

The Impact of Asymmetry on Expected Stock Returns: An Investigation of Alternative Risk Measures

Stephen P. Huffman* and Cliff R. Moll

Department of Finance and Business Law, College of Business, University of Wisconsin Oshkosh, Oshkosh, WI, USA

Abstract. We investigate the relation between various alternative risk measures and future daily returns using a sample of firms over the 1988-2009 time period. Previous research indicates that returns are not normally distributed and that investors seem to care more about downside risk than total risk. Motivated by these findings and mixed empirical evidence supporting theoretical positive risk-return relationship, we model the relation between future returns and risk measures and investigate the following questions: (1) Are investors compensated for total risk and/or asymmetric measures of risk? (2) How does the degree of risk aversion in the lower tail of the return distribution impact the predictability of future returns? (3) Is upside risk or downside risk a better predictor of future returns? We find that, although investors seem to be compensated for total risk, measures of downside risk, such as the lower partial moment, are better at explaining future returns. Further, when comparing downside risk to upside risk, we find that investors are more concerned about downside risk. That is, downside risk is a better predictor of future returns. Our results are robust to the addition of traditional control variables, including size, book-to-market ratio of equity (B/M), leverage, and market risk measures, including beta, downside beta and co-skewness. Our findings are an important contribution to the literature as we document a positive risk-return relationship, using both total and asymmetric measures of risk.

Keywords: Asymmetric risk measures, expected stock returns.

1. Introduction

We investigate the relation between alternative risk measures and future daily stock returns to determine if the information contained in past stock returns can predict future stock returns. Using a sample of individual companies, we examine stock return asymmetry to determine whether: (1) total and asymmetric risk measures are related to future stock returns, (2) downside risk measures are better at explaining future returns than are traditional total risk measures, and (3) upside risk is better than downside risk at predicting future returns.

The financial crisis of 2008 and the increased volatility in equity markets that followed have called into

question traditional asset pricing models developed in the academic literature. Markowitz (1959), in his seminal work, assumes investors base financial decisions solely on the expected return and variance of an investment. A common assumption of asset pricing research is that investment returns are normally distributed (or that investor behavior can be modeled as a quadratic utility function). However, numerous studies in the finance literature have reported that stock returns are not normally distributed. Specifically, studies have documented that daily stock return distributions are negatively skewed and have fatter tails than a normal distribution; e.g., Fama (1976), Kon (1984), and Brown and Warner (1985). Chen et al. (2001) simply state, “Aggregate stock market returns are asymmetrically distributed” (page 346).

Consider, as anecdotal evidence, the 2008 financial crisis. During the crisis, the U.S. stock markets

*Corresponding author: Stephen P. Huffman. E-mail: huffman@uwosh.edu

experienced three months in which returns were more than three standard deviations away from the 36 month trailing average return. Given normally distributed returns, the probability of a negative return more than three standard deviations from the mean is 0.135%, or the equivalent of once every 741 months. Therefore, both empirical and anecdotal evidence suggest that market crashes, such as the dot.com bubble of 2000 and the financial crisis of 2008, occur more often than would be expected with normally distributed stock returns.

2. Motivation and Scholarly Contribution to the Literature

Our investigation is motivated primarily by three issues that currently exist in the asset pricing literature: 1) the overwhelming evidence presented in existing literature indicating that stock returns are not normally distributed; 2) the conflicting empirical findings related to the direction of the relationship between risk measures and returns; and 3) the lack of empirical research related to risk aversion and investor utility functions¹.

The use of lower partial moments, LPMs, allows us to determine whether alternative levels of downside risk aversion (alternatively shaped investor utility functions) can better explain stock returns. For example, traditional capital market theory assumes that investor behavior can be modeled as a quadratic utility function (i.e., a risk coefficient of two), consistent with the use of variance or semi-variance as risk measures.² An LPM with a risk coefficient of two

¹As noted by Bali and Cakici (2008, p. 31), "Although some studies find a positive relationship between idiosyncratic volatility and expected returns at the firm or portfolio level, often the cross-sectional relation has been found to be insignificant, and sometimes even negative." Further, Fu (2009) finds that the results of other cross sectional studies (i.e., Ang et al. (2006b)) are subject to monthly stock return reversals.

²Markowitz (1959) demonstrates that when returns are normally distributed, both variance and semi-variance can produce efficient portfolio sets; however, if returns are not normally distributed, then only semi-variance produces a set of efficient portfolios. Roy (1952) developed the Safety First criterion, which was one of the first downside risk measures. As described by Nawrocki (1999), Roy's Safety First measure provides a measure of the chance of a security price falling below some target return (or disaster level). According to Nawrocki (1999), Roy's measure was developed because "he did not believe that a mathematical utility function could be derived for an investor" (page 10).

is identical to the semi-variance and is characteristic of a traditional risk averse investor. However, we are able to examine alternative degrees of risk aversion, risk neutrality and risk seeking investor behavior by changing the risk coefficient. For example, an LPM with a risk coefficient of one is considered risk neutral, and a risk coefficient below one is consistent with risk seeking behavior.³ We also include skewness and kurtosis measures and examine the robustness of our results by controlling for firm size, book-value to market-value (B/M), financial risk, market/systematic risk, downside beta and co-skewness.⁴ Our primary contributions to the literature are our evaluation of asymmetric risk measures (e.g., semi-variance and other lower partial moments) as predictors of future stock returns, our comparison of the explanatory power of these alternative risk measures, and our inclusion of downside and upside risk measures in our models.

3. Data

We investigate the relation between alternative risk measures and expected daily stock returns over the January 1, 1988 through December 31, 2009 sample period. Specifically, we examine whether information contained in past return data can explain future daily returns, while controlling for size, B/M, and debt-to-asset (TDA) ratio. To be included in the sample, firms must have daily price, shares outstanding, trading volume, and share code equity data available from CRSP. Sample firms also must have available quarterly Compustat data for current liabilities, long-term debt, total assets, total common/ordinary equity and preferred stock, which are used to calculate the TDA (i.e., book leverage) ratio and the B/M ratio.

4. Methodology

Given the aforementioned sample restrictions, we calculate numerous alternative risk measures for each firm using past stock return data and include important control variables found to be related to returns in previous research. To be included in the sample, firms must have available return data for both the

³ See Nawrocki (1999) and Grootveld and Hallerbach (1999).

⁴Although not presented in our tables, we also find that our primary results are robust when we control for other co-movement measures with the market, such as upside beta and co-kurtosis.

current day and the previous 100 days to calculate risk measures such as standard deviation, semi-deviation, mean absolute deviation, downside risk below zero, upside risk above zero, and various lower partial moments. We calculate the book leverage of each firm each quarter as the sum of current and long-term liabilities, divided by total assets. Meanwhile, using quarterly data, we calculate the B/M ratio of equity as the book value of equity divided by the market value of equity. Book value of equity is defined as the sum of total common/ordinary equity, deferred taxes, and investment tax credit less the value of preferred stock. Market value of equity is defined as the end-of-quarter common equity share price multiplied by the end-of-quarter common shares outstanding.

The purpose of our study is to gain a better understanding of the relation between alternative asymmetric risk measures and future returns. First, in isolation, we examine the relation between risk measures and future returns. Specifically, we test whether each risk measure contains any information content for future daily returns by estimating the following cross-sectional model each day:

$$R_{it} = \alpha_t + \gamma_t(Risk\ Measure_{it-1}) + \varepsilon_{it} \quad (1)$$

where R_{it} is firm i 's day t excess return (the firm's daily return less the 30-day Treasury bill rate), and $Risk\ Measure_{it-1}$ is firm i 's risk measure, observed at day $t - 1$, calculated using return observations over the previous 100 trading days (i.e., days $t - 1$ through $t - 100$).

After estimating equation (1) for each day, we perform a Fama and MacBeth (1973) methodology by averaging the cross-sectional coefficient estimates across time and test for statistical significance by computing standard errors from the time-series standard deviation of the cross-sectional coefficients. The Fama and MacBeth (1973) methodology, combined with Newey and West (1987) standard errors, mitigates statistical issues caused by cross-sectional and serial correlation. Our Fama and MacBeth results are used to determine whether the risk measures offer any information content for future daily returns. A significant relation between the risk measure(s) and future excess returns is shown by a statistically significant γ . Given that each of the risk variables is a measure of volatility from the mean (or zero), we generally expect a positive value for the coefficient, γ , which is consistent with positive risk-return tradeoff given by traditional capital market theory.

We examine whether the information contained in the alternative risk measures is subsumed by including traditional control variables. Specifically, we test the information content of the risk measures, relative to traditional control variables, by estimating the following cross-sectional model each day:

$$R_{it} = \alpha_t + \gamma_t(Risk\ Measure_{it-1}) + \delta_t \ln(size)_{it-1} + \lambda_t \ln(B/M)_{it-1} + \eta_t tda_{it-1} + \varepsilon_{it} \quad (2)$$

where R_{it} and $Risk\ Measure_{it-1}$ are defined as in equation (1), and $\ln(size)_{it-1}$, $\ln(B/M)_{it-1}$, and tda_{it-1} are the previous fiscal quarter-end values of natural log of size (market value of equity), natural log of book-to-market equity and total debt-to-asset ratio for firm i , respectively. Again, we use the Fama and MacBeth (1973) methodology to determine statistical significance. Consistent with our previous hypothesis, and a positive risk-return relation, we expect a positive coefficient on our risk measures.

We model the relation between risk measures and future daily returns using the following 100-day measures: standard deviation, semi-variance, semi-deviation, skewness, kurtosis, downside risk below zero, upside risk above zero, mean absolute deviation (MAD), and lower partial moment using four different investor utility functions (extreme risk aversion, risk aversion, risk neutrality and risk seeking).⁵ Definitions for each risk measure are provided next.

We investigate the relation between the annualized standard deviation of returns and future returns. This relation has been examined in previous work and serves as a baseline model. Goyal and Santa-Clara (2003) report that the monthly lagged equally-weighted standard deviations of stocks do not predict future returns. Bali et al. (2005) use a similar procedure, but with value-weighted stock volatility, and also fail to find an empirical link between risk and return. These results further motivate our analysis of alternative risk measures. We define the annualized sample standard deviation as:

$$Stdev_{it} = \left(\sqrt{252}\right) \sqrt{\left(\frac{1}{n-1}\right) \sum_{t=-1}^n (R_{it} - \bar{R}_i)^2} \quad (3)$$

⁵We only present the results of the risk neutral and risk seeking investors. However, the results for the other LPM functions (i.e., extreme risk averse and risk averse investors) are qualitatively similar.

where n is the number of return observations with non-missing data over the previous 100 days, R_{it} is the observed return observation for firm i on day t , and \bar{R}_i is the average return for firm i over the previous 100 days. Given a positive risk-return tradeoff, we expect a positive coefficient on standard deviation.

Next, we investigate the relation between another total risk measure, mean absolute deviation (MAD), and future daily returns. MAD, similar to standard deviation, is a measure of total risk. Both MAD and standard deviation can be used to form efficient frontiers. If the returns are not multivariate normal, then the mean-variance efficient frontier is not consistent with Von Neumann's principle of maximization of expected utility (MEU). However, the MAD efficient frontier is consistent with MEU "regardless of the distribution" (see Konno and Koshizuka, 2005, p. 894). Therefore, the relation between MAD and future returns is not dependent on the return distribution. Specifically, we calculate the MAD as:

$$MAD_{it} = \left[\left(\frac{1}{n} \right) \sum_{t=-1}^n |R_{it} - \bar{R}_i| \right] \quad (4)$$

where n , R_{it} and \bar{R}_i are defined as in equation (3). Given ample empirical evidence suggesting non-normal stock returns, it is possible that alternative risk measures, such as semi-deviation and mean absolute deviation, are more appropriate measures of risk. Given a positive risk-return tradeoff, we expect a positive coefficient on MAD.

Additionally, we include two higher order moment measures, skewness and kurtosis, in our model. Specifically, we investigate whether investors are compensated for skewness and kurtosis. We calculate skewness as in equation (5) and kurtosis as in equation (6):

$$Skew_i = \left[\frac{n}{(n-1)(n-2)} \right] \left[\frac{\sum_{t=1}^n (R_{it} - \bar{R}_i)^3}{s^3} \right] \quad (5)$$

$$Kurt_i = \left[\frac{n(n+1)}{(n-1)(n-2)(n-3)} \right] \left[\frac{\sum_{t=1}^n (R_{it} - \bar{R}_i)^4}{s^4} \right] - \left(\frac{3(n-1)^2}{(n-2)(n-3)} \right) \quad (6)$$

where s is the sample standard deviation, defined in equation (3), and R_{it} and \bar{R}_i are defined as in equation (3). Kraus and Litzenberger (1976) develop an asset pricing model that includes skewness. Additional models developed by Barberis and Huang (2008) and Brunnermeier et al. (2007) suggest that investors are willing to pay for skewness. Chen et al. (2001) provide an extensive list of empirical studies finding stock returns to be negatively skewed. Because negative skewness indicates a greater probability of downside risk, we expect a negative skewness coefficient (especially if the securities have large values for kurtosis).

Next, we investigate the relation between semi-deviation and future daily returns. The annualized sample semi-deviation, a measure of downside risk, is defined as:

$$SemiDev_{it} = \left(\sqrt{252} \right) \sqrt{\left(\frac{1}{n_B - 1} \right) \sum_{t=-1}^n (Min [0, R_{it} - \bar{R}_i])^2} \quad (7)$$

where $n = 100$, n_B is the number of non-missing return observations less than zero over the previous 100 days, and R_{it} and \bar{R}_i are defined as in equation (3). Given a positive relation between downside risk and expected returns, we expect a positive coefficient on semi-deviation.

Next, we investigate the information content of the lower partial moments contained in past returns for future daily returns. We examine the relation between several lower partial moments (LPM) about the mean and future daily returns and define the N^{th} degree LPM as:

$$LPM \text{ About Mean} = \frac{1}{n_B} \sum_{i=1}^{n_B} [\max(0, \bar{R}_i - R_{it})]^N \quad (8)$$

where \bar{R}_i and R_{it} are defined as in equation (3), and n_B is the number of non-missing return observations less than \bar{R}_i over the previous 100 days. Meanwhile, N is the degree of risk aversion. Risk averse behavior is signified by $N > 1$, whereas risk seeking behavior is indicated by $N < 1$. We model four distinct lower partial moment measures, where the difference between each measure lies in the shape of the investor utility function (i.e., level of risk aversion). First, we calculate the LPM for each firm using $N = 3$, which

indicates an extremely risk averse investor. Then, we calculate the LPM for each firm using $N = 3/2$, indicating a marginally risk averse investor. Next, using $N = 1$, we calculate the LPM for each firm, assuming a risk neutral investor. Finally, we model risk seeking investor behavior using N equal to $1/2$.

Finally, we investigate the relation between downside risk below zero and future daily returns. Our measure of downside risk, annualized downside risk below zero, is calculated as:

$$DRzero_{it} = \left(\sqrt{252} \right) \sqrt{\left(\frac{1}{n_D - 1} \right) \sum_{t=1}^n (Min [z, R_{it} - z])^2} \quad (9)$$

where n and R_{it} are defined as in equation (3). Meanwhile, z is the threshold and is equal to zero. Lastly, n_D is the number of non-missing return observations less than the threshold, z . Downside risk below zero increases as the number and/or magnitude of returns below zero increases. As such, a positive and significant coefficient on downside risk would imply risk aversion in the negative tail of the return distribution. Given investors' aversion to losses, we expect a positive coefficient on downside risk below zero.

We then investigate the relation between our measure of upside risk above zero (hereafter referred to as upside risk) and future daily returns. Specifically, we examine whether upside risk has any information content for future daily returns. Upside risk is similar to downside risk below zero, except we are examining the opposite side of the return distribution. Specifically, annualized upside risk above zero is calculated as:

$$URzero_{it} = \left(\sqrt{252} \right) \sqrt{\left(\frac{1}{n_U - 1} \right) \sum_{t=1}^n (Max [0, R_{it} - 0])^2} \quad (10)$$

where n_U is the number of non-missing return observations over the previous 100 days that are above zero, and R_{it} is defined as in equation (3). As depicted in the equation, we expect upside risk above zero to increase as the number and/or magnitude of returns above zero increases. A negative coefficient on the upside risk measure implies risk seeking behavior in

the upper tail of the return distribution, and a positive coefficient implies risk averse behavior.

As a robustness test, we also include measures of co-movement between the firm returns and the CRSP equally- and value-weighted indices. Specifically, we test whether our risk measures contain any information for future returns beyond that given by traditional control variables and measures of co-movement. Each day, we estimate the following cross-sectional model:

$$R_{it} = \alpha_t + \gamma_t (Risk\ Measure_{it-1}) + \delta_t \ln(size)_{it-1} + \lambda_t \ln(B/M)_{it-1} + \eta_t tda_{it-1} + \kappa_t (Co - movement_{it-1}) + \varepsilon_{it} \quad (11)$$

where R_{it} , $Risk\ Measure_{it-1}$, $\ln(size)_{it-1}$, $\ln(B/M)_{it-1}$ and tda_{it-1} are defined as in equation (2). Co-movement is firm i 's co-movement measure (e.g., beta, downside beta, co-skewness)⁶, observed at day $t - 1$, and calculated using return observations over the previous 100 trading days (i.e., days $t - 1$ through $t - 100$).

5. Results

We report the descriptive statistics for the alternative risk measures and control variables in Table 1. Meanwhile, in Table 2, we present the regression results for the models expressed in equation (1). In

⁶We estimate the following co-movement measures relative to the excess returns on a market index, R_{mt} . For the conditional beta measures (i.e., downside beta, β^- , and upside beta, β^+), co-skewness and co-kurtosis, we use the excess return of the value-weighted CRSP market relative to estimation period's mean excess market return as our conditional value, \tilde{r}_{mt} . Likewise, we use the demeaned value for each individual firm return in our calculations, which is expressed as \tilde{r}_{it} .

$$\beta_i = \frac{\sum \tilde{r}_{it} \tilde{r}_{mt}}{\sum \tilde{r}_{mt}^2}, \beta_i^- = \frac{\sum (\tilde{r}_{it}) [\min(0, \tilde{r}_{mt})]}{\sum [\min(0, \tilde{r}_{mt})]^2}$$

$$\beta_i^+ = \frac{\sum (\tilde{r}_{it}) [\max(0, \tilde{r}_{mt})]}{\sum [\max(0, \tilde{r}_{mt})]^2},$$

$$CoSkew_i = \frac{\frac{1}{n} \sum \tilde{r}_{it} \tilde{r}_{mt}^2}{\sqrt{\frac{1}{n} \sum \tilde{r}_{it}^2} \left(\frac{1}{n} \sum \tilde{r}_{mt}^2 \right)^{3/2}},$$

and $CoKurt_i = \frac{\frac{1}{n} \sum \tilde{r}_{it} \tilde{r}_{mt}^3}{\sqrt{\frac{1}{n} \sum \tilde{r}_{it}^2} \left(\frac{1}{n} \sum \tilde{r}_{mt}^2 \right)^{3/2}}$. We use the symbol β^{VW} to denote the beta calculated using market return using the value-weighted CRSP index. Otherwise β indicates that the equally-weighted CRSP market index returns are used in the estimation.

Table 1
Descriptive Statistics for Risk Measures and Control Variables

Variable ^a	Variable Name	Mean	Std Dev	25 th Pctl	Median	75 th Pctl
<i>R</i>	Daily Return	0.0001	0.0514	-0.0153	0.0000	0.0144
<i>ln(size)</i>	Size	11.8563	2.1391	10.3326	11.7175	13.2757
<i>ln(B/M)</i>	Ratio of Book Value to Market Value	-0.7112	0.9362	-1.2308	-0.6271	-0.1026
<i>Tda</i>	Ratio of Total Debt to Total Assets	0.2137	0.2008	0.0321	0.1722	0.3393
β	Beta, Equally-weighted	1.0778	1.1088	0.4014	0.9617	1.6254
β^{VW}	Beta Value-weighted	0.7163	0.8539	0.2065	0.6439	1.1652
β^-	Downside Beta	0.7625	0.9249	0.2389	0.6992	1.2330
β^+	Upside Beta	0.6611	0.9787	0.1189	0.5911	1.1566
<i>Co-Skew</i>	Co-skewness with Market	-0.0741	0.2530	-0.1895	-0.0549	0.0721
<i>Co-Kurt</i>	Co-kurtosis with Market	0.9658	1.2909	0.1966	0.7310	1.4703
<i>Stdev</i>	Annualized Standard Deviation	0.6314	0.4866	0.3292	0.5071	0.7834
<i>MAD</i>	Mean Absolute Deviation	0.0273	0.0194	0.0146	0.0223	0.0342
<i>Skew</i>	Skewness	0.4449	1.2768	-0.0408	0.3380	0.8232
<i>Kurt</i>	Excess Kurtosis	4.5794	8.6418	0.7706	1.9653	4.5630
<i>Semi-Dev</i>	Annualized Semi-Deviation	0.5772	0.4063	0.3043	0.4698	0.7283
<i>Risk Neutral LPM</i>	Risk Neutral Lower Partial Moment	0.0268	0.0200	0.0139	0.0215	0.0337
<i>Risk Seeking LPM</i>	Risk Seeking Lower Partial Moment	0.1390	0.0496	0.1044	0.1305	0.1646
<i>DRzero</i>	Annualized Downside Risk Measure	0.6395	0.4913	0.3236	0.5088	0.8003
<i>URzero</i>	Annualized Upside Risk Measure	0.8000	0.7990	0.3703	0.5895	0.9585

Notes: ^a R_{it} , the daily returns for firm *i* for each day *t*, are obtained from CRSP from January 1, 1988 through December 31, 2008 for 26,121,668 firm days. The variables $\ln(\text{size})_{it}$, $\ln(B/M)_{it}$, and *tda* are calculated as the fiscal quarter-end values of natural log of market value of equity, natural log of book-to-market equity, and total debt-to-asset ratio for firm *i*, respectively. $Stdev_{it}$ is the annualized sample standard deviation for

each firm *i*, calculated as: $Stdev_{it} = \left(\sqrt{252}\right) \sqrt{\left(\frac{1}{n-1}\right) \sum_{t=-1}^n (R_{it} - \bar{R}_i)^2}$, where \bar{R}_i is the average return for firm *i* over the previous 100 days, *n*, as presented in Equation (3). MAD_{it} is the mean absolute deviation for each firm *i*, over the previous 100 days, calculated as: $MAD_{it} = \left[\left(\frac{1}{n}\right) \sum_{t=-1}^n |R_{it} - \bar{R}_i|\right]$, where \bar{R}_i is the average return for firm *i* over the previous 100 days, *n*, as presented in Equation (4). $Skew_{it}$ is

the statistical measure for skewness for each firm *i* over the previous 100 days, calculated as: $Skew_{it} = \left[\frac{n}{(n-1)(n-2)}\right] \left[\frac{\sum_{t=-1}^n (R_{it} - \bar{R}_i)^3}{s^3}\right]$,

where *s* is the sample standard deviation, as presented in equation (5). $Kurt_{it}$ is the statistical measure for excess kurtosis for each firm *i* over the previous 100 days, calculated as: $Kurt_{it} = \left[\frac{n(n+1)}{(n-1)(n-2)(n-3)}\right] \left[\frac{\sum_{t=-1}^n (R_{it} - \bar{R}_i)^4}{s^4}\right] - \left(\frac{3(n-1)^2}{(n-2)(n-3)}\right)$, where *s* is the sample standard

deviation, as presented in equation (6). $Semi-Deviation_{it}$ is the annualized sample semi-deviation for each firm *i*, calculated as: $SemiDev_{it} = \left(\sqrt{252}\right) \sqrt{\left(\frac{1}{n_B-1}\right) \sum_{t=-1}^n (\text{Min}[0, R_{it} - \bar{R}_i])^2}$, where \bar{R}_i is the average return for firm *i* over the previous 100 days and n_B is the number

of non-missing return observations less than zero over the previous 100 days, as presented in Equation (7). The variables based on the lower partial moments, LPM, are calculated for each firm *i* over the previous 100 days as: $LPM \text{ About Mean} = \frac{1}{n_B} \sum_{i=1}^{n_B} [\max(0, \bar{R}_i - R_{it})]^N$,

where n_B is the number of non-missing return observations less than the mean, \bar{R}_i , over the previous 100 days and *N* depicts the degree of risk aversion, such that: *Extreme Risk Averse LPM* uses *N* = 3, *Risk Averse LPM* uses *N* = 3/2, *Risk Neutral LPM* uses *N* = 1 and *Risk Seeking LPM* uses *N* = 1/2. $DRzero$ is the annualized downside risk below zero for each firm *i*, over the previous 100 days, calculated as:

$DRzero_{it} = \sqrt{\left(\frac{1}{n_D-1}\right) \sum_{t=1}^n (\text{Min}[z, R_{it} - z])^2}$, where *z* is the threshold and is equal to zero and n_D is the number of non-missing return observations less than the threshold, *z*, as presented in equation (9). $URzero$ is the annualized upside risk below zero for each firm *i*, over

the previous 100 days, calculated as: $URzero_{it} = \sqrt{\left(\frac{1}{n_U-1}\right) \sum_{t=1}^n (\text{Max}[0, R_{it} - 0])^2}$, where n_U is the number of non-missing return observations over the previous 100 days that are above zero, as presented in equation (10).

The following co-movement measures are estimated relative to the excess returns on a market index, R_{mt} , where we use the equally-weighted CRSP market index as our market index. For the conditional beta measures (i.e., downside beta, β^- , and upside beta, β^+), co-skewness and

co-kurtosis, we use the excess return of the value-weighted CRSP market relative to estimation period's mean excess market return as our conditional value, \tilde{r}_{mt} . Similarly, the demeaned value for each individual firm return is expressed as \tilde{r}_{it} .

$$\beta_i = \frac{\sum \tilde{r}_{it} \tilde{r}_{mt}}{\sum \tilde{r}_{mt}^2}, \quad \beta_i^- = \frac{\sum (\tilde{r}_{it}) [\min(0, \tilde{r}_{mt})]}{\sum [\min(0, \tilde{r}_{mt})]^2}, \quad \beta_i^+ = \frac{\sum (\tilde{r}_{it}) [\max(0, \tilde{r}_{mt})]}{\sum [\max(0, \tilde{r}_{mt})]^2}, \quad CoSkew_i = \frac{\frac{1}{n} \sum \tilde{r}_{it} \tilde{r}_{mt}^2}{\sqrt{\frac{1}{n} \sum \tilde{r}_{it}^2 (\frac{1}{n} \sum \tilde{r}_{mt}^2)}}, \text{ and}$$

$$CoKurt_i = \frac{\frac{1}{n} \sum \tilde{r}_{it} \tilde{r}_{mt}^3}{\sqrt{\frac{1}{n} \sum \tilde{r}_{it}^2 (\frac{1}{n} \sum \tilde{r}_{mt}^2)^{3/2}}}.$$

β indicates that the equally-weighted CRSP market index returns are used in the estimation. We use the symbol β^{VW} to denote the beta calculated using market return using the value-weighted CRSP index

the single factor models presented in Table 2, we find that the co-movement risk measures β , β^- and coskew, in models (1), (2), and (3), respectively are not significant.⁷ In Table 2, we find that most of the total risk and alternative risk measures are significant predictors of future daily returns. In fact, we find that only the skewness and kurtosis measures are unrelated to future returns [model (6) and model (7)]. Overall, we find that the alternative risk measures display the anticipated signs.⁸

Unlike Goyal and Santa-Clara (2003) and Bali et al. (2005), we find an empirical link between risk and return. As our Table 2 results illustrate in model (4), we find a positive and significant relation between standard deviation and future daily returns, implying that investors are compensated for total risk.⁹ Looking at model (8), we also find a positive and significant

relation between the semi-deviation and future returns. Our measure of semi-deviation, as defined in equation (7), measures the volatility of return observations falling below the firm's mean return over the previous 100 days. Given the positive and significant coefficient on semi-deviation, our results indicate that investors are compensated for downside risk.

Our results from model (4) indicate that total risk matters, whereas our results from model (8) indicate that downside risk matters. We compare differences in the explanatory power given by the two models using adjusted R^2 . Given that both models (4) and (8) have the same dependent variable, we can directly compare the adjusted R^2 of the models. We find that, although model (4) has a higher adjusted R^2 than model (8), the difference is not statistically significant.¹⁰ Thus, it appears that investors care about both total and downside risk. In model (5) of Table 2, we find a positive and significant relation between the mean absolute deviation (MAD) and future daily returns, which confirms our findings and suggests that investors are compensated for total risk.

Our results from models (6) and (7) of Table 2 indicate that there is no clear relation between future daily returns and skewness and kurtosis, respectively. This is consistent with previous research by Kraus and Litzenberger (1976) and suggests that higher order moments do not seem to adequately measure underlying risk factors related to future returns.

In models (8), (9), and (10) of Table 2, we examine the relation between LPMs about the mean and future daily returns.¹¹ Each of our LPM models measures a

⁷In unreported results, we find that the coefficients on co-kurtosis and the upside beta, β^+ , are negative and significant (at the 5 percent level). Although we do find a positive and significant coefficient on the value-weighted beta, β , the relation disappears when the other control variables are included.

⁸When we use the one-month-ahead return as the dependent variable, none of the coefficients for our alternative risk measures are significant. Only when the return month is included in the estimation period does a coefficient for risk (i.e., standard deviation) become significant. We conjecture that the lack of significance for our alternative risk measures in predicting the next month's return is due to: (1) monthly stock return reversals, (2) time varying risk characteristics, and (3) investors being quick to recognize stocks with asymmetric risk characteristics, leading to rapid price changes.

⁹Although Ang et al. (2006a) find a strong negative relationship between idiosyncratic volatility and expected stock returns, other authors (e.g., Fu, 2009) show that idiosyncratic volatility varies over time. As noted by Bali and Cakici (2008, p. 31), "Although some studies find a positive relationship between idiosyncratic volatility and expected returns at the firm or portfolio level, often the cross-sectional relation has been found to be insignificant, and sometimes even negative." Haung et al. (2007) and Fu (2009, p. 25) show that the Ang et al. (2006a) "results are driven by monthly stock return reversals" and that the negative result disappears when the monthly price reversals are controlled for.

¹⁰Results of t-tests suggest no difference in the adjusted R^2 for models (4) and (8).

¹¹The semi-deviation is the LPM for a risk averse utility function with a degree of risk aversion of 2 (i.e., $N=2$). Although not reported in Table 2, LPMs for averse and extremely risk averse investors have risk aversions of 3/2 and 3, respectively (i.e., we examine 5 LPM with different degrees of risk aversion of: 1/2, 1, 2, 3/2 and 3).

Table 2

The Relationship between Risk Measures and Future Daily Stock Returns Parameter Estimates using Fama and MacBeth (1973) methodology of averaging the cross-sectional coefficient estimates of the model for $R_{i,t} = \alpha_t + \gamma_t(Risk\ Measure_{i,t-1}) + \varepsilon_{i,t}$, Equation (1) with Newey and West (1987) t-statistics in parentheses and the means of adjusted-R² values for the one-day ahead return for the 5,448 days from May 25, 1988 through December 31, 2009

Variable ^a	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9	Model 10	Model 11	Model 12
β^{EW}	0.0000 (0.61)											
β^-		-0.0001 (-0.94)										
Co-Skew			-0.0002 (-0.77)									
Stdev				0.0022*** (5.14)								
MAD					0.0564*** (5.60)							
Skew						-0.0000 (-1.27)						
Kurt							-0.000 (-1.18)					
Semi-Dev								0.0035*** (6.27)				
Risk									0.0766*** (6.15)			
Neutral LPM										0.0255*** (6.14)		
Risk											0.0027*** (6.52)	
Seeking LPM												0.0012*** (4.21)
DRzero												0.0052
URzero												
Mean Adj. R ²	0.0120	0.0109	0.0022	0.0055	0.0062	0.0005	0.0005	0.0056	0.0060	0.0056	0.0051	0.0052

^aSee Notes for Table 1, for variable definitions. *, **, and *** indicate significance at the 0.10, 0.05, and 0.01 levels, respectively.

slightly different type of downside risk based on how downside risk is perceived by investors. Specifically, we model downside risk using five different utility functions with N degrees: extremely risk averse with $N = 3$, semi-deviation with $N = 2$, risk averse with $N = 3/2$, risk neutral with $N = 1$ and risk seeking with $N = 1/2$. Although each of the LPM variables is significantly related to future daily returns, the relationship is the strongest when we assume investors are averse to downside risk. Specifically, the LPM models with risk averse investor utility functions have the highest average adjusted R^2 , indicating that they explain future daily stock returns the best. This result is not surprising, considering that we expect investors to be averse to risk in general, especially downside risk as quantified by our LPM measures.

We examine the relation between future daily returns and two additional asymmetric risk measures: downside risk below zero and upside risk above zero in models (11) and (12), respectively. As defined in equations (9) and (10), we use a threshold value of zero for these asymmetric risk measures. Downside and upside risk are included in order to examine whether investors view downside risk differently than they view upside risk. The results indicate that investors are averse to each measure of risk. Specifically, stocks with greater risk, either upside or downside, earn higher future returns. The positive coefficient on the downside risk measure in model (11) is consistent with risk aversion in the lower tail of the distribution. Meanwhile, the positive coefficient on upside risk in model (12) implies that investors demand more compensation (higher returns) for stocks with greater upside risk (greater probability of large positive return shocks).

In Table 3, we add the control variables to our single risk factor models. Consistent with prior research (Fama and French, 1992, 1993; Barber and Lyon, 1997), we document a positive relation between B/M and future returns in models (13) through (22). Additionally, we document a statistically significant negative relation between firm size and future daily returns in 7 of the 10 models presented in Table 3. The negative relation between size and returns is also consistent with prior research (Banz, 1981; Fama and French, 1992, 1993; Barber and Lyon (1997). However, we find no significant relation between book leverage and future daily returns. This is consistent with the results of Fama and French (1992), and suggests that the B/M effect subsumes the leverage effect.

In Table 3, we find that the positive relation between standard deviation and future daily returns remains highly significant in model (16), even after including the three control variables. Thus, it appears as though investors are compensated for total risk in addition to the B/M and size factors. Using model (16) as our baseline model, we examine which type(s) of risk and/or utility functions best explain future returns.

We find that the positive and significant relation between semi-deviation and future returns remains when the control variables are added in model (18). However, we find that size is no longer significantly related to returns. This finding is interesting because it appears as though the explanatory power offered by semi-deviation, a measure of downside risk, subsumes the explanatory power firm size and might indicate that firm size is a proxy for downside risk. In fact, our subsequent results show that nearly all of our measures of downside risk subsume the explanatory power given by firm size.

Our results for model (17), presented in Table 3, suggest a positive and statistically significant relation between future daily returns and the mean absolute deviation (MAD) of returns over the previous 100 days. This is consistent with our hypothesis and suggests that investors are averse to deviations from the mean return and demand compensation for the perceived risk inherent in these deviations. In fact, the explanatory power of model (17) is greater than that of model (16), indicating that MAD does a better job than standard deviation of explaining future returns.¹²

Our results for models (19) and (20), reported in Table 3, indicate a significant positive relation between downside risk, as measured by the LPM, and future returns, even after controlling for B/M, size, and leverage. This is consistent with investors demanding compensation for downside risk. That is, our results indicate that investors are pricing downside risk higher than total risk through their buying and selling behavior.

Consistent with the results of models (18) through (20), the results of model (21) also indicate a positive and significant relation between downside risk and future returns, as measured by deviations in returns that are at or below zero. Specifically, the positive coefficient estimate on downside risk in model (21) suggests that investors demand compensation, via

¹² Results of an unreported t-test indicate that the explained variation (Adjusted R^2) of model (17) is significantly greater (at the 1% level) than the explained variation of model (16).

Table 3

The Relationship between Risk Measures and Future Daily Stock Returns, Controlling for Size, B/M and Leverage Parameter Estimates using Fama and MacBeth (1973) methodology of averaging the cross-sectional coefficient estimates of the model for $R_{i,t} = \alpha_t + \gamma_t(Risk\ Measure)_{i,t-1} + \lambda_t \ln(B/M)_{i,t-1} + \eta_t \ln(Lev)_{i,t-1} + \varepsilon_{i,t}$, Equation (2) with Newy and West (1987) t-statistics in parentheses and the means of adjusted-R² values for the one-day ahead return for the 5,448 days from May 25, 1988 through December 31, 2009

Variable ^a	Model 13	Model 14	Model 15	Model 16	Model 17	Model 18	Model 19	Model 20	Model 21	Model 22
<i>ln(size)</i>	-0.0005*** (-7.26)	-0.0005*** (-7.31)	-0.0005*** (7.62)	-0.0002*** (-3.56)	-0.0002*** (-4.37)	0.000 (0.00)	-0.0000 (-0.12)	-0.0002*** (-3.48)	-0.0000 (-1.10)	-0.0003*** (-4.61)
<i>ln(B/M)</i>	0.0001** (2.28)	0.0001** (2.25)	0.0001* (1.80)	0.0002*** (3.03)	0.0002*** (3.23)	0.0003 (4.25)	0.0004*** (4.26)	0.0004*** (4.21)	0.0002*** (3.60)	0.0002*** (2.33)
<i>tda</i>	-0.0001 (0.47)	-0.0001 (-0.43)	-0.0001 (-0.33)	-0.0002 (-0.82)	-0.0002 (-0.78)	-0.0002 (-0.88)	-0.0002 (-0.75)	-0.000 (-0.18)	-0.0003 (-1.21)	-0.0002 (-0.83)
β	0.0002*** (2.63)									
β^-		0.0002*** (2.67)								
<i>Co-Skew</i>			-0.0002 (-1.03)							
<i>Stdev</i>				0.0019*** (3.92)						
<i>MAD</i>					0.0477*** (4.24)					
<i>Semi-Dev</i>						0.0037*** (5.33)				
<i>Risk Neutral LPM</i>							0.0805*** (5.28)			
<i>Risk Seeking LPM</i>								0.0246*** (5.48)		
<i>DRzero</i>									0.0026*** (5.64)	
<i>URzero</i>										0.0009*** (2.78)
Mean Adj. R ²	0.0154	0.0144	0.0081	0.0120	0.0130	0.0120	0.0127	0.0122	0.0115	0.0117

Notes: ^aSee Notes for Table 1, for variable definitions. *, **, and *** indicate significance at the 0.10, 0.05, and 0.01 levels, respectively.

higher future returns, for stocks with more frequent and/or extreme occurrences in the lower tail of the return distribution. However, although downside risk is significantly related to future returns, it does not seem to explain the variation in returns as well as other measures of total risk (i.e., standard deviation and mean absolute deviation).¹³ Finally, our results for model (22) indicate a positive and statistically significant relation between upside risk and future returns. This suggests that investors are compensated for upside risk and warrants further investigation. Note, though, that firm size is again negatively related to future returns. Thus, although it appears that downside risk is related to firm size, upside risk does not seem to be measuring the same risk factors/characteristics as firm size.

We include co-movement measures as additional control variables in the models presented in Panels A through C of Table 4. Overall, we find that including market risk, downside market risk and co-skewness does not affect our primary finding. That is, even after controlling for the aforementioned variables, total risk, downside risk and upside risk all still explain future returns.

In Panel A of Table 4, we find that the inclusion of market risk, β , does not affect the sign or significance of our risk measures. We find it interesting to note that the coefficient for beta is insignificant in all of the models which include a total risk measure [models (23) and (24)] or a downside measure relative to a zero return. However, for measures relative to the mean in those cases [models (25)–(27)], the sign on β is negative.¹⁴ This result is both interesting and puzzling. It suggests that after controlling for downside risk, as measured as deviations below the mean, beta becomes negatively related to future returns.

In Panel B of Table 4, downside market risk, β^- , is insignificant in all models, with the exception of model (36), which includes the upside risk measure.¹⁵

¹³In an unreported t-test, we find that the explained variation (Adjusted R^2) of model (16) is significantly greater (at the 10% level) than the explained variation of model (21).

¹⁴When we conduct the same analysis found in Table 4, Panel A, using the value-weighted market index, we find similar results. That is, all of the coefficients on the alternative risk measures remain positive and significant (at the 1 percent level) while the coefficient on beta is insignificant, with the exception of the models including the variables for: semi-deviation, risk neutral LPM, and risk seeking LPM. In those cases the sign on beta is negative, which is counterintuitive.

¹⁵When we conduct the same analysis found in Panel B of Table 4 using upside beta, we find that all of the coefficients on

Finally, in Panel C of Table 4, we present the results of models including our control variables and a measure of co-skewness. Although the coefficient for co-skewness is negative and significant (at the 5 percent level) in all models (except for risk seeking LPM), the coefficient for co-skewness was not significant in the single factor model or in the model with our three control variables.¹⁶ We are somewhat concerned that the robustness models with co-skewness and the alternative risk measures as independent variables may be subject to issues related to multicollinearity. However, it doesn't seem to be a significant issue, as our primary results hold even when co-movement measures are included in our models.

6. Conclusion

We document several interesting relations between various risk measures and future daily returns. First, we find evidence consistent with investors being compensated for total risk via higher future returns. However, we find that the mean absolute deviation has more explanatory power than standard deviation. This is important, given the potential limitations of standard deviation when returns are not normally distributed. Second, the results of the models including downside risk measures indicate that investors care more about downside risk than total risk. This is intuitively appealing and suggests what we have suspected all along, that investors care more about losses, as suggested by prospect theory.¹⁷ We also show that the relations between the alternative risk measures and future daily returns hold even after the inclusion of traditional control variables including size, B/M and leverage. Finally, we show that our primary results hold when measures of co-movement,

the alternative risk measures remain positive and significant (at the 1 percent level). However, the coefficient on beta is insignificant, with the exception of the models that include semi-deviation, risk neutral LPM, and risk seeking LPM. In those cases the sign on the coefficient of the upside beta is negative.

¹⁶When we conduct the same analysis found in Panel C of Table 4 using co-kurtosis measures, we find that all of the coefficients for the alternative risk measures remain positive and significant (at the 1 percent level) and that the coefficients on co-kurtosis is positive and significant (at the 1 percent level) in all models. However, the coefficient for co-kurtosis is negative and significant in the single factor model, which could indicate a multicollinearity issue in the robustness test models containing co-kurtosis as an independent variable.

¹⁷See Kahneman and Tversky (1979).

Table 4

Panel A: The Relationship between Risk Measures and Future Stock Returns, Controlling for Size, B/M, Leverage and the Equally-weight Beta as the Co-movement Measure
 Parameter Estimates using Fama and MacBeth (1973) methodology of averaging the cross-sectional coefficient estimates of the model for $R_{it} = \alpha_t + \gamma_t(Risk\ Measure_{it-1}) + \delta_t \ln(size)_{it-1} + \lambda_t \ln(B/M)_{it-1} + \eta_t tda_{it-1} + \kappa_t (Co - movement_{it-1}) + \varepsilon_{it}$, Equation (11) with Newy and West (1987) t-statistics in parentheses and the means of adjusted-R² values for the one-day ahead return for the 5,448 days from May 25, 1988 through December 31, 2009

Variable ^a	Model 23	Model 24	Model 25	Model 26	Model 27	Model 28	Model 29
$\ln(size)$	-0.0002*** (-3.45)	-0.0002*** (-4.39)	0.0000 (0.69)	0.0000 (0.65)	-0.0001*** (-3.19)	-0.0000 (-0.62)	-0.0003*** (-4.64)
$\ln(B/M)$	0.0002*** (3.06)	0.0002*** (3.20)	0.0003*** (4.19)	0.0004*** (4.21)	0.0003*** (4.17)	0.0002*** (3.62)	0.0002*** (2.54)
tda	-0.0003 (-1.36)	-0.0002 (-1.26)	-0.0003* (-1.72)	-0.0003 (-1.60)	-0.0002 (-0.77)	-0.0004* (-1.91)	-0.0002 (-1.27)
β^-	-0.0000 (-0.15)	-0.0002 (-1.26)	-0.0002** (-2.26)	-0.0001* (-1.95)	-0.0001* (-1.95)	-0.0001 (-1.20)	0.0001 (1.48)
$Stdev$	0.0020*** (4.14)						
MAD		0.0005*** (4.60)					
$Semi-Dev$			0.0040*** (5.90)				
$Risk\ Neutral\ LPM$				0.0876*** (5.89)			
$Risk\ Seeking\ LPM$					0.0269*** (6.22)		
$DRzero$						0.0028*** (6.31)	
$URzero$							0.0009*** (3.87)
Mean Adj. R ²	0.0191	0.0192	0.0184	0.0187	0.0177	0.0181	0.0193

^aSee Notes for Table 1, for variable definitions. *, **, and *** indicate significance at the 0.10, 0.05, and 0.01 levels, respectively.

Table 4

Panel B: The Relationship between Risk Measures and Future Stock Returns, Controlling for Size, B/M, Leverage and the Downside Beta as the Co-movement measure Parameter Estimates using Fama and MacBeth (1973) methodology of averaging the cross-sectional coefficient estimates of the model for $R_{it} = \alpha_t + \gamma_t (Risk_Measure_{it-1}) + \delta_t \ln(size)_{it-1} + \lambda_t \ln(B/M)_{it-1} + \eta_t tda_{it-1} + \kappa_t (Co - movement_{it-1}) + \varepsilon_{it}$, Equation (11) with Newy and West (1987) t-statistics in parentheses and the means of adjusted-R² values for the one-day ahead return for the 5,448 days from May 25, 1988 through December 31, 2009

Variable ^a	Model 30	Model 31	Model 32	Model 33	Model 34	Model 35	Model 36
$\ln(size)$	-0.0002*** (-3.60)	-0.0002*** (-4.55)	0.0000 (0.41)	0.0000 (-0.34)	-0.0001*** (-1.37)	-0.0004 (-0.96)	-0.0003*** (-4.09)
$\ln(B/M)$	0.0002*** (3.20)	0.0002*** (3.31)	0.0003*** (4.27)	0.0004*** (4.27)	0.0001*** (4.23)	0.0002*** (3.73)	0.0002*** (2.68)
tda	-0.0002 (-1.10)	-0.0002 (-1.01)	-0.0003 (-1.33)	-0.0002 (-1.20)	-0.0001 (-0.52)	-0.0003 (-1.57)	-0.0002 (-1.06)
β^-	-0.0001 (1.41)	0.0000 (0.77)	0.0000 (0.77)	-0.0001 (-1.59)	-0.0001 (-1.27)	-0.0000 (-0.23)	0.0002** (2.45)
$Stdev$	0.0019*** (3.90)						
MAD		0.0473*** (4.21)					
$Semi-Dev$			0.0038*** (5.62)				
$Risk\ Neutral\ LPM$				0.0837*** (5.57)			
$Risk\ Seeking\ LPM$					0.0256*** (5.78)		
$DRzero$						0.0027*** (5.93)	
$URzero$							0.0009*** (2.75)
Mean Adj. R ²	0.0186	0.0185	0.0176	0.0180	0.0170	0.0173	0.0184

^aSee Notes for Table 1, for variable definitions. *, **, and *** indicate significance at the 0.10, 0.05, and 0.01 levels, respectively.

Table 4

Panel C: The Relationship between Risk Measures and Future Stock Returns, Controlling for Size, B/M, Leverage and Co-skew as the Co-movement measure Parameter Estimates using Fama and MacBeth (1973) methodology of averaging the cross-sectional coefficient estimates of the model for $R_{it} = \alpha_t + \gamma_t(Risk\ Measure)_{it-1} + \delta_t \ln(size)_{it-1} + \lambda_t \ln(B/M)_{it-1} + \eta_t tda_{it-1} + \kappa_t (Co - movement)_{it-1} + \varepsilon_{it}$, Equation (11) with Newy and West (1987) t-statistics in parentheses and the means of adjusted-R² values for the one-day ahead return for the 5,448 days from May 25, 1988 through December 31, 2009

Variable ^a	Model 37	Model 38	Model 39	Model 40	Model 41	Model 42	Model 43
<i>ln(size)</i>	-0.0002*** (-3.96)	-0.0002*** (-4.90)	-0.0000 (-0.33)	-0.0000 (-0.44)	-0.0002*** (-3.98)	-0.0001 (-1.59)	-0.0003*** (-5.02)
<i>ln(B/M)</i>	0.0002*** (3.06)	0.0002*** (3.24)	0.0003*** (4.24)	0.0004*** (4.25)	0.0004*** (4.20)	0.0002*** (3.61)	0.0002*** (0.89)
<i>tda</i>	-0.0002 (-0.89)	-0.0002 (-0.84)	-0.0002 (-0.96)	-0.0002 (-0.83)	-0.0001 (-0.25)	-0.0003 (-1.30)	-0.0002 (-0.89)
<i>Co-Skew</i>	-0.0004** (-2.01)	-0.0004** (-1.59)	-0.0004** (-2.12)	-0.0004** (-2.18)	-0.0003 (-1.59)	-0.0004** (-2.24)	-0.0004** (-1.95)
<i>Stdev</i>	0.0020*** (4.00)						
<i>MAD</i>		0.0484*** (4.31)					
<i>Semi-Dev</i>			0.0037*** (5.34)				
<i>Risk Neutral LPM</i>				0.0809*** (5.30)			
<i>Risk Seeking LPM</i>					0.0247*** (5.53)		
<i>DRzero</i>						0.0026*** (5.65)	
<i>URzero</i>							0.0009*** (2.89)
Mean Adj. R ²	0.0132	0.0141	0.0136	0.0138	0.0132	0.0127	0.0129

^aSee Notes for Table 1, for variable definitions. *, **, and *** indicate significance at the 0.10, 0.05, and 0.01 levels, respectively.

such as beta, downside beta and co-skewness, are included in our models along with the aforementioned control variables. Overall, our findings document a positive risk-return relationship, using both total and asymmetric measures of risk.

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