account the risks involved and would lead to offering the option to the wrong PMs.

The core argument presented in this paper – that SR is a misleading index of whom a fund should hire or fire – seems at odds with standard business practices. The *SR indifference curve* shows that even PMs with a negative individual SR should be hired if they contribute enough diversification. Why is that not the case? Because of a *netting* problem: a typical business agreement is that PMs are entitled to a percentage of their individual performance, not a percentage of the fund's performance. Legal clauses may release the fund from having to pay a profitable PM if the overall fund has lost money, however that PM is unlikely to remain at the firm after a number of such events. This is a very unsatisfactory situation, for a number of reasons: First, funds are giving up the extra-performance predicted by the *SR indifference curve*. Second, funds are compelled to hire 'star-PMs', who may require a high portion of the performance fee. Third, funds are always under threat of losing to competitors their 'star-PMs', who may leave the firm with their trade secrets for a slightly better deal. In some firms, PMs' turnover is extremely high, with an average tenure of only one or two years.

A way to avoid this suboptimal outcome is to offer a business deal that pays the PM a percentage of the fund's overall performance. This would again create some tensions, as some 'star-PMs' could do better with their individual deals. However, Section 2.3 tells us that we can emulate the performance of a 'star-PM' by hiring a sufficient number of 'average-PMs' with low correlation to the fund's performance. A first advantage of doing so is that 'average-PMs' have no bargaining power, thus we can pay them a lower proportion of the performance fee. A second advantage is that, because of the relatively low SR, they are unlikely to be poached. A third advantage is that, if we hire 'average-PMs' whose performances have low correlation to the fund's, we can internalize a private value to which the PMs have no access (they cannot leave and "take" the low correlation to the firm with them). The average-PM's performance may exhibit a low correlation to a limited number of funds', but not to all. In other words, the fund can capture the extra-performance postulated by the *SR indifference curve* without having to pay for it.

A future can therefore be envisioned in which investment firms structure payments with the following features:

- Payout is arranged in terms of funds' overall performance, which may be superior to that of 'star-driven' funds.
- The hiring process targets PMs with relatively low SRs (in some cases even below zero, if their correlation is sufficiently negative), and therefore cheaper to find, keep and replace.
- There is a very low turnover of PMs, as they cannot take the 'low correlation to the fund' with them, and their low SR does not get them an individual deal.

This kind of business arrangement is particularly suitable to firms that engage in algorithmic strategies, because the prerequisite of 'sufficient number of average-PMs' can be easily fulfilled with average-performing trading systems. Since the SR required to put each system in production will be relatively low, they can be developed in large numbers. As long as each quant developer is involved in a limited number of those systems, their bargaining power will still be limited.

6. Conclusions

Ideally, if an investment firm counted with virtually uncorrelated strategies, no optimization would be required at all. Although an unrealistic scenario, it is nonetheless true that many of the problems associated with portfolio optimization could be avoided, to a great extent, with a proper procedure of strategy approval. The procedure discussed in this paper goes in that direction.

We have divided the capital allocation problem in two sequential phases: strategy approval and portfolio optimization. The goal of the strategy approval phase is to raise the naïve benchmark's performance, reducing the burden typically placed on the portfolio optimization phase. We have demonstrated that there is no fixed SR threshold that we should demand for strategy approval. Instead, there is an indifference curve of pairs (candidate strategy's SR, candidate's correlation to approved set) that keep the benchmark's SR constant. At the extreme, it may be preferable to approve a candidate's strategy with negative Sharpe if its correlation to the approved set is sufficiently negative.

These results are particularly relevant in the context of performance degradation, as they demonstrate that selecting strategies (or PMs) solely based on past SR may lead to suboptimal results, especially when we ignore the impact that these decisions will have on the average correlation of the portfolio. The practical implication is that firms could emulate the performance of "star-PMs" through uncorrelated low-SR PMs, who will not have individual bargaining power. This theoretical framework justifies setting up a legal compensation structure based on overall fund performance, which internalizes a private value to which the PMs have no access, namely their low correlation to the platform.

Appendix

A.1. Definitions and standing hypothesis

Suppose a collection of S strategies, jointly-distributed as a multivariate Normal. The marginal distribution of excess returns is

$$r_s \sim N(\mu_s, \sigma_s^2), \quad for\ s = 1, \ldots, S \tag{2}$$

The portfolio composed of these S strategies, characterized by a vector of weightings $\{\omega_s\}$, has excess returns *r*, which follow a distribution

$$r \sim N\left(\sum_{s=1}^{S} \omega_s \mu_s, \sum_{s=1}^{S}\left(\omega_s^2 \sigma_s^2 + 2\sum_{t=s+1}^{S} \omega_s \omega_t \rho_{s,t}\right)\right) \tag{3}$$

The SR for such portfolio can then be computed as

$$SR = \frac{\sum_{s=1}^{S} \omega_s \mu_s}{\sqrt{\sum_{s=1}^{S}\left(\omega_s^2 \sigma_s^2 + 2\sum_{t=s+1}^{S} \omega_s \omega_t \rho_{s,t}\right)}} \tag{4}$$

We would like to investigate the variables that affect the risk-adjusted performance of such portfolio of strategies.

A.2. Benchmark portfolio

Let's set the benchmark portfolio to be the result of a naïve equal volatility weighting allocation,

$$\omega_s = \frac{1}{S\sigma_s}, \quad for\ s = 1, \ldots, S \tag{5}$$

Then, it is immediate to show that the SR of this benchmark portfolio is

$$SR_B = \overline{SR}\sqrt{\frac{S}{1 + \frac{2}{S}\sum_{s=1}^{S}\sum_{t=s+1}^{S} \rho_{s,t}}} \tag{6}$$

where $\overline{SR} = \frac{1}{S}\sum_{s=1}^{S} SR_s = \frac{1}{S}\sum_{s=1}^{S} \frac{\mu_s}{\sigma_s}$, the average SR across the strategies. The average correlation across off-diagonal elements is $\bar{\rho} = \frac{2\sum_{s=1}^{S}\sum_{t=s+1}^{S} \rho_{S,t}}{S(S-1)}$. We can compute the SR of the benchmark portfolio as

$$SR_B = \overline{SR}\sqrt{\frac{S}{1 + (S-1)\,\bar{\rho}}} \tag{7}$$

A.3. Sensitivity to performance degradation

Let's compute the partial derivative of Eq. (7) with respect to $\overline{SR}$ and $\bar{\rho}$

$$\frac{\partial SR_B}{\partial \overline{SR}} = \sqrt{\frac{S}{1 + (S-1)\,\bar{\rho}}} = \frac{SR_B}{\overline{SR}} \tag{8}$$

$$\frac{\partial SR_B}{\partial \bar{\rho}} = -\frac{\overline{SR}}{2}(S-1)\sqrt{S\,(1 + (S-1)\,\bar{\rho})} \tag{9}$$

$$\frac{\partial SR_B}{\partial \overline{SR}\partial \bar{\rho}} = -\frac{S-1}{2}\sqrt{S\,(1 + (S-1)\,\bar{\rho})} \tag{10}$$

Therefore, SR_B is a linear function of the average performance degradation, but a decreasing convex function of the average correlation increase.

A.4. Diversification

It is interesting to discuss diversification in the context of this benchmark portfolio because we do not assume a skillful capital allocation process. If the capital allocation process is skillful and $\bar{\rho} < 1$, then the portfolio's Sharpe ratio (*SR*) will surely beat the benchmark's (SR_B). However, if $\bar{\rho} = 1$, then $SR = SR_B = \overline{SR}$ and the capital allocation process cannot benefit from diversification.

We apply Taylor's expansion on Eq. (7) with respect to *S*, to the first order.

$$\Delta SR_B = \frac{\partial SR_B}{\partial S}\Delta S + \sum_{i=2}^{\infty} \frac{\frac{\partial^i SR_B}{\partial S^i}}{i!}\Delta S^i$$
$$\approx \frac{\overline{SR}\,(1-\bar{\rho})}{2\sqrt{S}\,[(1 + (S-1)\,\bar{\rho})]^{\frac{3}{2}}}\Delta S \tag{11}$$

Only when $\bar{\rho} = 0$, the SR can be expanded without limit by increasing *S*. But otherwise, SR gains become gradually smaller until eventually SR_B converges to the asymptotic limit

$$\lim_{S\to\infty} SR_B = \frac{\overline{SR}}{\sqrt{\bar{\rho}}} \tag{12}$$

A.5. Impact of candidate strategies on the benchmark

Equation (12) tells us that the *maximum* SR for the benchmark portfolio is a function of two variables: The average Sharpe ratio among strategies ($\overline{SR}$) and the average off-diagonal correlation ($\bar{\rho}$). It also shows that we should accept a below-average SR strategy if it adds diversification. Going back to Eq. (7), the value of SR_B after adding a new strategy can be updated as

$$SR_B^* = \frac{\left(\overline{SR}\cdot S + SR_{S+1}\right)}{\sqrt{S[\bar{\rho}(S-1) + 2\bar{\rho}_{S+1} + 1] + 1}} \tag{13}$$

where

- SR_{S+1} is the SR associated with the candidate strategy.
- $\bar{\rho}_{S+1} = \frac{1}{S}\sum_{s=1}^{S}\rho_{s,S+1}$.
- $\rho_{s,S+1}$ are the pairwise correlations between the candidate strategy and the set of *S* approved strategies.

A.6. Indifference curve and strategy approval

From Eq. (13), we can isolate an indifference curve for preserving the benchmark's SR, i.e. that imposes the condition $SR_B^* = SR_B$.

$$\bar{\rho}_{S+1} = \frac{1}{2}\left[\frac{(\overline{SR}\cdot S + SR_{S+1})^2}{S\cdot {SR_B}^2} - \frac{S+1}{S} - \bar{\rho}(S-1)\right] \tag{14}$$

This in turn leads to

$$\frac{\partial\bar{\rho}_{S+1}}{\partial SR_{S+1}} = \frac{\overline{SR}\cdot S + SR_{S+1}}{S\cdot {SR_B}^2} \tag{15}$$

And inserting Eq. (7) we derive the equilibrium condition,

$$\frac{\partial\bar{\rho}_{S+1}}{\partial SR_{S+1}} = \frac{(\overline{SR}\cdot S + SR_{S+1})(1+(S-1)\bar{\rho})}{S^2\cdot \overline{SR}^2} \tag{16}$$

References

Bailey, D., Lopez de Prado, M., 2012. The Sharpe ratio efficient frontier. J. Risk 15 (2), http://ssrn.com/abstract=1821643.

Bodie, Z., Kane, A., Marcus, A., 1995. Investments, third ed. Irwin Series in Finance, McGraw-Hill Companies.

Best, M., Grauer, R., 1991, January. On the sensitivity of Mean-Variance-Efficient portfolios to changes in asset means: Some analytical and computational results. Rev. Financ. Stud. 4 (2), 315–342.

DeMiguel, V., Garlappi, L., Uppal, R., 2009, May. Optimal versus Naive Diversification: How inefficient is the 1/N portfolio strategy? Rev. Financ. Stud. 22 (5), 1915–1953.

De Souza, C., Gokcan, S., 2004, Spring. Hedge fund investing: A quantitative approach to hedge fund manager selection and de-selection. J. Wealth Manag. 6 (4), 52–73.

Gregoriou, G., Huebner, G., Papageorgiou, N., Rouah, F., 2011. Hedge Funds: Insights in Performance Measurement, Risk Analysis and Portfolio Allocation. Wiley Finance, John Wiley and Sons, New York, USA.

Ineichen, A., 2003. Absolute Returns: The Risk and Opportunities of Hedge Fund Investing, p. 128. Wiley Finance, John Wiley and Sons, New York, USA.

Jensen, M., 1968. The performance of mutual funds in the period 1945–1964. J. Financ. 23, 389–416.

L'habitant, F., 2004. Hedge Funds: Quantitative Insights, pp. 267–295. Wiley Finance, John Wiley and Sons, New York, USA.

Markowitz, H.M., 1952. Portfolio selection. J. Financ. 7 (1), 77–91.

Sharpe, W., 1966. Mutual fund performance. J. Bus. 39 (1), 119–138.

Sharpe, W., 1975, Winter. Adjusting for risk in portfolio performance measurement. J. Portf. Manag. 1 (2), 29–34.

Sharpe, W., 1994, Fall. The Sharpe ratio. J. Portf. Manag. 21 (1), 49–58.

Sortino, F., van der Meer, R., 1991. Downside risk. J. Portf. Manag. 17 (4), 27–31.

Tobin, J., 1958. Liquidity preference as behavior towards risk. Rev. Econ. Stud. 25 (2), 65–86.

Treynor, J., 1966. How to rate management investment funds. Harv. Bus. Rev. 43, 63–75.

Treynor, J., Black, F., 1973. How to use security analysis to improve portfolio selection. J. Bus. 46, 66–86.